PRINT NAME: ___________________________________

LGIC 010 & PHIL 005 Practice Final Examination Spring Term, 2018

1. Let S be the conjunction of the following schemata.

   - $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
   - $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$
   - $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$

   (a) (15 points) Construct a maximal length succinct list l of structures such that each structure listed on l is a member of $\mathsf{mod}(S, 4)$.

(Additional space for your solution to 1.(a), as necessary)

(b) (15 points) For each structure A on your list l and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.

2. Let A be the structure interpreting a single triadic predicate letter P with $U^A = \mathbb{N}$ and $P^A = \{\langle i, j, k\rangle \mid i \cdot j = k\}$, the multiplication relation on $\mathbb{N}$.

   (a) (15 points) Let $X_1 = \{0, 1\}$. Is X_1 definable in A? If it is, write down a schema $S_1(x)$ such that $S_1[A] = X_1$; if not, specify an $h_1 \in \mathsf{Aut}(A)$ such that $h_1[X_1] \neq X_1$.

   (b) (15 points) Let $X_2 = \{0, 2, 4, 6, \ldots\}$ (the set of even non-negative integers). Is X_2 definable in A? If it is, write down a schema $S_2(x)$ such that $S_2[A] = X_2$; if not, specify an $h_2 \in \mathsf{Aut}(A)$ such that $h_2[X_2] \neq X_2$.

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3. (40 points) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a rigorous proof to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz), (\forall x)\neg Lxx\}$
   $S : (\forall x)(\forall y)(Lxy \supset \neg Lyx)$

   $A : U^A =$

   $L^A =$

Rigorous Proof

(b) $X : \{(\exists x)(\exists y)Lxy, (\forall x)((\exists y)Lxy \supset (\forall z)Lzx)\}$
$S : (\forall x)(\forall y)Lxy$

$B : U^B =$

$L^B =$

Rigorous Proof

(c) $X : \{(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z), (\forall x)(\exists y)Lyx\}$
$S : (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$

$C : U^C =$

$L^C =$

Rigorous Proof

(d) For each $n \geq 2$, let R^n be the schema

$$(\exists x_1)\dots(\exists x_n) \bigwedge_{1\leq i<j\leq n} x_i \neq x_j.$$

$X : \{(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz), (\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)), (\forall x)\neg Lxx,$
$(\exists x)(\forall y)\neg Lyx, (\exists x)(\forall y)\neg Lxy, (\forall x)((\exists y)Lxy \supset (\exists y)(Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy))),$
$(\forall x)((\exists y)Lyx \supset (\exists y)(Lyx \wedge (\forall z)\neg(Lyz \wedge Lzx)))\} \cup \{R^n \mid n \geq 2\}$
$S : (\forall x)(\forall y)Lxy$

$D : U^D =$

$L^D =$

Rigorous Proof

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

Name (printed)

Signature

Date

LGIC 010 & PHIL 005
Definitions for Final Examination
Spring Term, 2017

- $[n] = \{1, \ldots, n\}$.
- $\mathsf{mod}(S, n) = \{A \mid A \models S \text{ and } U^A = [n]\}$.
- $A \cong B$ if and only if A is *isomorphic to* B.
- A list l of structures is *succinct* if and only if for every pair of distinct structures A and B appearing on l, $A \ncong B$.
- $\mathsf{Aut}(A) = \{h \mid h \text{ is an automorphism of } A\}$.
- $\mathsf{orb}(a, A) = \{h(a) \mid h \in \mathsf{Aut}(A)\}$.
- $\mathsf{Orbs}(A) = \{O \mid \text{for some } a \in U^A,\ O = \mathsf{orb}(a, A)\}$.
- Let $S(x)$ be a schema with a single free variable x and A a structure.

$$S[A] = \{a \in U^A \mid A \models S[x|a]\}.$$

- $\mathbb{N}$ is the set of non-negative integers $\{0, 1, 2, 3, \ldots\}$.