PRINT NAME: ______________________________________

LGIC 010 & PHIL 005 Practice Final Examination Spring Term, 2016

1. Let S be the conjunction of the following schemata.

    - $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
    - $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)) \vee (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$

    (a) (15 points) Construct a maximal length succinct list l of structures such that each structure listed on l is a member of $\mathsf{mod}(S, 3)$.

(b) (15 points) For each structure A on your list l and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.

2. Let A be a structure interpreting a single triadic predicate letter P and a single dyadic predicate letter L with $U^A = \mathbb{Z}$, $P^A = \{\langle i, j, k\rangle \mid i + j = k\}$, the sum relation on $\mathbb{Z}$, and $L^A = \{\langle i, j\rangle \mid j = |i|\}$, the absolute-value relation on $\mathbb{Z}$.

   (a) (15 points) Let $X_1 = \{i \in \mathbb{Z} \mid i < 0\} = \{-1, -2, -3, \ldots\}$. Is X_1 definable in A? If it is, write down a schema $S_1(x)$ such that $S_1[A] = X_1$; if not, specify an $h_1 \in \mathsf{Aut}(A)$ such that $h_1[X_1] \neq X_1$.

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   (b) (15 points) Let $X_2 = \{i \in \mathbb{Z} \mid i > 0 \text{ and there is a } j \in \mathbb{Z} \text{ such that } i = 5j + 3\} = \{3, 8, 13, \ldots\}$. Is X_2 definable in A? If it is, write down a schema $S_2(x)$ such that $S_2[A] = X_2$; if not, specify an $h_2 \in \mathsf{Aut}(A)$ such that $h_2[X_2] \neq X_2$.

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3. (40 points) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists y)(\forall x)(Fx \equiv x = y), (\exists y)(\forall x)(\neg Fx \equiv x = y)\}$
   $S : (\exists x)(\exists y)(x \neq y \wedge (\forall z)(z = x \vee z = y))$

   $A : U^A =$

   $F^A =$

Deduction

(b) $X : \{(\exists x)(Fx \land Gx)\}$
$S : (\exists x)Fx \land (\exists x)Gx$

$B : U^B =$

$F^B =$

$G^B =$

Deduction

(c) $X : \{(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z), (\forall x)(\exists y)Lyx\}$
$S : (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$

$C : U^C =$

$L^C =$

Deduction

(d) For each $n \geq 2$, let R^n be the schema

$$(\exists x_1) \ldots (\exists x_n) \bigwedge_{1 \leq i < j \leq n} (x_i \neq x_j \wedge (\exists y) Lyx_i \wedge (\forall z)(Lzx_i \supset (\exists w)(Lzw \wedge Lwx_i)).$$

$X : \{(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz), (\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)), (\forall x)\neg Lxx,$
$(\forall x)(\exists y)(Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy))\} \cup \{R^n \mid n \geq 2\}$
$S : (\forall x)(\exists y)Lyx$

$D : U^D =$

$L^D =$

Deduction