PRINT NAME:

LGIC 010 & PHIL 005
Practice Examination I
Spring Term, 2016

1. We call a set of numbers X *good* if and only if no member of X evenly divides another member of X, that is, X is good if and only if for all i and j, if $i, j \in X$ and $i \neq j$, then for every k, $i \cdot k \neq j$.

   (a) (10 points) What is the maximum size of a good set X contained in $\{1, 2, \ldots, 100\}$?

   (b) (15 points) Give an example of a maximum size good set $X \subseteq \{1, 2, \ldots, 100\}$ and explain why there is no larger such set.

2. (15 points) How many truth-assignments to the sentence letters $p_1, \ldots, p_5$ satisfy the following truth-functional schema?

$$(((p_1 \supset p_2) \supset p_3) \supset p_4) \supset p_5$$

3. For the purposes of this problem, we restrict attention to truth-functional schemata all of whose sentence letters are among p_1, p_2, p_3, and p_4. We employ the following terminology.

   - A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
   - A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
   - The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.

   (a) (15 points) What is the length of a longest succinct list of schemata, all of the same power, that all imply $(((p_1 \equiv p_2) \equiv p_3) \equiv p_4$?

   (b) (15 points) What is the largest number n such that there is a satisfiable schema of power n and every disjunction of two inequivalent schemata of power n has the same power?

   (c) (15 points) What is the maximum power and what is the minimum power that can be achieved by a conjunction of two inequivalent schemata of power 64?

4. (15 points) For the purposes of this problem, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters. We employ the following terminology.

   - If S and T are MQ-schemata we say that a structure A is a *counterexample* to the claim that S implies T if and only if $A \models S$ and $A \not\models T$.

   Let S be the schema
   $$(\forall x)(Fx \oplus Gx)$$
   and let T be the schema
   $$(\forall x)Fx \oplus (\forall x)Gx.$$
   How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ are counterexamples to the claim that S implies T?