PRINT NAME:

LGIC 010 & PHIL 005
Practice Examination I
Spring Term, 2019

1. We call a set of numbers X *harmonious* if and only if **NO** number in X is divisible by some other number in X.

   (a) (10 points) What is the maximum size of a harmonious set X contained in $\{1, 2, \ldots, 18\}$?

   (b) (15 points) Give an example of a maximum size harmonious set $X \subseteq \{1, 2, \ldots, 18\}$ and explain why there is no larger such set.

2. (15 points) How many truth-assignments to the sentence letters p_1, p_2, p_3 satisfy the following truth-functional schema?

$$(p_1 \equiv p_2) \vee (p_1 \equiv p_3) \vee (p_2 \equiv p_3)$$

3. For the purposes of this problem, we restrict attention to truth-functional schemata all of whose sentence letters are among p_1, p_2, p_3, and p_4. We employ the following terminology.

   - A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
   - A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
   - The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.

   (a) (15 points) What is the length of a longest succinct list of schemata all of which have the same power?

   (b) (15 points) What is the length of a longest list of schemata none of which have the same power?

   (c) (15 points) Suppose that S_1, S_2, S_3 is a succinct list of schemata such that S_1 implies S_2, and S_2 implies S_3. Let k be the difference between the power of S_1 and the power of S_3. What is the maximum possible value of k that can be achieved by such a list?

4. (15 points) For the purposes of this problem, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters. We employ the following terminology.

   - If S and T are MQ-schemata we say that a structure A is a *counterexample* to the claim that S implies T if and only if $A \models S$ and $A \not\models T$.

   Let S be the schema

   $$(\forall x)(Fx \supset Gx),$$

   and let T be the schema

   $$(\forall x)(Gx \vee Fx)$$

   How many structures with universe of discourse $\{1, 2, 3, 4\}$ are counterexamples to the claim that S implies T?