**PRINT NAME:** ________________________________________

**LGIC 010 & PHIL 005 Practice Final Examination Spring Term, 2013**

1. (30 points) Taking the universe of discourse to be the set of non-negative integers $\{0, 1, 2, \ldots\}$ and using the triadic predicate letter "$P$" to express the relation $\boxed{3}$ is the product of $\boxed{1}$ and $\boxed{2}$, express the following statements in quantificational notation. (The boxed numerals indicate the order of argument places to the predicate letters.) You may need to use the symbol for identity in your paraphrases.

   (a) $x = 0$.

   (b) $x = 1$.

   (c) $x$ is a prime power, that is, $x$ is $p^n$ for some prime number $p$ and some integer $n \geq 1$.

2. (40 points) Let $S$ be the following schema.

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx)$$

   (a) How long a list of distinct structures $A$ with universe of discourse $\{1, 2, 3\}$ satisfy the schema $S$?

   (b) How long a list of pairwise non-isomorphic structures with universe of discourse $\{1, 2, 3\}$ satisfy the schema $S$?

   (c) How long a list of distinct structures $A$ with universe of discourse $\{1, 2, 3, 4\}$ satisfy the condition: $A \models S$ and $|\mathsf{Aut}(\mathsf{A})| = 1$?

   (d) How long a list of distinct structures $A$ with universe of discourse $\{1, 2, 3\}$ satisfy the condition: $A \models S$ and $|\mathsf{Def}(\mathsf{A})| = 2$?

3. (30 points) For each of the following pairs consisting of a set of schemata $X$ and a schema $S$ determine whether $X$ implies $S$. If so, provide a deduction to establish the implication. If not, specify a structure which makes $S$ false and all the schemata in $X$ true.

   (a) $X : \{(\forall x)((\exists y)Lxy \supset (\forall z)Lzx), (\exists x)(\exists y)Lxy\}$
   $S : (\forall u)(\forall z)Lzu$

   $A : U^A =$

   $L^A =$

Deduction

(b) $X : \{(\forall x)(Fx \supset (\exists y)(\neg Fy \wedge (\forall z)(Rxz \equiv z = y))), (\forall x)(\neg Fx \supset (\exists y)(Fy \wedge (\forall z)(Rzx \equiv z = y))), (\forall x)(\forall y)(\forall z)((Pxy \wedge Pxz) \supset y = z), (\forall x)(\exists y)(Fy \wedge Pyx)\}$
$S : p \wedge \neg p$

$B : U^B =$

$F^B =$

$P^B =$

$R^B =$

Deduction

(c) $X : \{(\forall x)Rxx, \neg(\forall x)(\forall y)Rxy, (\exists x)(\forall y)x = y\}$
$S : p \wedge \neg p$

$C : U^C =$

$R^C =$

Deduction