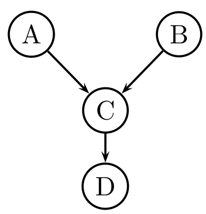
CIS620 MIDTERM – GRAPHICAL MODELS – SPRING 2009

NAME: ....

1. I-Equivalence

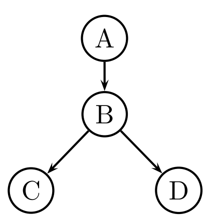
1.1. **Y-structure.** Consider the 4-node BN below. Does it have other BNs I-equivalent to it?

If yes, draw all of them.



1.2. **λ-structure.** Consider the 4-node BN below. Does it have other BNs I-equivalent to it?

If yes, draw all of them.



2. One given all

We want to compute the distribution over a variable given some assignment to all the other variables in the network, without constructing the whole joint. Specifically, we want the conditional probability: $P(X_1|x_2,\ldots,x_n)$. Describe the computation needed and give its complexity for BNs and MNs, assuming all nodes have k values:

2.1. **BN.** For a Bayes net $P(X) = \prod_{i=1}^{n} P(X_i|Pa_{X_i})$. Express your complexity in terms of k and network complexity, e.g., the number of variables (n), maximum number of parents (p) or children (c).

2.2. **MN.** For a Markov net $P(X) = \frac{1}{Z}\prod_{j=1}^{m} \phi_j(C_j)$. Express your complexity in terms of k and network complexity, e.g., the number of variables (n), the number of cliques (m), maximum number of variables in a clique (c) or maximum number of cliques containing a variable (v).

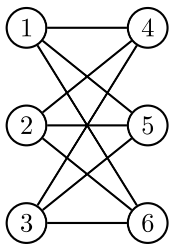
3. Moral Equivalence

Let G be a Bayes net structure and H be a Markov net structure over $\mathcal{X}$ such the undirected skeleton of G is H. Show that if G has no immoralities, then $I(G) = I(H)$.

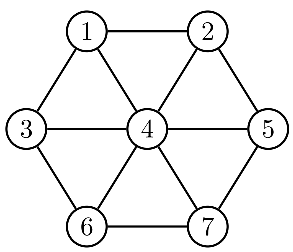
4. Chordality

For each graph below, state if the graph is chordal. If not, name a chordless cycle and add the smallest number of edges to triangulate the graph.

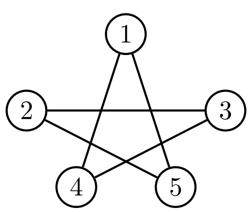
4.1. $K_{3,3}$. Complete bipartite graph:



4.2. **Hex.** Hexagonal graph:

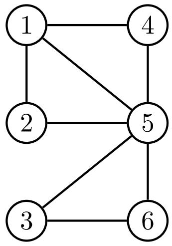


4.3. **Star.** Five-star graph:



5. Chordal Graph to Junction Tree

For the chordal graph below, list all maximal cliques and draw *all* valid junction trees on these maximal cliques.



6. Every Pair

Suppose that the junction tree T over $\mathcal{X}$ is such that for every pair $X, Y \in \mathcal{X}$, T has a clique that contains X, Y. Show that T is a single clique.