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LGIC 010 & PHIL 005
Practice Final Examination
Spring Term, 2009

1. Let S be the schema $(\forall x)\neg Lxx$.

   (a) (10 points) How long a list of distinct structures with universe of discourse $\{1, 2, 3\}$ satsify the schema S. 64

   (b) (10 points) How long a list of pairwise nonisomorphic structures with universe of discourse $\{1, 2, 3\}$ satsify the schema S. 16

   (c) (10 points) Give an example of a structure A with the following properties:

   - $A \models S$;
   - $U^A = \{1, 2, 3\}$;
   - A has exactly three automorphisms;
   - exactly two subsets of $\{1, 2, 3\}$ are definable in A.

   $L^A = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle\}$

2. (70 pts.) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists x)(Px \vee Qx)\}$
   $S : (\exists x)Px \vee (\exists x)Qx$

   $A : U^A =$

   $P^A =$

   $Q^A =$

Deduction

$\{1\}$	(1)	$(\exists x)(Px \vee Qx)$	P
$\{1, 2\}$	(2)	$Px \vee Qx$	(1)x EII
$\{3\}$	(3)	Px	P
$\{4\}$	(4)	Qx	P
$\{3\}$	(5)	$(\exists x)Px$	(3) EG
$\{4\}$	(6)	$(\exists x)Qx$	(4) EG
$\{\}$	(7)	$Px \supset (\exists x)Px$	(5)$\{3\}$ D
$\{\}$	(8)	$Qx \supset (\exists x)Qx$	(6)$\{4\}$ D
$\{1, 2\}$	(9)	$(\exists x)Px \vee (\exists x)Qx$	(2)(7)(8) TF
$\{1\}$	(10)	$(\exists x)Px \vee (\exists x)Qx$	(9)$\{2\}$ EIE

(b) $X : \{(\forall x)(Px \vee Qx)\}$
$S : (\forall x)Px \vee (\forall x)Qx$

$B : U^B = \{0, 1\}$

$P^B = \{0\}$

$Q^B = \{1\}$

Deduction

(c) $X : \{(\forall x)Rxx, \neg(\forall x)(\forall y)Rxy\}$
$S : \neg(\exists x)(\forall y)x = y$

$C : U^C =$

$R^C =$

Deduction

$\{1\}$	(1) $(\forall x)Rxx$	P
$\{2\}$	(2) $\neg(\forall x)(\forall y)Rxy$	P
$\{3\}$	(3) $(\exists x)(\forall y)x = y$	P
$\{3, 4\}$	(4) $(\forall y)u = y$	(3)u EII
$\{1\}$	(5) Ruu	(1) UI
$\{3, 4\}$	(6) $u = y$	(4) UI
$\{\}$	(7) $u = y \supset (Ruu \equiv Ruy)$	III
$\{3, 4\}$	(8) $u = x$	(4) UI
$\{\}$	(9) $u = x \supset (Ruy \equiv Rxy)$	III
$\{1, 3,$ ~~4~~$\}$	(10) Rxy	(5)(6) TF; (7)(8) $\{4\}$ EIE (9)
$\{1, 3\}$	(11) $(\forall y)Rxy$	(10) UG
$\{1, 3\}$	(12) $(\forall x)(\forall y)Rxy$	(11) UG
$\{1, 2, 3\}$	(13) $p \wedge \neg p$	(2)(12) TF
$\{1, 2\}$	(14) $(\exists x)(\forall y)x = y \supset (p \wedge \neg p)$	$\{3\}$(13) D
$\{1, 2\}$	(15) $\neg(\exists x)(\forall y)x = y$	(14) TF

(d) $X : \{(\forall x)(Fx \supset (\exists y)(\neg Fy \wedge (\forall z)(Rxz \equiv z = y))), (\forall x)(\neg Fx \supset (\exists y)(Fy \wedge (\forall z)(Rzx \equiv z = y))), (\forall x)(\forall y)(\forall z)((Pxy \wedge Pxz) \supset y = z), (\forall x)(\exists y)(Fy \wedge Pyx)\}$
$S : p \wedge \neg p$

$D : U^D = \{0, 1, 2, \ldots\}$

$F^D = \{0, 2, 4, \ldots\}$

$R^D = \{\langle 2i, 2i+1\rangle \mid 0 \leq i\}$

$P^D = \{\langle 2i, i\rangle \mid 0 \leq i\}$

Deduction

(e) $X : \{(\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \supset Rxz), (\forall x)\neg Rxx, (\forall x)(\forall y)(Rxy \vee Ryx \vee x = y), (\forall x)(\exists y)(\forall z)(Pxz \equiv z = y), (\forall x)(\exists y)Pyx, (\forall x)(\forall y)(Pxy \supset Rxy)\}$
$S : p \wedge \neg p$

$E : U^E = \{\ldots - 3, -2, -1, 0, 1, 2, 3, \ldots\}$

$P^E = \{\langle i, i+1\rangle \mid i \in U^E\}$

$R^E = \{\langle i, j\rangle \mid i < j\}$

Deduction

(f) $X : \{(\exists x)(\forall y)((\forall z)Rzy \equiv y = x)\}$
$S : (\forall x)(\exists y)(\forall z)(Rxz \equiv z = y)$

$F : U^F = \{0, 1\}$

$R^F = \{\langle 0, 0\rangle, \langle 0, 1\rangle, \langle 1, 0\rangle\}$

Deduction

(g) $X : \{(\forall x)(\exists z)(\forall w)(Rxw \equiv w = z),$
$(\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset y = x)\}$
$S : (\forall z)(\exists x)Rxz$

$G : U^G = \{0, 1, 2, 3, \ldots\}$

$R^G = \{\langle i, i+1\rangle \mid i \in U^G\}$

Deduction