PRINT NAME: ___________________________________

LGIC 010 & PHIL 005
Practice Examination II
Spring Term, 2014

1. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent, and every schema on the list implies $(\forall x)(Fx \oplus Gx)$? 8

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent and each schema on the list is satisfied by exactly 228 structures with universe of discourse $\{1, 2, 3, 4\}$? 16

3. Let S_1 be the following schema.

$$(\exists x)\neg Lxx \land (\forall x)(\forall y)(Lxy \supset Lyx).$$

   (a) (10 points) Specify a structure A_1 of size at least 4 which satisfies S_1, that is, U^{A_1} has at least 4 members and $A_1 \models S_1$.

   $U^{A_1} = \{1, 2, 3, 4\}$

   $L^{A_1} = \emptyset$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_1? 960

4. Let S_2 be the following schema.

$$(\forall x)(\exists y)Lxy \land (\forall x)(\exists y)\neg Lxy.$$

   (a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2.

   $U^{A_2} = \{1, 2, 3, 4\}$

   $L^{A_2} = \{\langle i, i\rangle \mid 1 \leq i \leq 4\}$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2? 38,416

5. Let S_3 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y) \wedge (\forall x)(\forall y)(Rxy \supset \neg Ryx) \wedge (\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \supset Rzx).$$

(a) (10 points) Specify a structure A_3 of size at least 4 which satisfies S_3.

$U^{A_3} = \{1, 2, 3, 4, 5, 6\}$

$R^{A_3} = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle, \langle 4, 5\rangle, \langle 5, 6\rangle, \langle 6, 4\rangle\}$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5, 6\}$ satisfy S_3? 40

6. We say that a schema S *admits* a positive natural number n if and only if there is a structure A of size n which satisfies S.

(a) (10 points) Write down a schema S involving only the dyadic predicate letter "L" and the identity predicate such that S admits n if and only if n is divisible by two, and S implies

$$(\forall x)Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)).$$

Let S be the conjunction of the following two schemata.

- $(\forall x)Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$
- $(\forall x)(\exists y)(\forall z)((Lxz \wedge x \neq z) \equiv z = y)$

(b) (10 points) Write down a schema S involving only the dyadic predicate letter "R" and the identity predicate such that S admits n if and only if n is divisible by three, and S implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y).$$

Let S be the conjunction of the following two schemata.

- $(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y)$
- $(\forall x)(\forall y)(Rxy \supset \neg Ryx) \wedge (\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \supset Rzx)$