PRINT NAME:

LGIC 010 & PHIL 005
Practice Examination I
Spring Term, 2018

1. We call a set of numbers X *good* if and only if **NO** number in X is double some other number in X.

   (a) (10 points) What is the maximum size of a good set X contained in $\{1, 2, \ldots, 10\}$?

   (b) (15 points) Give an example of a maximum size good set $X \subseteq \{1, 2, \ldots, 10\}$ and explain why there is no larger such set.

2. (15 points) How many truth-assignments to the sentence letters p_1, p_2, p_3 satisfy the following truth-functional schema?

$$(p_1 \equiv p_2) \vee (p_1 \equiv p_3) \vee (p_2 \equiv p_3)$$

3. For the purposes of this problem, we restrict attention to truth-functional schemata all of whose sentence letters are among p_1, p_2, p_3, p_4, and p_5. We employ the following terminology.

- A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
- A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
- The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.

(a) (15 points) What is the length of a longest succinct list of schemata all of which have the same power?

(b) (15 points) What is the length of a longest list of schemata none of which have the same power?

(c) (15 points) What is the maximum power and what is the minimum power that can be achieved by an exclusive disjunction (that is, $\oplus$) of two inequivalent schemata of power 128?

4. (15 points) For the purposes of this problem, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters. We employ the following terminology.

- If S and T are MQ-schemata we say that a structure A is a *counterexample* to the claim that S implies T if and only if $A \models S$ and $A \not\models T$.

Let S be the schema
$$(\forall x)Fx \equiv (\forall x)Gx.$$
and let T be the schema
$$(\forall x)(Fx \equiv Gx)$$
How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ are counterexamples to the claim that S implies T?