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LGIC 010 & PHIL 005
Practice Examination II
Spring Term, 2013

1. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent, and no schema on the list is implied by $(\exists x)(Fx \oplus Gx)$?
   32,760

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent and each schema on the list is satisfied by exactly 28 structures with universe of discourse $\{1, 2, 3, 4\}$?
   16

3. Let S_1 be the following schema.

   $$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lzx) \wedge (\forall x)(\exists y)(\forall z)(Lxz \equiv y = z)$$

   (a) (10 points) Specify a structure A_1 of size at least 6 which satisfies S_1, that is, U^{A_1} has at least 6 members and $A_1 \models S_1$.

   $U^{A_1} = \{1, 2, 3, 4, 5, 6\}$

   $L^{A_1} = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle, \langle 4, 5\rangle, \langle 5, 6\rangle, \langle 6, 4\rangle$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5, 6\}$ satisfy S_1?
   40

4. Let S_2 be the conjunction of the following schemata.

   - $(\forall x)(\forall y)(Lxy \supset Lyx)$
   - $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$
   - $(\forall x)Lxx$

   (a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2, that is, U^{A_2} has at least 4 members and $A_2 \models S_2$.

   $U^{A_2} = \{1, 2, 3, 4\}$

$L^{A_2} = \{\langle i, i\rangle \mid 1 \leq i \leq 4\}$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2?
15

5. Let S_3 be the conjunction of the following schemata.

- $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
- $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$
- $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$
- $(\forall x)(\exists y)(Lxy \wedge (\forall w)(Lxw \supset (w = y \vee Lyw))$
- $(\exists x)((\exists y)Lyx \wedge (\forall y)(Lyx \supset (\exists z)(Lyz \wedge Lzx)))$

(a) (10 points) Specify a structure A_3 of size at least 4 which satisfies S_3, that is, U^{A_3} has at least 4 members and $A_3 \models S_3$.

$U^{A_3} = \{1, 2, 3, \ldots\}$

$L^{A_3} = \{\langle i, j\rangle \mid (\mathsf{parity}(i) = \mathsf{parity}(j) \text{ and } i < j) \text{ or } \mathsf{parity}(i) < \mathsf{parity}(j)\}$,
where $\mathsf{parity}(k) = 0$, if k is even and $\mathsf{parity}(k) = 1$, if k is odd.

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_3?
0

6. We say that a schema S *admits* a positive integer n if and only if there is a structure A of size n which satisfies S; the spectrum of S is the set of positive integers n such that S admits n.

(a) (10 points) What is the spectrum of the conjunction of the following schemata?

- $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
- $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$
- $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$
- $(\forall x)((\forall y)\neg Lyx \vee (\forall y)\neg Lxy) \supset Fx)$
- $(\forall x)(\forall y)((Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy)) \supset (Fx \equiv \neg Fy))$

the set of odd positive integers

(b) (10 points) What is the spectrum of the conjunction of the following schemata?

- $(\forall x)\neg Lxx$

- $(\forall x)(\forall y)(Lxy \supset Lyx)$
- $(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z)$

the set of even positive integers