22 Lecture 04.18

On, 04.18, we extended the deduction rules to include existential generalization and existential instantiation which allow us to mirror common informal forms of argument involving the existential quantifier. The following deduction gives an example of their use.

$$\{(\forall x)((\exists y)Lxy \supset (\forall z)Lzx), (\exists x)(\exists y)Lxy\} \text{ implies } (\forall v)(\forall z)Lvz.$$

$\{1\}$	(1) $(\exists x)(\exists y)Lxy$	P
$\{1,2\}$	(2) $(\exists y)Lwy$	(1)w EII
$\{3\}$	(3) $(\forall x)((\exists y)Lxy \supset (\forall z)Lzx)$	P
$\{3\}$	(4) $(\exists y)Lwy \supset (\forall z)Lzw$	(3) UI
$\{1,2,3\}$	(5) $(\forall z)Lzw$	(2)(4) TF
$\{1,2,3\}$	(6) Lvw	(5) UI
$\{1,\cancel{2},3\}$	(7) $(\exists y)Lvy$	(5) EG; $\{2\}$ EIE
$\{3\}$	(8) $(\exists y)Lvy \supset (\forall z)Lzv$	(3) UI
$\{1,3\}$	(9) $(\forall z)Lzv$	(7)(8) TF
$\{1,3\}$	(10) $(\forall v)(\forall z)Lzv$	(9) UG

We next added rules for deriving schemata involving the identity predicate and illustrated their use with the following deduction.

$$\{(\forall x)Rxx, \neg(\forall x)(\forall y)Rxy\} \text{ implies } \neg(\exists x)(\forall y)x = y.$$

$\{1\}$	(1) $(\forall x)Rxx$	P
$\{2\}$	(2) $\neg(\forall x)(\forall y)Rxy$	P
$\{3\}$	(3) $(\exists x)(\forall y)x = y$	P
$\{3,4\}$	(4) $(\forall y)u = y$	(3)u EII
$\{1\}$	(5) Ruu	(1) UI
$\{3,4\}$	(6) $u = y$	(4) UI
$\{\}$	(7) $u = y \supset (Ruu \equiv Ruy)$	III
$\{3,4\}$	(8) $u = x$	(4) UI
$\{\}$	(9) $u = x \supset (Ruy \equiv Rxy)$	III
$\{1,3,\cancel{4}\}$	(10) Rxy	(5)(6) TF; (7)(8) $\{4\}$ EIE (9)
$\{1,3\}$	(11) $(\forall y)Rxy$	(10) UG
$\{1,3\}$	(12) $(\forall x)(\forall y)Rxy$	(11) UG
$\{1,2,3\}$	(13) $p \wedge \neg p$	(2)(12) TF
$\{1,2\}$	(14) $(\exists x)(\forall y)x = y \supset (p \wedge \neg p)$	$\{3\}$(13) D
$\{1,2\}$	(15) $\neg(\exists x)(\forall y)x = y$	(14) TF

We briefly touched on the Soundness and Completeness Theorems and the Church-Turing Theorem. I will include a discussion of these results in a forthcoming memoir.