

# Datalog and Negation

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CIS 700: Advanced Topics in Databases

MW 1:30-3

Towne 309

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## Homework for this week

- Sign up to present a paper (the Google doc [link](#) was sent on Friday)
- Class [schedule](#) is being updated based on this.
- Think about a course [project](#), and come speak with me about it. You should send me a written proposal by Feb. 9.



## Last time: Datalog<sup>+</sup> with recursion

- Evaluating non-recursive IDB predicates: ordering
- Naïve and semi-naïve evaluation of recursive predicates
- Negation can be tricky...



# Example: Recursion and Negation

EDB:

$\text{red}(X,Y)$  = There is a red bus from  $X$  to  $Y$

$\text{green}(X,Y)$  = There is a green bus from  $X$  to  $Y$



IDB:

$\text{greenPath}(X,Y)$  = You can get from  $X$  to  $Y$  using only green buses.

$\text{monopoly}(X,Y)$  = Red has a bus from  $X$  to  $Y$ , but you can't get there on Green, even changing buses.



# Example: Recursion and Negation

EDB:

red(X,Y)= There is a red bus from X to Y

green(X,Y)= There is a green bus from X to Y



IDB:

greenPath(X,Y) :- green(X,Y)

greenPath(X,Y) :- greenPath(X,Z) , greenPath(Z,Y)

monopoly(X,Y) :- red(X,Y) , NOT greenPath(X,Y)



## Two Minimal Models

1. EDB + greenPath(1,2) + monopoly(2,3)
2. EDB + greenPath(1,2) + greenPath(2,3) + greenPath(1,3)

greenPath(X,Y) :- green(X,Y)

greenPath(X,Y) :- greenPath(X,Z), greenPath(Z,Y)

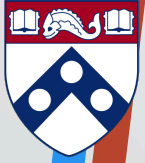
monopoly(X,Y) :- red(X,Y), NOT greenPath(X,Y)





# Stratified Models

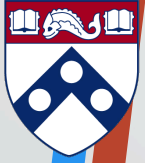
1. *Dependency graph* describes how IDB predicates depend negatively on each other.
2. *Stratified Datalog* = no recursion involving negation.
3. *Stratified model* is a particular model that “makes sense” for stratified Datalog programs.



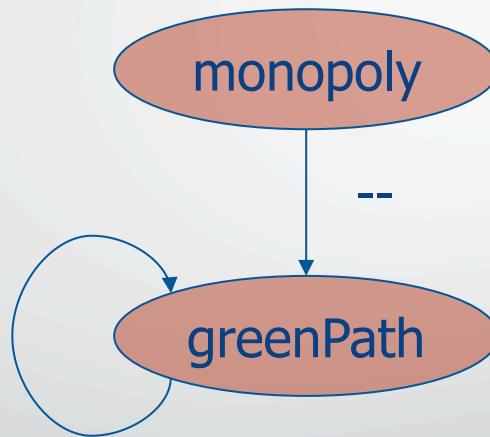
# Dependency Graph

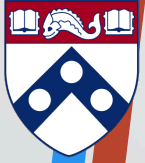
- Nodes = IDB predicates.
- Arc  $p \rightarrow q$  iff there is a rule for  $p$  that has a subgoal with predicate  $q$ .
- Arc  $p \rightarrow q$  labeled “-” iff there is a subgoal with predicate  $q$  that is negated.





# Monopoly Example





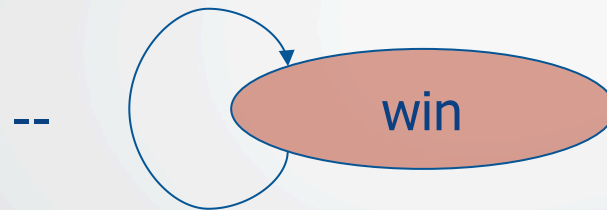
## Another Example: “Win”

`win(X) :- move(X,Y), NOT win(Y)`

- Represents games where you win by forcing your opponent to a position where they have no move.



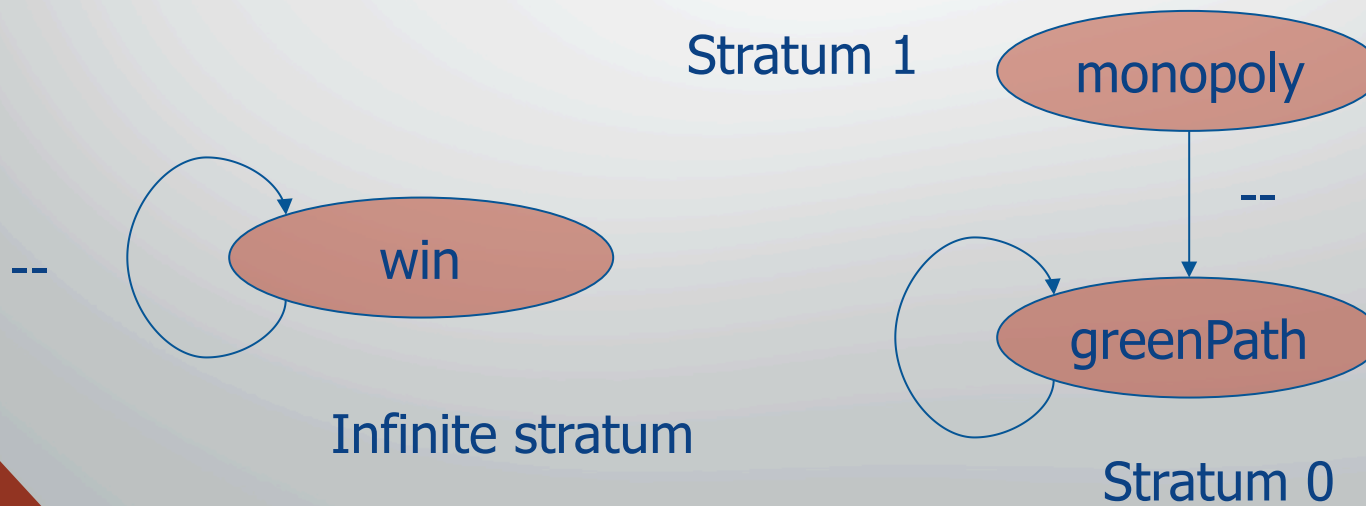
# Dependency Graph for “Win”

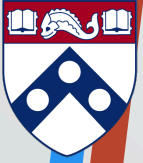




# Strata

- The stratum of an IDB predicate is the largest number of –'s on a path from that predicate, in the dependency graph.
- Examples:





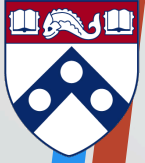
# Stratified Programs

- If all IDB predicates have finite strata, then the Datalog program is *stratified*.
- If any IDB predicate has an infinite stratum, then the program is *unstratified*, and no stratified model exists.



# Stratified Model

- Evaluate strata  $0, 1, \dots$  in order.
- If the program is stratified, then any negated IDB subgoal has already had its relation evaluated.
  - Safety assures that we can “subtract it from something.”
  - Treat it as EDB.
- Result is the stratified model.



# Examples

- For “Monopoly,” greenPath is in stratum 0: compute it (the transitive closure of green).
- Then, monopoly is in stratum 1: compute it by taking the difference of red and greenPath.
- Result is first model proposed.
- “Win” is not stratified, thus no stratified model.



## Putting it all together...

$p(x,y):- q(x,y), \text{ NOT } r(x,y)$   
 $r(x):- s(x,y), \text{ not } t(y)$   
 $r(x):- s(x,y), r(y)$

EDB predicates are  $q, t, s$ .

$Q=\{ab, bc, cd, de\}$

$S=\{ab, bc, ac\}$

$T=\{a, b, c\}$

- What are the strata of the IDB predicates  $p$  and  $r$ ?
- What is the translation of the rules for  $p$  and  $r$  into relational algebra?





# Summary

- Datalog captures the core of many query languages:
  - Unions of conjunctive queries
  - Recursion
  - Negation
- The syntax for conjunctive queries is used for a number of different practical problems
  - Query optimization
  - Data integration
  - Data citation
  - ...