Preview of Lecture 02.22

On 02.22, we will continue our discussion of the expressive power of polyadic quantification theory. We will start by introducing a new logical dyadic relation, identity, which will allow us to "put the quant into quantification." The identity relation "=" has a uniform interpretation over all structures A namely $=^A$ is equal to $\{\langle a, a\rangle \mid a \in U^A\}$. Since the interpretation of the identity relation is uniform, we omit mention of it when we specify structures. By making use of the identity relation, we can introduce, for each integer $k \geq 1$, the quantifiers there are at least k x's such that $S(x)$, there are at most k x's such that $S(x)$, and there are exactly k x's such that $S(x)$ as follows.

$$\begin{array}{ll}
(\exists^{k\leq} x)S(x): & (\exists x_1)\ldots(\exists x_k)(\bigwedge_{1\leq i<j\leq k} x_i \neq x_j \wedge \bigwedge_{1\leq i\leq k} S(x_i)) \\
(\exists^{\leq k} x)S(x): & \neg(\exists^{k+1\leq} x)S(x) \\
(\exists^{=k} x)S(x): & (\exists^{\leq k} x)S(x) \wedge (\exists^{k\leq} x)S(x)
\end{array}$$

We will explore the use of these quantifiers to define regular simple graphs, functional relations, tournaments, and orderings.