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LGIC 010 & PHIL 005
Practice Examination II
Spring Term, 2009

1. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that no two schemata on the list are equivalent, and no schema on the list is implied by $(\forall x)(Fx \equiv Gx)$?
   $2^{15} - 2^{12}$

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters "F" and "G" can be constructed so that each schema on the list implies the next schema on the list, but is not implied by it, and no schema on the list implies $(\forall x)(Fx \equiv Gx)$?
   15

3. Let S_1 be the following schema.

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y)$$

   (a) (10 points) Specify a structure A_1 of size at least 4 which satisfies S_1, that is, U^{A_1} has at least 4 members and $A_1 \models S_1$.

   $U^{A_1} = \{1, 2, 3, 4\}$

   $L^{A_1} = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle, \langle 4, 1\rangle, \langle 3, 1\rangle, \langle 4, 2\rangle\}$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_1?
   2^6

4. Let S_2 be the following schema.

$$(\forall x)(\forall y)(\exists w)(\forall z)(Rxyz \equiv z = w).$$

   (a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2.

   $U^{A_2} = \{1, 2, 3, 4\}$

   $R^{A_2} = \{\langle i, j, 1\rangle \mid 1 \leq i, j \leq 4\}$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2?
   4^{16}

5. We say that a schema S *admits* a positive natural number n if and only if there is a structure A of size n which satisfies S.

   (a) (10 points) Write down a schema S involving only the dyadic predicate letter "L," the monadic predicate letters F and G, and the identity predicate such that S admits n, if and only if, n is divisible by four, and S implies

   $$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y).$$

   S is the conjunction of the following three schemata.

   $$\begin{array}{l} (\forall x)\neg Lxx \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y) \\ (\forall x)(((\forall y)\neg Lyx \supset (Fx \wedge Gx)) \wedge ((\forall y)\neg Lxy \supset (\neg Fx \wedge \neg Gx))) \\ (\forall x)(\forall y)((Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy)) \supset \\ \quad (((Fx \wedge Gx) \equiv (\neg Fy \wedge Gy)) \wedge ((\neg Fx \wedge Gx) \equiv (Fy \wedge \neg Gy)) \\ \quad \wedge((Fx \wedge \neg Gx) \equiv (\neg Fy \wedge \neg Gy)) \wedge ((\neg Fx \wedge \neg Gx) \equiv (Fy \wedge Gy)))) \end{array}$$

   (b) (10 points) Write down a schema S involving only the dyadic predicate letter "L," and the identity predicate such that S admits n, if and only if, n is divisible by three, and S implies

   $$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)(\exists y)(\forall z)(Lzx \equiv z = y).$$

   S is the conjunction of the following two schemata.

   $$\begin{array}{r} (\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)(\exists y)(\forall z)(Lzx \equiv z = y) \\ (\forall x)\neg Lxx \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lzx) \end{array}$$

6. Taking the universe of discourse to be the set of positive integers $\{1, 2, \ldots\}$ and using the triadic predicate letter "P" to express the relation $\boxed{3}$ is the sum of $\boxed{1}$ and $\boxed{2}$, express the following statements in quantificational notation. (The boxed numerals indicate the order of argument places to the predicate letters.) You may need to use the symbol for identity in your paraphrases.

   (a) (10 points) x is divisible by eight.

   $$(\exists w)(\exists y)(\exists z)(Pyyz \wedge Pzzw \wedge Pwwx)$$

   (b) (10 points) $x = y + 1$.

   $$(\exists z)(Pyzx \wedge (\forall v)(\forall w)\neg Pvwz)$$