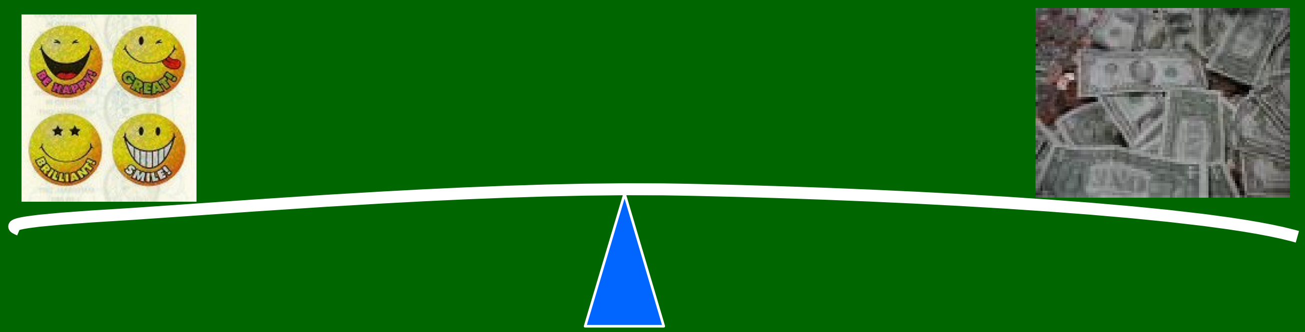
Quantifying the Tension Between Efficiency and Cost Recovery



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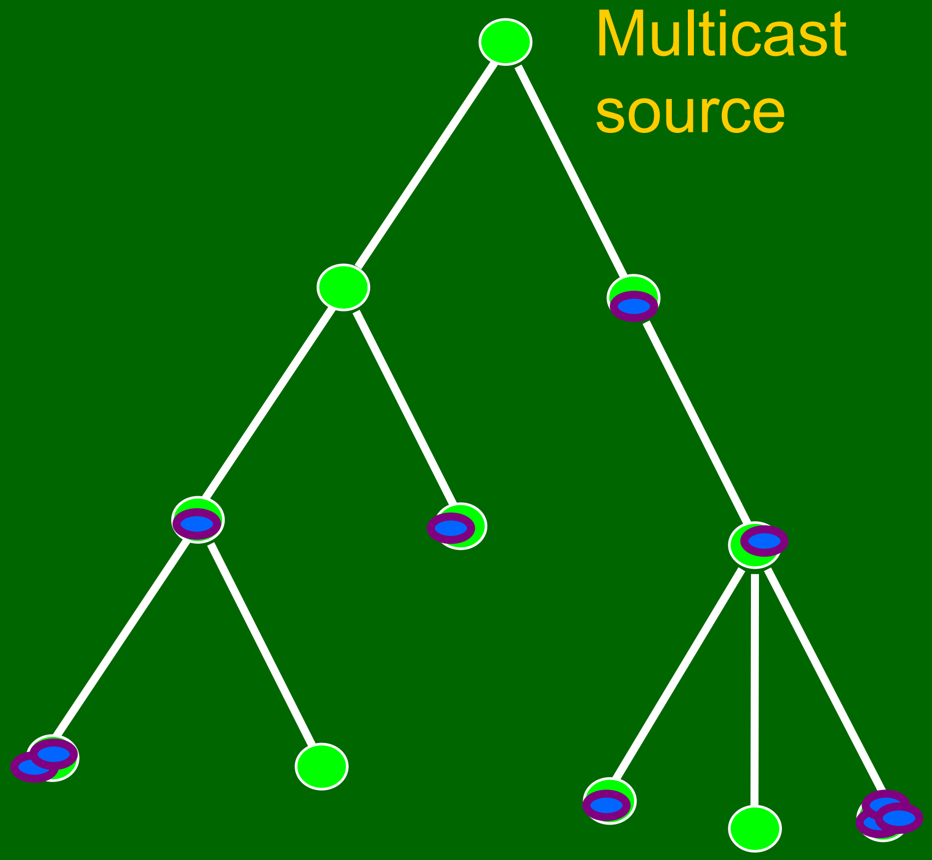
The Multicast Routing (MCR) Mechanism Design Problem



Terms:

1. Valuations
2. Bids
3. Allocation
4. Payments
5. Cost of Allocation
6. Utility

Results by People in This Group

- Feigenbaum- Papadimitriou-Shenker
  - Communication Complexity of fixed tree case
  - Optimizing efficiency in the graph case is NP Hard
- Archer-Feigenbaum-Krishnamurthy-Sami-Shenker
  - Lower bounds on communication complexity
- Archer-Feigenbaum- Sami-Shenker
  - collusion, trade cost recovery for comm. comp.
- Mitchell-Teague
  - Strategic node model

Our Mechanism Design Desiderata

- Maximize Efficiency
  - Valuation(S) - Cost(S)
- Balance Budget (BB)
  - $C(S) \geq$ total charges $\geq \alpha\, C(S)$
- Maintain Incentive properties
  - Strategyproofness
  - Groupstrategyproofness

Can we have it all?

The Economists Said,

- "Can't achieve budget balance, efficiency and truthfulness simultaneously"
  - Green-Laffont, Roberts
- So, two types of mechanisms were designed:

#1

Efficient

Strategyproof

#2

BB

Groupstrategyproof

The Computer Scientists Said,

- "Why don't we approximate?"
- "Can't approximate efficiency and budget balance, while maintaining truthfulness"
  - [Feigenbaum- Krishnamurthy- Sami- Shenker 02]

Two Possibilities

Can't approximate simultaneously because:

3. Fundamental tension between BB and efficiency

Or

2. Artifact of modeling assumption

We Reformulate Efficiency

- Old definition: Maximize Efficiency
  - Valuation(S) - Cost(S)
- If Valuation(OPT) - Cost(OPT) ≈ 0
  - Then approximating efficiency forces us to optimize it
- New definition: Minimize inefficiency
  - Valuation(U-S) + Cost(S)
  - Sum of excluded valuations + cost
  - Standard model, [e.g. see GW 95]

This Allows Us to Formally…

1. Quantify the efficiency of mechanisms that were known to be budget balanced
2. Quantify the tension between these objectives
3. Demonstrate that we can trade cost recovery for efficiency
4. Indicate how to design BB mechanisms for efficiency

#1: Quantify the Efficiency of Well Known BB Mechanisms

- Shapley for fixed tree multicast is ln(k) efficient
- Shapley for submodular cost functions is ln(k) efficient
- Jain-Vazirani cost sharing for the Steiner tree game is $\ln^2(k)$ factor efficient
  - Matches lower bound of all known methods

#2: Quantifying the Tension

- How efficient can you get if you want to recover an α ($\alpha<1$) fraction of the cost?
- At best, max $(1/\alpha, \alpha \ln(k))$ efficient for submodular cost functions
- Similar results for all known Steiner tree methods

#3: Trading the Objectives

- Can we improve efficiency by sacrificing cost-recovery?
- Subsidize: run mechanism with scaled down costs
- Caveat: subsidizing indefinitely does not help
  - See previous slide

#4: Clean Characterization of the Efficiency of Cost Sharing

- Clean, closed-form characterization of the loss of efficiency of BB cost-sharing methods
  - Necessary and sufficient
- Indicates how we can design B.B. Mechanisms for efficiency

Current Work [1]

- BB, groupstrategyproof mechanisms exist for a variety of applications
- Facility Location, Steiner forest, Set Cover …
- How efficient are these?

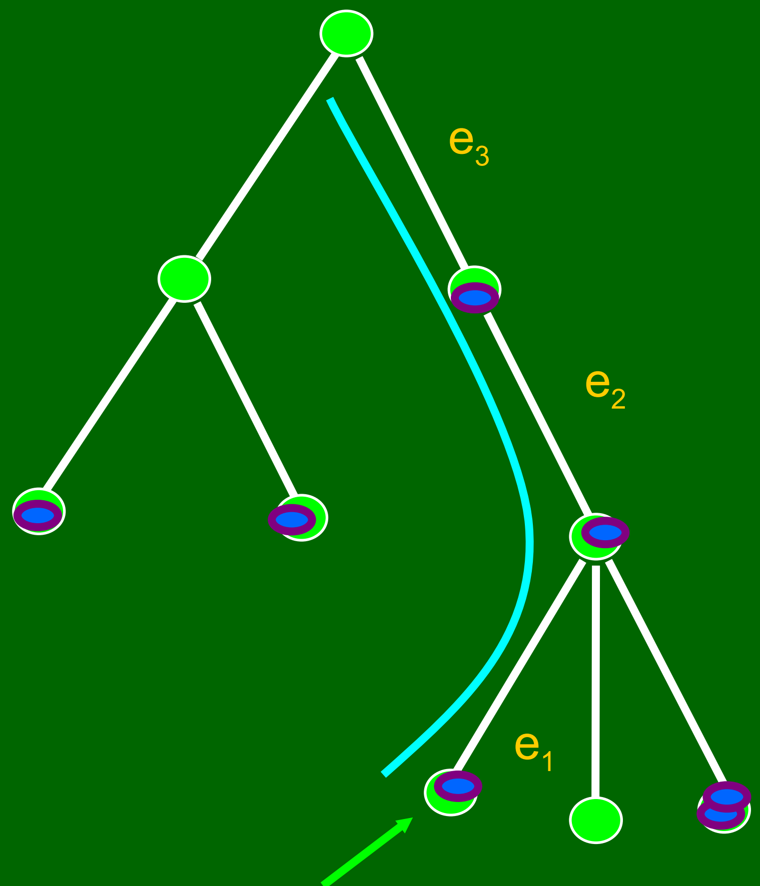
Current Work [2]

- Shapley has bad n/w complexity [Feigenbaum- Krishnamurthy- Sami- Shenker]
- We can tradeoff n/w complexity for BB [Archer –Feigenbaum- Krishnamurthy – Sami – Shenker]

Can we tradeoff n\w complexity v\s BB v\s efficiency?

Techniques

Shapley Cost Sharing

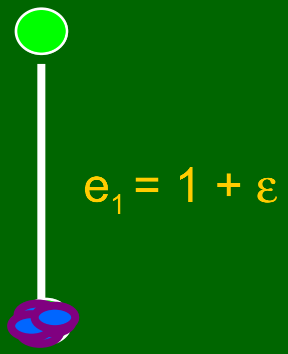


Cost share = $c(e_1) + c(e_2)/4 + c(e_3)/5$

Shapley Carving Mechanism -SCM

- Accept bids
- Let $S_0 = U$
- Repeat iteratively
  - If all players in S_i can pay shares, stop
  - Delete a player p, $bid_p < Sh(p, S_i)$
    - $S_{i+1} = S_i \setminus \{p\}$
- Serve S_f, charge players Shapley shares

Shapley Is Not Optimal

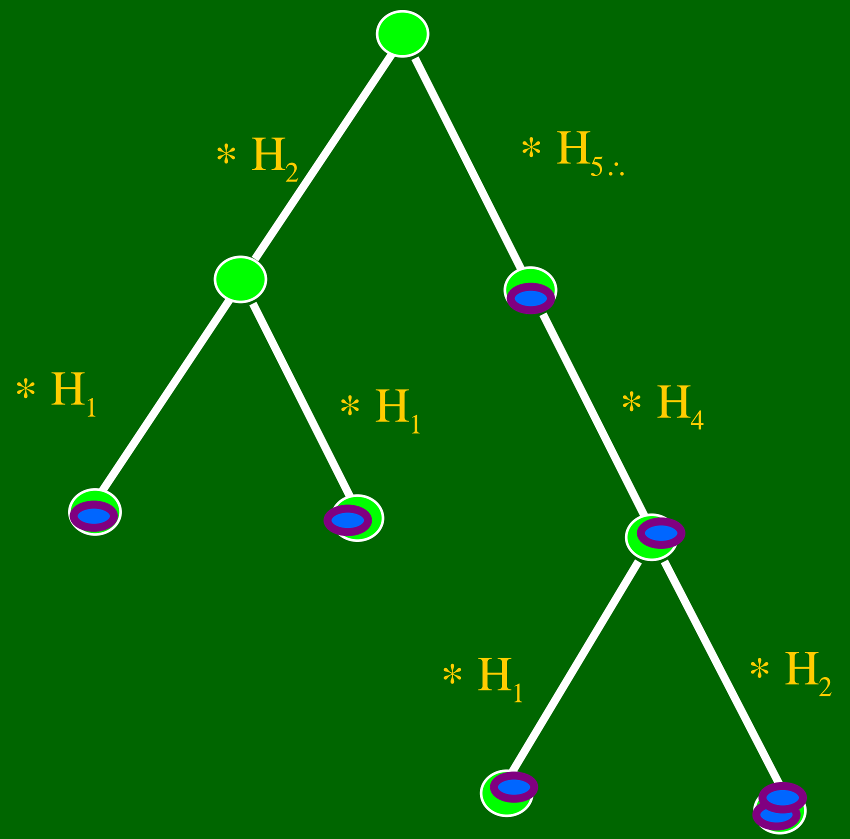


k players with valuations :1,1/2, 1/3, … , 1/k

Efficiency: C(S) + Valuations(U\S)

Potential Function (P) [Hart, Mas-Colell 89]

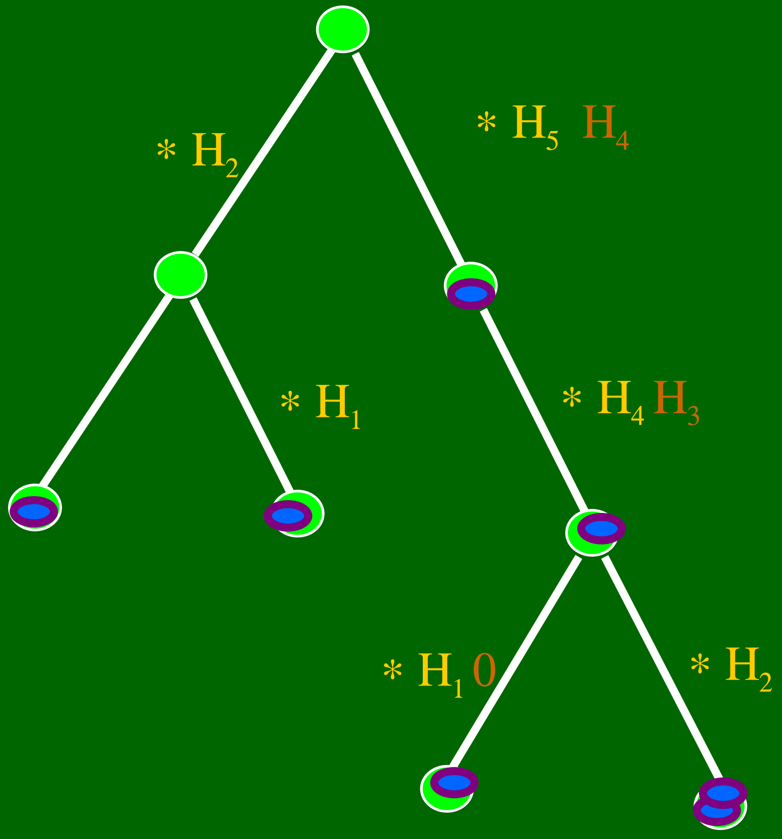
H_i harmonic number



Modified objective: P(S) + Valuations(U\S)

Modified objective v/s inefficiency?

What Does Carving Do to the Modified Objective?



$P(S_i)$ Falls , Valuations$(U\backslash S_i)$ rises

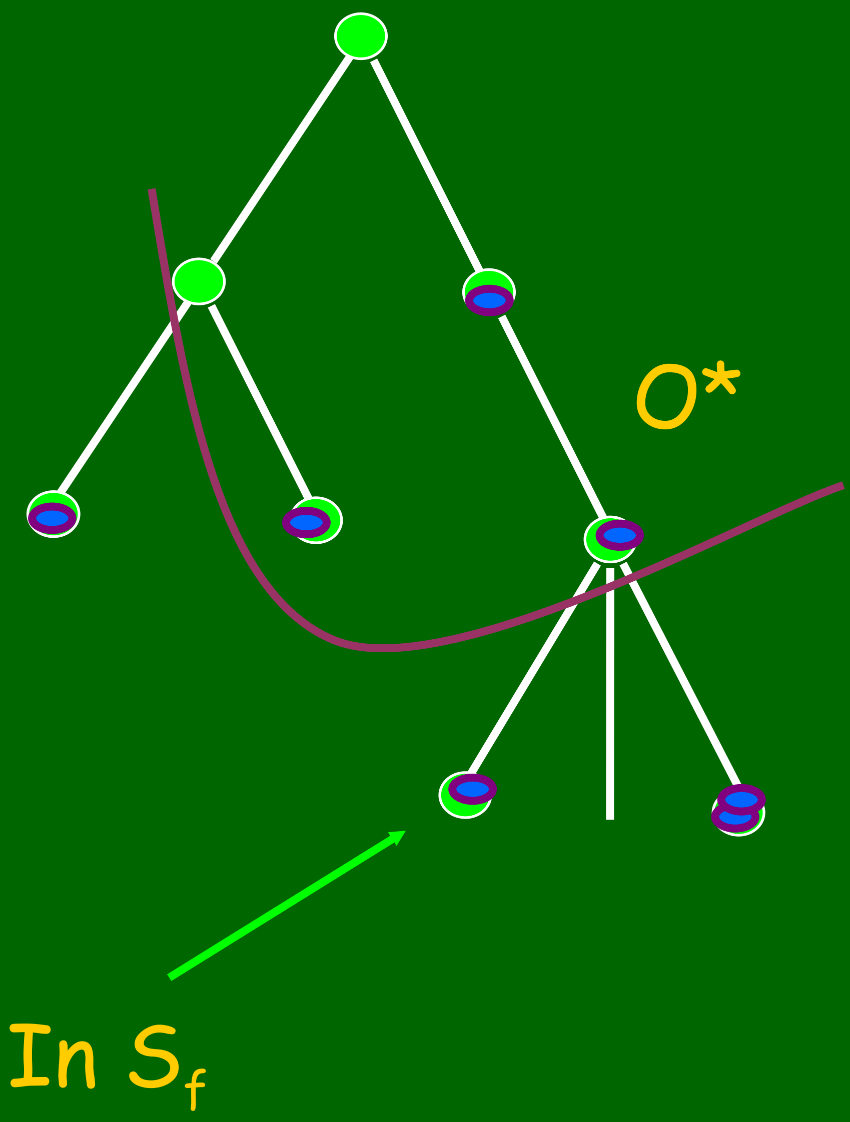
Fall $\geq$ Rise

Modified objective: $P(S)$ + Valuations$(U\backslash S)$

What Does Carving .... Contd.

- Carving performs descent on modified objective
- Finds local minima
- We would have been happy with a global minima
- Fortunately,

S_f Is a Subset of O*



Issues α-subsidized Shapley

- Subset property breaks
- But, players in $S_f \setminus O^*$ pay a α fraction of the cost.
- Charge these players to the objective separately
- Potential > Cost breaks too!
- Lose an additive term here.

Issues With Steiner Tree

- JV is not a potential game
- Big Problem!
- Idea: distill property of Hart Mas-Colell Potential
- α-Summability of cost-shares over all deletion orderings
- JV is $\ln^2 k$ summable!