Preview of Lecture 01.20

On 01.20, we will begin our systematic treatment of truth-functional logic. You should read sections 1-9 of *Deductive Logic* to prepare for class.

Per our conversation last time, we will use sentence letters to schematize statements, that is, sentences (of natural language) which are true or false. We will study ways of forming compound statements from simpler statements; insofar as we will restrict our study to the formation of compound statements whose truth value depends only on the truth value of the simpler statements out of which they are composed, we will be able to interpret these schemata via truth assignments to sentence letters, and retain full access to their logical powers thereby. Thus the term, *truth-functional* logic.

Consider again schematizing the statements "i loves j", $1 \leq i, j, \leq 4$, using the sentence letters p_{ij}; for example, the sentence letter p_{11} schematizes the statement "1 loves 1", or briefly, "1 is a narcissist." Suppose we wish to write down truth-functional schemata using these sentence letters, thus interpreted, that are true just in case

1. all of 1, 2, 3, and 4 are narcissists;
2. none of 1, 2, 3, and 4 are narcissists;
3. at least one of 1, 2, 3, and 4 is a narcissist;
4. an odd number of 1, 2, 3, and 4 are narcissists.

In order to do so, we introduce the following truth-functional connectives. For each connective, we display its truth-functional interpretation via a table indicating the truth value of the compound schema as a function of the truth values of its components.

- Conjunction:

p	q	$p \wedge q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

- Negation:

p	$\neg p$
$\top$	$\bot$
$\bot$	$\top$

- Inclusive Disjuntion

p	q	$p \vee q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

- Exclusive Disjunction

p	q	$p \oplus q$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

We can now schematize conditions 1 – 4 above as follows.

S1: $((p_{11} \wedge p_{22}) \wedge p_{33}) \wedge p_{44}$

S2: $((\neg p_{11} \wedge \neg p_{22}) \wedge \neg p_{33}) \wedge \neg p_{44}$

S3: $((p_{11} \vee p_{22}) \vee p_{33}) \vee p_{44}$

S4: $((p_{11} \oplus p_{22}) \oplus p_{33}) \oplus p_{44}$

The first three are quite straightforward to verify; the fourth requires some explanation, which we will offer in class on Wednesday.