Preview of Lecture 02.17

On 02.17, we will commence our study of polyadic quantification theory. This topic will remain our focus through the end of the Term. As opposed to truth-functional and monadic logic which, as we've seen, are of limited expressive power, polyadic quantification theory allows for faithful schematization of vast tracts of scientific discourse. But we begin, not with science, but with literature.

Consider the sentences

- Romeo loves Juliet.
- Someone loves Juliet.
- Romeo loves someone.

The first sentence implies the second and the third sentence. We can schematize the second, by making use of the monadic predicate "$\bigcirc$ loves Juliet" thus

$$(\exists x)(x \text{ loves Juliet}).$$

And we can schematize the third, by making use of the monadic predicate "Romeo loves $\bigcirc$" thus

$$(\exists x)(\text{Romeo loves } x).$$

But if we wish to schematize the sentence "someone loves someone," which is also implied by the first sentence above, we need to expand our resources to include *dyadic predicates*.

- $\boxed{1}$ loves $\boxed{2}$
- $\langle \text{Romeo}, \text{Juliet} \rangle$ is in the extension of "$\boxed{1}$ loves $\boxed{2}$."
- $(\exists x)(\exists y)(x \text{ loves } y)$

The extension of a dyadic predicate is a set of *ordered* pairs.

- $\langle 45, 47 \rangle$ is in the extension of "$\boxed{1} \leq \boxed{2}$."
- $\langle 45, 47 \rangle$ is not in the extension of "$\boxed{2} \leq \boxed{1}$."
- $\langle 47, 45 \rangle$ is in the extension of "$\boxed{2} \leq \boxed{1}$."

Similarly, the extension of a triadic predicate, such as "$\boxed{1}$ is further from $\boxed{2}$ than it is from $\boxed{3}$," is a set of ordered triples.

Consider the following statements involving alternation of quantifiers.

1. Everyone loves someone (or other).

$$(\forall x)(\exists y)(x \text{ loves } y).$$

2. There is someone whom everyone loves.

$$(\exists y)(\forall x)(x \text{ loves } y).$$

3. Everyone is loved by someone.

$$(\forall y)(\exists x)(x \text{ loves } y).$$

4. Someone loves everyone.

$$(\exists x)(\forall y)(x \text{ loves } y).$$

The second statement implies the first, and the fourth implies the third. We will give counterexamples to show that no other implications obtain.