20 Lecture 04.11

We commenced our study of the deductive apparatus for polyadic quantification theory expounded in Warren Golfarb's text *Deductive Logic*. We gave examples of deductions using the rules described on pages 183 – 185 of the text. We began with a simple to deduction to show that if a relation is asymmetric, then it is irreflexive.

$$\{(\forall x)(\forall y)(Lxy \supset \neg Lyx)\} \text{ implies } (\forall x)\neg Lxx.$$

$\{1\}$	(1)	$(\forall x)(\forall y)(Lxy \supset \neg Lyx)$	P
$\{1\}$	(2)	$(\forall y)(Lxy \supset \neg Lyx)$	(1) UI
$\{1\}$	(3)	$Lxx \supset \neg Lxx$	(2) UI
$\{1\}$	(4)	$\neg Lxx$	(3) TF
$\{1\}$	(5)	$(\forall x)\neg Lxx$	(4) UG

We next showed that if a relation is transitive and irreflexive, then it's asymmetric.

$$\{(\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)), (\forall x)\neg Lxx\} \text{ implies}$$
$$(\forall x)(\forall y)(Lxy \supset \neg Lyx).$$

$\{1\}$	(1)	$(\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$	P
$\{1\}$	(2)	$(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$	(1) UI
$\{1\}$	(3)	$(\forall z)(Lxy \supset (Lyz \supset Lxz))$	(2) UI
$\{1\}$	(4)	$Lxy \supset (Lyx \supset Lxx)$	(3) UI
$\{5\}$	(5)	$(\forall x)\neg Lxx$	P
$\{5\}$	(6)	$\neg Lxx$	(5) UI
$\{1, 5\}$	(7)	$(Lxy \supset \neg Lyx)$	$(4, 6)$ TF
$\{1, 5\}$	(8)	$(\forall y)(Lxy \supset \neg Lyx)$	(7) UG
$\{1, 5\}$	(9)	$(\forall x)(\forall y)(Lxy \supset \neg Lyx)$	(8) UG