1. (10 points) How long a list of pure monadic schemata involving only the predicate letters “F” and “G” can be constructed so that no two schemata on the list are equivalent, and no schema on the list is implied by \((\forall x)(Fx \equiv Gx)\)?

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters “F” and “G” can be constructed so that each schema on the list implies the next schema on the list, but is not implied by it, and no schema on the list implies \((\forall x)(Fx \equiv Gx)\)?

3. Let \(S_1\) be the following schema.
\[
(\forall x)(\forall y)(Lxy \supset \neg Lyx) \land (\forall x)(\forall y)(Lxy \lor Lyx \lor x = y)
\]
(a) (10 points) Specify a structure \(A_1\) of size at least 4 which satisfies \(S_1\), that is, \(U^{A_1}\) has at least 4 members and \(A_1 \models S_1\).

\[U^{A_1} = \]

\[L^{A_1} = \]

(b) (10 points) How many structures with universe of discourse \(\{1, 2, 3, 4\}\) satisfy \(S_1\)?

4. Let \(S_2\) be the following schema.
\[
(\forall x)(\forall y)(\exists w)(\forall z)(Rxyz \equiv z = w).
\]
(a) (10 points) Specify a structure \(A_2\) of size at least 4 which satisfies \(S_2\).

\[U^{A_2} = \]

\[R^{A_2} = \]

(b) (10 points) How many structures with universe of discourse \(\{1, 2, 3, 4\}\) satisfy \(S_2\)?
5. We say that a schema $S$ admits a positive natural number $n$ if and only if there is a structure $A$ of size $n$ which satisfies $S$.

(a) (10 points) Write down a schema $S$ involving only the dyadic predicate letter “$L$,” the monadic predicate letters $F$ and $G$, and the identity predicate such that $S$ admits $n$, if and only if, $n$ is divisible by four, and $S$ implies

$$(\forall x)\neg Lxx \land (\forall x)(\forall y)(\forall z)((Lxy \land Lyz) \supset Lxz) \land (\forall x)(\forall y)(Lxy \lor Lyx \lor x = y).$$

(b) (10 points) Write down a schema $S$ involving only the dyadic predicate letter “$L$,” and the identity predicate such that $S$ admits $n$, if and only if, $n$ is divisible by three, and $S$ implies

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \land (\forall x)(\exists y)(\forall z)(Lzx \equiv z = y).$$

6. Taking the universe of discourse to be the set of positive integers $\{1, 2, \ldots \}$ and using the triadic predicate letter “$P$” to express the relation $[3]$ is the sum of $[1]$ and $[2]$, express the following statements in quantificational notation. (The boxed numerals indicate the order of argument places to the predicate letters.) You may need to use the symbol for identity in your paraphrases.

(a) (10 points) $x$ is divisible by eight.

(b) (10 points) $x = y + 1.$