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LGIC 010 & PHIL 005 Practice Final Examination Spring Term, 2019

1. (10 points) Write down a schema S involving only the triadic predicate letter "P," the monadic predicate letters "F" and "G," and the identity predicate such that

   - $\mathsf{Spec}(S) = \{2^n \mid n \in \mathbb{Z}^+\}$, and
   - S implies

   $$(\forall x)(\forall y)(Fy \supset (\exists z)(\forall w)(Pxyw \equiv z = w)) \wedge (\forall x)(\forall y)(\forall z)(Pxyz \supset Gz).$$

2. (10 points) Let T be the conjunction of the following schemata.

   - $(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z))$
   - $(\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$
   - $(\exists x)(\forall y)(Lyy \equiv y = x)$
   - $(\forall x)(\forall y)(Lxy \supset Lyx)$

   Specify the spectrum of T.

   $\mathsf{Spec}(T) =$

3. Let S be the conjunction of the following schemata.

- $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx)$
- $(\exists x)(\exists y)(\exists z)(Lxy \wedge Lyz \wedge Lxz)$

(a) (15 points) Construct a maximal length succinct list l of structures such that each structure listed on l is a member of $\mathsf{mod}(S, 4)$.

(Additional space for your solution to 3.(a), as necessary)

(b) (15 points) For each structure A on your list l and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.

(**Additional space for your solution to 3.(b), as necessary**)

4. Let A be the structure interpreting a single triadic predicate letter P with $U^A = \mathbb{Z}$ and $P^A = \{\langle i, j, k\rangle \mid i \cdot j = k\}$, the multiplication relation on $\mathbb{Z}$.

   (a) (10 points) Let $X_1 = \{-1\}$. Is X_1 definable in A? If it is, write down a schema $S_1(x)$ such that $S_1[A] = X_1$; if not, specify an $h_1 \in \mathsf{Aut}(A)$ such that $h_1[X_1] \neq X_1$.

   .

   (b) (10 points) Let $X_2 = \{1, -1, 8, -8, 27, -27, \ldots\}$ (the set of perfect "cubes"). Is X_2 definable in A? If it is, write down a schema $S_2(x)$ such that $S_2[A] = X_2$; if not, specify an $h_2 \in \mathsf{Aut}(A)$ such that $h_2[X_2] \neq X_2$.

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5. For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a cogent argument to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) (15 points)
   $X : \{(\forall x)\neg Lxx, (\forall x)(\forall y)(Lxy \supset (Lyz \supset Lxz)),$
   $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx), (\forall x)(\exists y)(Lxy \wedge (\forall z)\neg(Lzy \wedge Lxz))),$
   $(\forall x)(\exists y)(Lyx \wedge (\forall z)\neg(Lyz \wedge Lzx)), (\forall x)(\exists y)(Fy \wedge Lyx), (\forall x)(\exists y)(Fy \wedge Lxy),$
   $(\forall x)(\forall y)((Fx \wedge Fy \wedge Lxy) \supset (\exists z)(Fz \wedge Lxz \wedge Lzy))\}$
   $S : (\forall x)(Fx \wedge \neg Fx)$

   $A : U^A =$

   $F^A =$

   $L^A =$

Cogent Argument

(b) (15 points)

$X : \{(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z),$
$(\forall x)(\exists y)(\exists z)(y \neq z \wedge Lyx \wedge Lzx)\}$
$S : (\forall y)y \neq y$

$B : U^B =$

$L^B =$

Cogent Argument

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

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Name (printed)

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Signature

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Date

LGIC 010 & PHIL 005
Definitions for Final Examination
Spring Term, 2019

- $[n] = \{1, \ldots, n\}$.
- $\mathsf{mod}(S, n) = \{A \mid A \models S \text{ and } U^A = [n]\}$.
- $\mathsf{Spec}(S) = \{n \mid \mathsf{mod}(S, n) \neq \emptyset\}$.
- $A \cong B$ if and only if A is *isomorphic to* B.
- A list l of structures is *succinct* if and only if for every pair of distinct structures A and B appearing on l, $A \ncong B$.
- $\mathsf{Aut}(A) = \{h \mid h \text{ is an automorphism of } A\}$.
- $\mathsf{orb}(a, A) = \{h(a) \mid h \in \mathsf{Aut}(A)\}$.
- $\mathsf{Orbs}(A) = \{O \mid \text{for some } a \in U^A,\ O = \mathsf{orb}(a, A)\}$.
- Let $S(x)$ be a schema with a single free variable x and A a structure.

$$S[A] = \{a \in U^A \mid A \models S[x|a]\}.$$

- $\mathbb{Z}$ is the set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.