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LGIC 010 & PHIL 005
Practice Examination II
Spring Term, 2012

1. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent, and no schema on the list is implied by $(\exists x)(Fx \wedge Gx)$?
   $2^{15} - 2^7$

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent and each schema on the list is satisfied by exactly 38 structures with universe of discourse $\{1, 2, 3, 4\}$?
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3. Let S_1 be the following schema.

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx).$$

   (a) (10 points) Specify a structure A_1 of size at least 4 which satisfies S_1, that is, U^{A_1} has at least 4 members and $A_1 \models S_1$.

   $U^{A_1} = \{1, 2, 3, 4\}$

   $L^{A_1} = \emptyset$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_1?
   3^6

4. Let S_2 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge \neg(\forall x)(\exists y)(\forall z)(Lzx \equiv z = y).$$

   (a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2.

   $U^{A_2} = \{1, 2, 3, 4\}$

   $L^{A_2} = \{\langle 1, 1\rangle, \langle 2, 1\rangle, \langle 3, 1\rangle, \langle 4, 1\rangle\}$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2?
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5. We say that a schema S *admits* a positive natural number n if and only if there is a structure A of size n which satisfies S.

   (a) (10 points) Write down a schema S involving only the dyadic predicate letter "L" and the identity predicate such that S admits n, if and only if, n is divisible by three, and S implies
$$(\forall x)(\forall y)(Lxy \supset \neg Lyx).$$

$$(\forall x)\neg Lxx \wedge (\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lzx)$$

   (b) (10 points) Write down a schema S involving only the dyadic predicate letter "L," and the identity predicate such that S admits n, if and only if, n is divisible by two, and S implies
$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge \neg(\forall x)(\exists y)(\forall z)(Lzx \equiv z = y).$$

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)((\exists y)Lyx \supset (\exists y)(\exists z)(Lyx \wedge Lzx \wedge y \neq z)) \wedge$$
$$(\forall x)(\forall y)(\forall z)((Lyx \wedge Lzx) \supset (x = y \vee x = z \vee y = z))$$

6. Taking the universe of discourse to be the set of positive integers $\{1, 2, \ldots\}$ and using the triadic predicate letter "P" to express the relation $\boxed{3}$ is the product of $\boxed{1}$ and $\boxed{2}$ and the dyadic predicate letter "Q" to express the relation $\boxed{1}$ is less than $\boxed{2}$, express the following statements in quantificational notation. (The boxed numerals indicate the order of argument places to the predicate letters.) You may need to use the symbol for identity in your paraphrases.

   (a) (10 points) $x = y^4$.

$$(\exists z)(Pyyz \wedge Pzzx)$$

   (b) (10 points) $x = 2y + 1$.

$$(\exists u)((\exists v)(\forall w)(Qwu \equiv w = v) \wedge (\exists z)(Puyz \wedge Qzx \wedge (\forall t)\neg(Qzt \wedge Qtx)))$$