AUCTION-DRIVEN COORDINATION FOR PLANTWIDE OPTIMIZATION

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Abstract

Model predictive control strategies generally focus on controlling plant outputs to setpoints; in industry, however, a more desirable goal is maximizing a plant's profitability. In principle, this can be done by creating a plant model and maximizing profit with respect to the market prices of the plant's inputs and outputs, but in practice, such centralized approaches often cannot effectively be applied at the operations time scale due to the size and complexity of the problem. One solution is to use decentralized optimization at the unit operations level by tearing process streams and coordinating the resulting pieces. Such optimization, however, requires that unit inputs and outputs be priced. We show that a traditional Lagrangean-based approach to this pricing fails for simple systems. Instead, we define slack resources over the torn process streams and price them using auctions. Unlike Lagrange multipliers, slack resource prices contain useful information and can be used to make decisions regarding capital improvements, thus providing a strong tie between the operations and management layers in chemical plants.

Keywords

auctions, distributed optimization, resource prices, process decomposition, optimal coordination, penalty-based methods, Lagrangean decomposition

Introduction

In principle, one can construct a detailed model of a plant and use it to centrally maximize the plant's profit by finding the "best" or most profitable setpoints for the process streams. In reality, such large-scale problems are quite difficult, and, at least for the foreseeable future, intractable, especially for large refineries. The assumption that one could assemble and use all the information from each unit is also somewhat strong. Often units are designed and run separately by different sets of people in different organizations; even the control systems are fine-tuned individually for each unit operation.

Plantwide optimization problems have a definite structure which can be exploited for decomposition: chemical plants are composed of separate units linked together by process streams. This component-wise structure can be used to attack these optimization problems, both by simplifying them and by solving them in their "natural form". In other words, unit operations are essentially separate pieces and have separate control systems and operators; there are benefits to preserving this as much as possible.

One approach to problems with this type of structure is process decomposition followed by some form of penalty-based coordination (Findstein, 1980; Mesarovic, 1970; Morari, 1980). Here, one tears process streams and locally optimizes the units assuming they are all independent; this generates stream value proposals from each unit. If the stream value proposals across each tear are all the same, there is no need for coordination—the units' locally optimal solutions taken together are globally optimal as well. If, on the other hand, there are differences (which is most often the case), the units have conflicts, and the local solutions are not feasible. In order to generate feasible solutions, one applies a penalty to the units so that they alter their proposed stream states. The
goal of this approach is to reach an optimal, system-wide solution by repeatedly adjusting the applied penalty.

If the system-wide objective is to maximize profit, it is natural to view the applied penalties in terms of prices. Starting from an optimization perspective, one might try to apply some form of Lagrangean relaxation technique to the problem and use the Lagrange multipliers to coordinate units. We show, however, that this method is inadequate for the typical situation where units in one part of a plant are indifferent to what happens in another part. We claim that instead of starting with the Lagrangean, starting with a clear definition of system resources is a better strategy. This leads to what we call a slack resource auction: a price-based penalty method that can optimally coordinate a broad class of systems including those for which Lagrangean-based methods are inadequate.

One Approach: Lagrangean-based Coordination

Consider a plant consisting of two units, A and B, connected by a process stream (Fig. 1). Each unit has a set of local constraints and a local objective or utility function. In addition, suppose each unit is indifferent to the stream’s state except for the temperature. After decomposing this system by tearing the stream (Fig. 2), the goal of the plant manager is to solve the following problem:

\[
\text{Maximize } u_A(T_A) + u_B(T_B) \\
\text{subject to:} \\
T_A = T_B \\
A \text{’s constraints} \\
B \text{’s constraints}
\]

where \(T_A\) and \(T_B\) are the proposed stream temperatures coming from Unit A and going into Unit B, and where \(u_A\) and \(u_B\) are their utility functions. Applying Lagrangean relaxation to the linking constraint moves it out of the constraint set and introduces it into the objective as a soft constraint:

\[
\text{Maximize } u_A(T_A) + u_B(T_B) - \lambda(T_A - T_B) \\
\text{subject to:} \\
A \text{’s constraints} \\
B \text{’s constraints}
\]

subject to:

1. Set \(\lambda\) to zero.
2. Have subproblems submit their proposed temperatures \(T_A\) and \(T_B\).
3. If \(T_A\) and \(T_B\) are sufficiently close, stop.
4. If \(T_A > T_B\), increase \(\lambda\) and go to Step 2.
5. If \(T_A < T_B\), decrease \(\lambda\) and go to Step 2.

Although the Lagrangean auction appears to be reasonable, it is unable to coordinate the case where one of the units is indifferent to the temperature.

Example 1: One Indifferent Unit

Consider the pair of units depicted in Fig. 2, and suppose that their utility functions over the stream temperature are described by Fig. 3. Because \(B\) is indifferent to the temperature, the system-wide solution will lie at \(A\)’s locally optimal point at 30 °F. Applying a Lagrangean-based decomposition results in the system described by Eqn. 3. The goal of this method is to vary \(\lambda\) until \(T_A = T_B\) at the system solution. This approach, however, does not converge (see Fig. 4).

The Lagrangean auction procedure is then
temperatures that minimum feasible temperature—these are the only will not converge.

It is crucial that we define meaningful resources.

Construct an effective auction with meaningful prices, it is any stream temperature they both agree to. In order to do this, the system resource seems to be the point where the units can select what does it mean to have a supply of temperature? In example above, the system resource seems to be the coordinating subproblems stems directly from a failure to make capital investments. Consider, for instance, a plant manager who has the option of placing a heat exchanger somewhere in the plant. If temperature prices could be meaningfully compared, the manager would place the heat exchanger at the point where the system would benefit the most—at the stream with the highest temperature price. Lagrangean-based methods cannot help here.

There are other problems with Lagrangean-based methods. Not only are Lagrangean methods unsuitable for coordinating systems where some of the units are indifferent, they are also unable to coordinate systems where at least one of the units has a piecewise linear utility function—the indifferent unit being a special case of this. In addition, prices in one part of a plant cannot be meaningfully compared with prices in another part. Thus, the Lagrange multipliers provide no useful information with regard to making capital investments. Consider, for instance, a plant manager who has the option of placing a heat exchanger somewhere in the plant. If temperature prices could be meaningfully compared, the manager would place the heat exchanger at the point where the system would benefit the most—at the stream with the highest temperature price. Lagrangean-based methods cannot help here.

A Better Approach to Coordination: Slack Resource Auctions

The difficulty the “Lagrangean auction” has in coordinating subproblems stems directly from a failure to define precisely what the auctioned resources are. In the example above, the system resource seems to be the stream temperature. This is awkward at best. For instance, what does it mean to have a supply of temperature? In principle, this resource is unbounded—the units can select any stream temperature they both agree to. In order to construct an effective auction with meaningful prices, it is crucial that we define meaningful resources.

System resources, in the usual sense, are quantities like steam, electricity, or cooling water. These resources have a definite supply, and there is a definite way of dividing them between units. In fact, this is the characterizing difference between traditional, divisible resources and linking stream resources. From the system’s point of view, divisible resources can always be expressed in the form

$$D_i(x_1) + D_2(x_2) + \cdots + D_n(x_n) \leq S$$

where $x_i$ is Agent $i$’s decision variable vector, $D_i$ maps Agent $i$’s activities into a demand for system resources, and $S$ is the supply. The left-hand-side of this constraint represents the aggregate demand for resources, and each of its terms is one of the units’ individual demands. The temperature of a linking stream, on the other hand, cannot be divided between units; it is an indivisible value which must be shared. Linking constraints have the form

$$T_A = T_B$$

One way of re-expressing this constraint so that it looks like a divisible resource constraint is

$$|T_A - T_B| \leq S$$

or

$$\frac{1}{2}|T_A - T_B| + \frac{1}{2}|T_A - T_B| \leq S$$

where each term on the left-hand-side of Eqn. 7 represents one of the units’ demand for the slack in temperature, and $S$ is the supply of “temperature slack” or the system’s tolerance for a difference in the temperatures of the torn linking stream. Taking this temperature slack as our resource leads to the following decomposition of the above problem:

Max $u_A(T_A) - p[T_A - T_B^0]$ 
subject to: $A$’s constraints

Max $u_B(T_B) - p[T_B^0 - T_B]$ 
subject to: $B$’s constraints

where $T_A^0$ and $T_B^0$ are parameters from the auctioneer defining the “slack poles” of the stream temperature, and $p$ is the slack resource price. This leads to the following slack resource auction procedure:

1. Decompose the system by breaking the appropriate linking streams.
2. For each relevant stream variable $x$ which joins Units $i$ and $j$, define the slack resource $R_{ij}$.
3. For each slack resource $R_i$, define a price $p_i$ and a slack supply $S_i$.
4. Set all prices to zero.
5. Request proposed stream states from units.
6. For each slack resource \( R_x \), compute the slack demand \( D_x \) and the excess demand \( E_x = D_x - S_x \).
7. For each resource, if the excess demand is negative, lower the price and go to Step 5.
8. For each resource, if the excess demand is positive, raise the price and go to Step 5.
9. If the excess demand is zero for all slack resources, stop.

Example 2: One indifferent unit revisited

Consider the system of units described in Example 1 and apply a slack resource auction to it by introducing \(|T_A - T_B|\) as a resource. This yields a coordination problem in the form of Eqn. 4. Unlike the Lagrangean procedure, the slack resource auction converges to the system-wide solution at an equilibrium price of 0 (see Fig. 5). Since prices measure conflict, this is correct—Unit B is indifferent to the temperature, and so there is no conflict at 30 °F.

![Figure 5: Slack resource auction price dynamics](image)

Example 3: Three Units with Recycle

Now consider a slightly more complicated system consisting of three units joined by process streams and a recycle loop (Fig. 6). For the purposes of discussion, suppose that each linking stream has a product stream draw which defines the value of that stream as a function of temperature (see Fig. 7).

![Figure 6: Three units with recycle](image)

![Figure 7: Product stream values](image)

Table 1: Heating parameters

<table>
<thead>
<tr>
<th>Unit</th>
<th>Ideal ( \Delta T ) (°F)</th>
<th>Heat Cost ($/°F)</th>
<th>Cool. Cost ($/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.0</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>10.0</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>C</td>
<td>20.0</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>
The goal of this problem is to determine the triple \((T_1; T_2; T_3)\) which maximizes the sum of the units’ utility functions. Following the slack resource auction procedure, one tears the process streams, defines a slack temperature resource over each of them, and adjusts prices based on demand. The final, system-wide optimal result is

<table>
<thead>
<tr>
<th>Stream</th>
<th>Temp. (°F)</th>
<th>Slack Price ($/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stream 1</td>
<td>150.0</td>
<td>0.41</td>
</tr>
<tr>
<td>Stream 2</td>
<td>150.0</td>
<td>0.30</td>
</tr>
<tr>
<td>Stream 3</td>
<td>160.0</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**Discussion**

It is natural to ask what the relationship is between the equilibrium prices of a slack resource auction and the dual prices from the original centralized problem. There is a well-known market-based interpretation of the primal and dual problems in optimization: given a primal problem, the dual problem can be viewed as a “malevolent market” whose goal is to drive the net profit of the primal problem to as low a value as possible by manipulating resource prices (Bradley, 1977). This malevolent market is assumed to be complete—composed of an infinite number of infinitely varied agents. Now consider the case where we have a plant which is decomposed into separate units. Although the slack resources are the same, this market consists only of the component subproblems. Therein lies the difference: the coordinating auction prices result from an imperfect market. Auction prices, therefore, generalize the notion of dual prices: as the number of units increases, we expect the auction prices to approach the dual prices. This same distinction can be seen when coordinating LPs using Dantzig-Wolfe decomposition—the prices generated by the master problem are never equal to the dual prices of the corresponding constraints.

Slack resource auctions are not limited to the simple coordination problems treated here. In particular, they can coordinate several unit operations with recycle and units with certain types of non-convex utility functions. With a slight modification to the pricing, we can guarantee that slack resource auctions will always coordinate units with concave objectives and convex feasible regions. The idea is this: augment the amount normally charged each unit, \(p|T_1-T_2|\), with a small quadratic term, \(q(T_1-T_2)^2\), where \(q\) is some small positive constant. The existence of an optimal resource price vector can be demonstrated (see Jose, 1997) by casting this augmented slack resource auction as a nonlinear complementarity problem:

\[
\text{Find } p \text{ such that:} \\
-\Delta(p; q) \geq 0 \\
p^T(-\Delta(p; q)) = 0
\]

where \(\Delta(p; q)\) is the excess demand vector and \(p\) is the slack price vector.

The price adjustment mechanism we have described here is very simple. Viewed from a control perspective, we are essentially using PI controllers to drive the demand for slack resources to the level of their supply. This works well for simple examples but can be slow for larger systems. Faster convergence may be obtained by viewing the price adjustment procedure as following a homotopy path along some implicitly defined demand curve towards a point where the excess demand is zero. With appropriate normalization of the resource prices, standard fixed-point algorithms might also prove useful in adjusting price vectors in a more sophisticated and more efficient manner.

**Conclusion**

If the plantwide objective is to maximize profit and if it is necessary to decompose a chemical plant into pieces, it is natural to consider price-based penalty methods for coordinating the units. Standard Lagrangean-based approaches are ineffective not only because the Lagrange multipliers lack meaningful interpretation as prices, but also because they fail to coordinate simple, but common, systems. Defining meaningful resources suggests slack resource auctions as an alternative which yield meaningful prices, and, more importantly, are able to coordinate systems for which Lagrangean auctions are unsuitable.

Although the focus on coordination was motivated by the difficulty of solving plantwide optimization problems centrally, there are other benefits. Prices from a slack resource auction provide plant managers with useful information regarding the “hot spots” or bottlenecks in a plant. Slack prices not only identify bottlenecks, but they also give a measure of their severity, or equivalently, a measure of how much the system as a whole would benefit by allowing more slack there. For instance, when dealing with stream temperatures, slack resource prices can help
answer the question, “How much would we benefit by adding a heat exchanger there?” Seeing where these bottlenecks are and being able to evaluate their severity are important tools for connecting the operations and management layers in chemical plants.

References


