Noninterference Theorem for Dependency Core Calculus (DCC)

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This report is automatically generated by the tool `lf2tex` from the semantics specification `ni.elf` (Twelf source) and the syntactic specification `ni.tex` (LaTeX source)[1].

Contents

[1] Last update: April 7, 2005

$$\frac{\ell \preceq \mathtt{t}_2}{\ell \preceq (\mathtt{t}_1 \rightarrow \mathtt{t}_2)} \quad \text{(pt-fun)}$$

$$\frac{\ell_2 \preceq \mathtt{t}}{\ell_2 \preceq (\mathtt{t}[\ell_1])} \quad \text{(pt-lab1)}$$

$$\frac{\ell_2 \preceq \ell_1}{\ell_2 \preceq (\mathtt{t}[\ell_1])} \quad \text{(pt-lab2)}$$

$$\boxed{\Gamma \vdash \mathtt{e} : \mathtt{t}} \quad \boxed{\text{ty}}$$

$$\Gamma \vdash \mathtt{true} : \mathtt{bool} \quad \text{(ty-true)}$$

$$\Gamma \vdash \mathtt{false} : \mathtt{bool} \quad \text{(ty-false)}$$

$$\frac{\Gamma \vdash \mathtt{e}_1 : \mathtt{bool} \quad \Gamma \vdash \mathtt{e}_2 : \mathtt{t} \quad \Gamma \vdash \mathtt{e}_3 : \mathtt{t}}{\Gamma \vdash (\mathtt{if}\ \mathtt{e}_1\ \mathtt{e}_2\ \mathtt{e}_3) : \mathtt{t}} \quad \text{(ty-if)}$$

$$(\Gamma, \mathtt{x}{:}\mathtt{t}) \vdash (\mathtt{x}) : \mathtt{t} \quad \text{(ty-var1)}$$

$$\frac{\Gamma \vdash (\mathtt{x}_2) : \mathtt{t}_2}{(\Gamma, \mathtt{x}_1{:}\mathtt{t}_1) \vdash (\mathtt{x}_2) : \mathtt{t}_2} \quad \text{(ty-var2)}$$

$$\frac{(\Gamma, \mathtt{x}{:}\mathtt{t}_1) \vdash \mathtt{e} : \mathtt{t}_2}{\Gamma \vdash (\lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}) : (\mathtt{t}_1 \rightarrow \mathtt{t}_2)} \quad \text{(ty-fun)}$$

$$\frac{\Gamma \vdash \mathtt{e}_1 : (\mathtt{t}_1 \rightarrow \mathtt{t}_2) \quad \Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1}{\Gamma \vdash (\mathtt{e}_1\ \mathtt{e}_2) : \mathtt{t}_2} \quad \text{(ty-app)}$$

$$\frac{\Gamma \vdash \mathtt{e} : \mathtt{t}}{\Gamma \vdash (\mathtt{e}[\ell]) : (\mathtt{t}[\ell])} \quad \text{(ty-lab)}$$

$$\frac{\Gamma \vdash \mathtt{e}_1 : (\mathtt{t}_1[\ell]) \quad (\Gamma, \mathtt{x}{:}\mathtt{t}_1) \vdash \mathtt{e}_2 : \mathtt{t}_2 \quad \ell \preceq \mathtt{t}_2}{\Gamma \vdash (\mathtt{bind}\ \mathtt{x} = \mathtt{e}_1\ \mathtt{in}\ \mathtt{e}_2) : \mathtt{t}_2} \quad \text{(ty-bind)}$$

$$\boxed{\mathtt{e}[\mathtt{e}/\mathtt{x}] = \mathtt{e}} \quad \boxed{\text{sub}}$$

$$\mathtt{true}[\mathtt{e}/\mathtt{x}] = \mathtt{true} \quad \text{(sub-true)}$$

$$\mathtt{false}[\mathtt{e}/\mathtt{x}] = \mathtt{false} \quad \text{(sub-false)}$$

$$\frac{\mathtt{e}_1[\mathtt{e}/\mathtt{x}] = \mathtt{e}_4 \quad \mathtt{e}_2[\mathtt{e}/\mathtt{x}] = \mathtt{e}_5 \quad \mathtt{e}_3[\mathtt{e}/\mathtt{x}] = \mathtt{e}_6}{(\mathtt{if}\ \mathtt{e}_1\ \mathtt{e}_2\ \mathtt{e}_3)[\mathtt{e}/\mathtt{x}] = (\mathtt{if}\ \mathtt{e}_4\ \mathtt{e}_5\ \mathtt{e}_6)} \quad \text{(sub-if)}$$

$$(\mathtt{x})[\mathtt{e}/\mathtt{x}] = \mathtt{e} \quad \text{(sub-var1)}$$

$$(\mathtt{x}_1)[\mathtt{e}/\mathtt{x}_2] = (\mathtt{x}_1) \quad \text{(sub-var2)}$$

$$\frac{\mathtt{e}_1[\mathtt{e}/\mathtt{x}_2] = \mathtt{e}_2}{(\lambda \mathtt{x}_1{:}\mathtt{t}.\mathtt{e}_1)[\mathtt{e}/\mathtt{x}_2] = (\lambda \mathtt{x}_1{:}\mathtt{t}.\mathtt{e}_2)} \quad \text{(sub-fun)}$$

$$\frac{\mathtt{e}_1[\mathtt{e}/\mathtt{x}] = \mathtt{e}_3 \quad \mathtt{e}_2[\mathtt{e}/\mathtt{x}] = \mathtt{e}_4}{(\mathtt{e}_1\ \mathtt{e}_2)[\mathtt{e}/\mathtt{x}] = (\mathtt{e}_3\ \mathtt{e}_4)} \quad \text{(sub-app)}$$

$$\frac{\mathtt{e}_1[\mathtt{e}/\mathtt{x}] = \mathtt{e}_2}{(\mathtt{e}_1[\ell])[\mathtt{e}/\mathtt{x}] = (\mathtt{e}_2[\ell])} \quad \text{(sub-lab)}$$

$$\frac{\mathtt{e}_1[\mathtt{e}/\mathtt{x}_2] = \mathtt{e}_3 \quad \mathtt{e}_2[\mathtt{e}/\mathtt{x}_2] = \mathtt{e}_4}{(\mathtt{bind}\ \mathtt{x}_1 = \mathtt{e}_1\ \mathtt{in}\ \mathtt{e}_2)[\mathtt{e}/\mathtt{x}_2] = (\mathtt{bind}\ \mathtt{x}_1 = \mathtt{e}_3\ \mathtt{in}\ \mathtt{e}_4)} \quad \text{(sub-bind)}$$

$$\boxed{\mathtt{e} \Downarrow \mathtt{e}} \qquad \boxed{\text{ev}}$$

$$\mathtt{true} \Downarrow \mathtt{true} \tag{ev-true}$$

$$\mathtt{false} \Downarrow \mathtt{false} \tag{ev-false}$$

$$\frac{\mathtt{e}_1 \Downarrow \mathtt{true} \quad \mathtt{e}_2 \Downarrow \mathtt{e}_4}{(\mathtt{if}\ \mathtt{e}_1\ \mathtt{e}_2\ \mathtt{e}_3) \Downarrow \mathtt{e}_4} \tag{ev-if1}$$

$$\frac{\mathtt{e}_1 \Downarrow \mathtt{false} \quad \mathtt{e}_3 \Downarrow \mathtt{e}_4}{(\mathtt{if}\ \mathtt{e}_1\ \mathtt{e}_2\ \mathtt{e}_3) \Downarrow \mathtt{e}_4} \tag{ev-if2}$$

$$(\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}) \Downarrow (\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}) \tag{ev-fun}$$

$$\frac{\mathtt{e}_1 \Downarrow (\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}_3) \quad \mathtt{e}_2 \Downarrow \mathtt{e}_4 \quad \mathtt{e}_3[\mathtt{e}_4/\mathtt{x}] = \mathtt{e}_5 \quad \mathtt{e}_5 \Downarrow \mathtt{e}_6}{(\mathtt{e}_1\ \mathtt{e}_2) \Downarrow \mathtt{e}_6} \tag{ev-app}$$

$$\frac{\mathtt{e}_1 \Downarrow \mathtt{e}_2}{(\mathtt{e}_1[\ell]) \Downarrow (\mathtt{e}_2[\ell])} \tag{ev-lab}$$

$$\frac{\mathtt{e}_1 \Downarrow (\mathtt{e}_3[\ell]) \quad \mathtt{e}_2[\mathtt{e}_3/\mathtt{x}] = \mathtt{e}_4 \quad \mathtt{e}_4 \Downarrow \mathtt{e}_5}{(\mathtt{bind}\ \mathtt{x} = \mathtt{e}_1\ \mathtt{in}\ \mathtt{e}_2) \Downarrow \mathtt{e}_5} \tag{ev-bind}$$

$$\boxed{\mathtt{val}\ \mathtt{e}} \qquad \boxed{\text{val}}$$

$$\mathtt{val}\ \mathtt{true} \tag{val-true}$$

$$\mathtt{val}\ \mathtt{false} \tag{val-false}$$

$$\mathtt{val}\ (\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}) \tag{val-fun}$$

$$\frac{\mathtt{val}\ \mathtt{e}}{\mathtt{val}\ (\mathtt{e}[\ell])} \tag{val-lab}$$

$$\boxed{\gamma ::= \cdots} \qquad \boxed{\text{h}}$$

$$\gamma ::= \cdot \tag{hz}$$

$$\gamma ::= (\gamma, \mathtt{x} \mapsto \mathtt{e}) \tag{hx}$$

$$\boxed{\gamma(\mathtt{e}) = \mathtt{e}} \qquad \boxed{\text{subh}}$$

$$\cdot(\mathtt{e}) = \mathtt{e} \tag{subh-z}$$

$$\gamma(\mathtt{true}) = \mathtt{true} \tag{subh-true}$$

$$\gamma(\mathtt{false}) = \mathtt{false} \tag{subh-false}$$

$$\frac{\gamma(\mathtt{e}_1) = \mathtt{e}_4 \quad \gamma(\mathtt{e}_2) = \mathtt{e}_5 \quad \gamma(\mathtt{e}_3) = \mathtt{e}_6}{\gamma((\mathtt{if}\ \mathtt{e}_1\ \mathtt{e}_2\ \mathtt{e}_3)) = (\mathtt{if}\ \mathtt{e}_4\ \mathtt{e}_5\ \mathtt{e}_6)} \tag{subh-if}$$

$$((\gamma, \mathtt{x} \mapsto \mathtt{e}))((\mathtt{x})) = \mathtt{e} \tag{subh-var1}$$

$$\frac{\gamma((\mathtt{x}_2)) = \mathtt{e}_2}{((\gamma, \mathtt{x}_1 \mapsto \mathtt{e}_1))((\mathtt{x}_2)) = \mathtt{e}_2} \tag{subh-var2}$$

$$\frac{\gamma(\mathtt{e}_1) = \mathtt{e}_2}{\gamma((\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}_1)) = (\lambda \mathtt{x}:\mathtt{t}.\mathtt{e}_2)} \tag{subh-fun}$$

$$\frac{\gamma(\mathtt{e}_1) = \mathtt{e}_3 \quad \gamma(\mathtt{e}_2) = \mathtt{e}_4}{\gamma((\mathtt{e}_1\ \mathtt{e}_2)) = (\mathtt{e}_3\ \mathtt{e}_4)} \tag{subh-app}$$

$$\frac{\gamma(e_1) = e_2}{\gamma((e_1[\ell])) = (e_2[\ell])} \quad \text{(subh-lab)}$$

$$\frac{\gamma(e_1) = e_3 \quad \gamma(e_2) = e_4}{\gamma((\texttt{bind } x = e_1 \texttt{ in } e_2)) = (\texttt{bind } x = e_3 \texttt{ in } e_4)} \quad \text{(subh-bind)}$$

$$\boxed{\frac{((\gamma, x_1 \mapsto e_1))((x_2)) = e_2}{e_2[e_1/x_1] = e_2}} \quad \boxed{\text{subhneq}}$$

$$\boxed{\gamma \models \Gamma} \quad \boxed{\text{mod}}$$

$$\cdot \models \cdot \quad \text{(mod-z)}$$

$$\frac{\gamma \models \Gamma \quad \cdot \vdash e : t}{((\gamma, x \mapsto e)) \models (\Gamma, x{:}t)} \quad \text{(mod-x)}$$

$$\boxed{\zeta \equiv \ell} \quad \boxed{\text{z}}$$

$$\boxed{e \sim_\zeta e : t} \quad \boxed{\text{qv}}$$

$$\boxed{e \approx_\zeta e : t} \quad \boxed{\text{qe}}$$

$$\boxed{\forall (e \sim_\zeta e : t).\ (e\ e) \approx_\zeta (e\ e) : t} \quad \boxed{\text{qx}}$$

$$\boxed{\frac{}{e_1 \sim_\zeta e_2 : t}} \quad \boxed{\text{assume}}$$

$$\boxed{\frac{\forall (e_3 \sim_\zeta e_4 : t_1).\ (e_1\ e_3) \approx_\zeta (e_2\ e_4) : t_2 \quad e_3 \sim_\zeta e_4 : t_1}{(e_1\ e_3) \approx_\zeta (e_2\ e_4) : t_2}} \quad \boxed{\text{apply}}$$

$$\boxed{\frac{\texttt{assume } e_3 \sim_\zeta e_4 : t_1 \quad (e_1\ e_3) \approx_\zeta (e_2\ e_4) : t}{\forall (e_3 \sim_\zeta e_4 : t_1).\ (e_1\ e_3) \approx_\zeta (e_2\ e_4) : t_2}} \quad \boxed{\text{discharge}}$$

$$\texttt{true} \sim_\zeta \texttt{true} : \texttt{bool} \quad \text{(qv-true)}$$

$$\texttt{false} \sim_\zeta \texttt{false} : \texttt{bool} \quad \text{(qv-false)}$$

$$\frac{\texttt{val } e_1 \quad \texttt{val } e_2 \quad \cdot \vdash e_1 : (t_1 \to t_2) \quad \cdot \vdash e_2 : (t_1 \to t_2) \quad \forall (e_3 \sim_\zeta e_4 : t_1).\ (e_1\ e_3) \approx_\zeta (e_2\ e_4) : t_2}{e_1 \sim_\zeta e_2 : (t_1 \to t_2)} \quad \text{(qv-fun)}$$

$$\frac{e_1 \sim_\zeta e_2 : t}{(e_1[\ell]) \sim_\zeta (e_2[\ell]) : (t[\ell])} \quad \text{(qv-lab1)}$$

$$\frac{\texttt{val } e_1 \quad \texttt{val } e_2 \quad \cdot \vdash e_1 : (t[\ell]) \quad \cdot \vdash e_2 : (t[\ell]) \quad \ell \not\preceq \zeta}{e_1 \sim_\zeta e_2 : (t[\ell])} \quad \text{(qv-lab2)}$$

$$\frac{e_1 \Downarrow e_3 \quad e_2 \Downarrow e_4 \quad e_3 \sim_\zeta e_4 : t}{e_1 \approx_\zeta e_2 : t} \quad \text{(qe-ev)}$$

$$\boxed{\gamma \sim_\zeta \gamma : \Gamma} \quad \boxed{\text{qh}}$$

$$\cdot \sim_\zeta \cdot : \cdot \quad \text{(qhz)}$$

$$\frac{\gamma_1 \sim_\zeta \gamma_2 : \Gamma \quad e_1 \sim_\zeta e_2 : t}{((\gamma_1, x \mapsto e_1)) \sim_\zeta ((\gamma_2, x \mapsto e_2)) : (\Gamma, x{:}t)} \quad \text{(qhx)}$$

$$\boxed{\frac{}{e_1[e/x] = e_2}} \quad \boxed{\text{sub-total}}$$

$$\boxed{\frac{}{\gamma(e_1) = e_2}}$$ subh-total

$$\boxed{\frac{}{\texttt{assume } e_1 \sim_\ell e_2 : t}}$$ assume-total

$$\boxed{\frac{((\gamma, x \mapsto e_1))(e_2) = e_3 \quad \gamma(e_2) = e_4}{e_4[e_1/x] = e_3}}$$ subhsub

(subhsub-true)

1:	$((\gamma, x \mapsto e))(\texttt{true}) = \texttt{true}$	*given
2:	$\gamma(\texttt{true}) = \texttt{true}$	*given
3:	$\texttt{true}[e/x] = \texttt{true}$	sub-true

(subhsub-false)

1:	$((\gamma, x \mapsto e))(\texttt{false}) = \texttt{false}$	*given
2:	$\gamma(\texttt{false}) = \texttt{false}$	*given
3:	$\texttt{false}[e/x] = \texttt{false}$	sub-false

(subhsub-if)

1:	$((\gamma, x \mapsto e_1))((\texttt{if } e_8\ e_5\ e_2)) = (\texttt{if } e_9\ e_6\ e_3)$	*given
2:	$((\gamma, x \mapsto e_1))(e_8) = e_9$	↓subh-if: 1
3:	$\gamma((\texttt{if } e_8\ e_5\ e_2)) = (\texttt{if } e_{10}\ e_7\ e_4)$	*given
4:	$\gamma(e_8) = e_{10}$	↓subh-if: 3
5:	$e_{10}[e_1/x] = e_9$	subhsub: 2,4
6:	$((\gamma, x \mapsto e_1))(e_5) = e_6$	↓subh-if: 1
7:	$\gamma(e_5) = e_7$	↓subh-if: 3
8:	$e_7[e_1/x] = e_6$	subhsub: 6,7
9:	$((\gamma, x \mapsto e_1))(e_2) = e_3$	↓subh-if: 1
10:	$\gamma(e_2) = e_4$	↓subh-if: 3
11:	$e_4[e_1/x] = e_3$	subhsub: 9,10
12:	$(\texttt{if } e_{10}\ e_7\ e_4)[e_1/x] = (\texttt{if } e_9\ e_6\ e_3)$	sub-if: 5,8,11

(subhsub-var1)

1:	$((\gamma, x \mapsto e))((x)) = e$	*given
2:	$\gamma((x)) = (x)$	*given
3:	$(x)[e/x] = e$	sub-var1

(subhsub-var2)

1:	$((\gamma, x_1 \mapsto e_1))((x_2)) = e_2$	*given
2:	$\gamma((x_2)) = e_2$	↓subh-var2: 1
3:	$e_2[e_1/x_1] = e_2$	subhneq: 1

(subhsub-fun)

1:	$((\gamma, x_1 \mapsto e_1))((\lambda x_2 : t.e_2)) = (\lambda x_2 : t.e_3)$	*given
2:	$((\gamma, x_1 \mapsto e_1))(e_2) = e_3$	↓subh-fun: 1
3:	$\gamma((\lambda x_2 : t.e_2)) = (\lambda x_2 : t.e_4)$	*given
4:	$\gamma(e_2) = e_4$	↓subh-fun: 3
5:	$e_4[e_1/x_1] = e_3$	subhsub: 2,4
6:	$(\lambda x_2 : t.e_4)[e_1/x_1] = (\lambda x_2 : t.e_3)$	sub-fun: 5

(subhsub-app)

1:	$((\gamma, x \mapsto e_1))((e_5\ e_2)) = (e_6\ e_3)$	*given
2:	$((\gamma, x \mapsto e_1))(e_5) = e_6$	↓subh-app: 1
3:	$\gamma((e_5\ e_2)) = (e_7\ e_4)$	*given
4:	$\gamma(e_5) = e_7$	↓subh-app: 3
5:	$e_7[e_1/x] = e_6$	subhsub: 2,4

6 :	$((\gamma, x \mapsto e_1))(e_2) = e_3$	↓subh-app: 1
7 :	$\gamma(e_2) = e_4$	↓subh-app: 3
8 :	$e_4[e_1/x] = e_3$	subhsub: 6,7
9 :	$(e_7\ e_4)[e_1/x] = (e_6\ e_3)$	sub-app: 5,8

(subhsub-lab)

1 :	$((\gamma, x \mapsto e_1))((e_2[\ell])) = (e_3[\ell])$	*given
2 :	$((\gamma, x \mapsto e_1))(e_2) = e_3$	↓subh-lab: 1
3 :	$\gamma((e_2[\ell])) = (e_4[\ell])$	*given
4 :	$\gamma(e_2) = e_4$	↓subh-lab: 3
5 :	$e_4[e_1/x] = e_3$	subhsub: 2,4
6 :	$(e_4[\ell])[e_1/x] = (e_3[\ell])$	sub-lab: 5

(subhsub-bind)

1 :	$((\gamma, x_1 \mapsto e_1))((\mathtt{bind}\ x_2 = e_5\ \mathtt{in}\ e_2)) = (\mathtt{bind}\ x_2 = e_6\ \mathtt{in}\ e_3)$	*given
2 :	$((\gamma, x_1 \mapsto e_1))(e_5) = e_6$	↓subh-bind: 1
3 :	$\gamma((\mathtt{bind}\ x_2 = e_5\ \mathtt{in}\ e_2)) = (\mathtt{bind}\ x_2 = e_7\ \mathtt{in}\ e_4)$	*given
4 :	$\gamma(e_5) = e_7$	↓subh-bind: 3
5 :	$e_7[e_1/x_1] = e_6$	subhsub: 2,4
6 :	$((\gamma, x_1 \mapsto e_1))(e_2) = e_3$	↓subh-bind: 1
7 :	$\gamma(e_2) = e_4$	↓subh-bind: 3
8 :	$e_4[e_1/x_1] = e_3$	subhsub: 6,7
9 :	$(\mathtt{bind}\ x_2 = e_7\ \mathtt{in}\ e_4)[e_1/x_1] = (\mathtt{bind}\ x_2 = e_6\ \mathtt{in}\ e_3)$	sub-bind: 5,8

$$\frac{\gamma(e_1) = e_2 \quad e_2[e_3/x] = e_4}{((\gamma, x \mapsto e_3))(e_1) = e_4}$$

subsubh

(subsubh-true)

1 :	$\gamma(\mathtt{true}) = \mathtt{true}$	*given
2 :	$\mathtt{true}[e/x] = \mathtt{true}$	*given
3 :	$((\gamma, x \mapsto e))(\mathtt{true}) = \mathtt{true}$	subh-true

(subsubh-false)

1 :	$\gamma(\mathtt{false}) = \mathtt{false}$	*given
2 :	$\mathtt{false}[e/x] = \mathtt{false}$	*given
3 :	$((\gamma, x \mapsto e))(\mathtt{false}) = \mathtt{false}$	subh-false

(subsubh-if)

1 :	$\gamma((\mathtt{if}\ e_8\ e_5\ e_1)) = (\mathtt{if}\ e_9\ e_6\ e_2)$	*given
2 :	$\gamma(e_8) = e_9$	↓subh-if: 1
3 :	$(\mathtt{if}\ e_9\ e_6\ e_2)[e_3/x] = (\mathtt{if}\ e_{10}\ e_7\ e_4)$	*given
4 :	$e_9[e_3/x] = e_{10}$	↓sub-if: 3
5 :	$((\gamma, x \mapsto e_3))(e_8) = e_{10}$	subsubh: 2,4
6 :	$\gamma(e_5) = e_6$	↓subh-if: 1
7 :	$e_6[e_3/x] = e_7$	↓sub-if: 3
8 :	$((\gamma, x \mapsto e_3))(e_5) = e_7$	subsubh: 6,7
9 :	$\gamma(e_1) = e_2$	↓subh-if: 1
10 :	$e_2[e_3/x] = e_4$	↓sub-if: 3
11 :	$((\gamma, x \mapsto e_3))(e_1) = e_4$	subsubh: 9,10
12 :	$((\gamma, x \mapsto e_3))((\mathtt{if}\ e_8\ e_5\ e_1)) = (\mathtt{if}\ e_{10}\ e_7\ e_4)$	subh-if: 5,8,11

(subsubh-var1)

1 :	$\gamma((x)) = (x)$	*given
2 :	$(x)[e/x] = e$	*given
3 :	$((\gamma, x \mapsto e))((x)) = e$	subh-var1

(subsubh-var2)

1 :	$(x_2)[e/x_3] = (x_2)$	*given
2 :	$\gamma((x_1)) = (x_2)$	*given
3 :	$((\gamma, x_3 \mapsto e))((x_1)) = (x_2)$	subh-var2: 2

(subsubh-fun)

1 :	$\gamma((\lambda x_2 : t.e_1)) = (\lambda x_2 : t.e_2)$	*given
2 :	$\gamma(e_1) = e_2$	↓subh-fun: 1
3 :	$(\lambda x_2 : t.e_2)[e_3/x_1] = (\lambda x_2 : t.e_4)$	*given
4 :	$e_2[e_3/x_1] = e_4$	↓sub-fun: 3
5 :	$((\gamma, x_1 \mapsto e_3))(e_1) = e_4$	subsubh: 2,4
6 :	$((\gamma, x_1 \mapsto e_3))((\lambda x_2 : t.e_1)) = (\lambda x_2 : t.e_4)$	subh-fun: 5

(subsubh-app)

1 :	$\gamma((e_5\ e_1)) = (e_6\ e_2)$	*given
2 :	$\gamma(e_5) = e_6$	↓subh-app: 1
3 :	$(e_6\ e_2)[e_3/x] = (e_7\ e_4)$	*given
4 :	$e_6[e_3/x] = e_7$	↓sub-app: 3
5 :	$((\gamma, x \mapsto e_3))(e_5) = e_7$	subsubh: 2,4
6 :	$\gamma(e_1) = e_2$	↓subh-app: 1
7 :	$e_2[e_3/x] = e_4$	↓sub-app: 3
8 :	$((\gamma, x \mapsto e_3))(e_1) = e_4$	subsubh: 6,7
9 :	$((\gamma, x \mapsto e_3))((e_5\ e_1)) = (e_7\ e_4)$	subh-app: 5,8

(subsubh-lab)

1 :	$\gamma((e_1[\ell])) = (e_2[\ell])$	*given
2 :	$\gamma(e_1) = e_2$	↓subh-lab: 1
3 :	$(e_2[\ell])[e_3/x] = (e_4[\ell])$	*given
4 :	$e_2[e_3/x] = e_4$	↓sub-lab: 3
5 :	$((\gamma, x \mapsto e_3))(e_1) = e_4$	subsubh: 2,4
6 :	$((\gamma, x \mapsto e_3))((e_1[\ell])) = (e_4[\ell])$	subh-lab: 5

(subsubh-bind)

1 :	$\gamma((\texttt{bind}\ x_2 = e_5\ \texttt{in}\ e_1)) = (\texttt{bind}\ x_2 = e_6\ \texttt{in}\ e_2)$	*given
2 :	$\gamma(e_5) = e_6$	↓subh-bind: 1
3 :	$(\texttt{bind}\ x_2 = e_6\ \texttt{in}\ e_2)[e_3/x_1] = (\texttt{bind}\ x_2 = e_7\ \texttt{in}\ e_4)$	*given
4 :	$e_6[e_3/x_1] = e_7$	↓sub-bind: 3
5 :	$((\gamma, x_1 \mapsto e_3))(e_5) = e_7$	subsubh: 2,4
6 :	$\gamma(e_1) = e_2$	↓subh-bind: 1
7 :	$e_2[e_3/x_1] = e_4$	↓sub-bind: 3
8 :	$((\gamma, x_1 \mapsto e_3))(e_1) = e_4$	subsubh: 6,7
9 :	$((\gamma, x_1 \mapsto e_3))((\texttt{bind}\ x_2 = e_5\ \texttt{in}\ e_1)) = (\texttt{bind}\ x_2 = e_7\ \texttt{in}\ e_4)$	subh-bind: 5,8

$$\frac{\forall(e_3 \sim_\zeta e_4 : t_1).\ (e_1\ e_3) \approx_\zeta (e_2\ e_4) : t_2}{\forall(e_3 \sim_\zeta e_4 : t_1).\ (e_2\ e_3) \approx_\zeta (e_1\ e_4) : t_2} \quad \boxed{\text{qxsym}}$$

$$\frac{e_1 \sim_\zeta e_2 : t}{e_2 \sim_\zeta e_1 : t} \quad \boxed{\text{qvsym}}$$

(qvsym-true)

1 :	$\texttt{true} \sim_\ell \texttt{true} : \texttt{bool}$	*given

(qvsym-false)

1 :	$\texttt{false} \sim_\ell \texttt{false} : \texttt{bool}$	*given

(qvsym-fun)

1:	$e_1 \sim_\ell e_2 : (t_1 \to t_2)$	*given
2:	$\mathtt{val}\ e_2$	↓qv-fun: 1
3:	$\mathtt{val}\ e_1$	↓qv-fun: 1
4:	$\cdot \vdash e_2 : (t_1 \to t_2)$	↓qv-fun: 1
5:	$\cdot \vdash e_1 : (t_1 \to t_2)$	↓qv-fun: 1
6:	$\forall (e_3 \sim_\ell e_4 : t_1).\ (e_1\ e_3) \approx_\ell (e_2\ e_4) : t_2$	↓qv-fun: 1
7:	$\forall (e_3 \sim_\ell e_4 : t_1).\ (e_2\ e_3) \approx_\ell (e_1\ e_4) : t_2$	qxsym: 6
8:	$e_2 \sim_\ell e_1 : (t_1 \to t_2)$	qv-fun: 2,3,4,5,7

(qvsym-lab1)

1:	$(e_1[\ell_2]) \sim_{\ell_1} (e_2[\ell_2]) : (t[\ell_2])$	*given
2:	$e_1 \sim_{\ell_1} e_2 : t$	↓qv-lab1: 1
3:	$e_2 \sim_{\ell_1} e_1 : t$	qvsym: 2
4:	$(e_2[\ell_2]) \sim_{\ell_1} (e_1[\ell_2]) : (t[\ell_2])$	qv-lab1: 3

(qvsym-lab2)

1:	$e_1 \sim_{\ell_1} e_2 : (t[\ell_2])$	*given
2:	$\mathtt{val}\ e_2$	↓qv-lab2: 1
3:	$\mathtt{val}\ e_1$	↓qv-lab2: 1
4:	$\cdot \vdash e_2 : (t[\ell_2])$	↓qv-lab2: 1
5:	$\cdot \vdash e_1 : (t[\ell_2])$	↓qv-lab2: 1
6:	$\ell_2 \not\preceq \ell_1$	↓qv-lab2: 1
7:	$e_2 \sim_{\ell_1} e_1 : (t[\ell_2])$	qv-lab2: 2,3,4,5,6

$$\frac{e_1 \sim_\zeta e_2 : t}{\mathtt{val}\ e_1}$$ qvval1

(qvval1-true)

1:	$\mathtt{true} \sim_\ell \mathtt{true} : \mathtt{bool}$	*given
2:	$\mathtt{val}\ \mathtt{true}$	val-true

(qvval1-false)

1:	$\mathtt{false} \sim_\ell \mathtt{false} : \mathtt{bool}$	*given
2:	$\mathtt{val}\ \mathtt{false}$	val-false

(qvval1-fun)

1:	$(\lambda x{:}t_1.e_1) \sim_\ell e_2 : (t_2 \to t_3)$	*given
2:	$\mathtt{val}\ e_2$	↓qv-fun: 1
3:	$\cdot \vdash (\lambda x{:}t_1.e_1) : (t_2 \to t_3)$	↓qv-fun: 1
4:	$\cdot \vdash e_2 : (t_2 \to t_3)$	↓qv-fun: 1
5:	$\forall (e_3 \sim_\ell e_4 : t_2).\ ((\lambda x{:}t_1.e_1)\ e_3) \approx_\ell (e_2\ e_4) : t_3$	↓qv-fun: 1
6:	$\mathtt{val}\ (\lambda x{:}t_1.e_1)$	↓qv-fun: 1

(qvval1-labl1)

1:	$(e_1[\ell_2]) \sim_{\ell_1} (e_2[\ell_2]) : (t[\ell_2])$	*given
2:	$e_1 \sim_{\ell_1} e_2 : t$	↓qv-lab1: 1
3:	$\mathtt{val}\ e_1$	qvval1: 2
4:	$\mathtt{val}\ (e_1[\ell_2])$	val-lab: 3

(qvval1-labl2)

1:	$e_1 \sim_{\ell_1} e_2 : (t[\ell_2])$	*given
2:	$\mathtt{val}\ e_2$	↓qv-lab2: 1
3:	$\cdot \vdash e_1 : (t[\ell_2])$	↓qv-lab2: 1
4:	$\cdot \vdash e_2 : (t[\ell_2])$	↓qv-lab2: 1
5:	$\ell_2 \not\preceq \ell_1$	↓qv-lab2: 1
6:	$\mathtt{val}\ e_1$	↓qv-lab2: 1

$$\boxed{\frac{\mathtt{e}_1 \sim_{\zeta} \mathtt{e}_2 : \mathtt{t}}{\mathtt{val}\ \mathtt{e}_2}}$$ qvval2

(qvval2-z)

1:	$\mathtt{e}_2 \sim_{\ell} \mathtt{e}_1 : \mathtt{t}$	*given
2:	$\mathtt{e}_1 \sim_{\ell} \mathtt{e}_2 : \mathtt{t}$	qvsym: 1
3:	$\mathtt{val}\ \mathtt{e}_1$	qvval1: 2

$$\boxed{\frac{\mathtt{e}_1 \sim_{\zeta} \mathtt{e}_2 : \mathtt{t}}{\cdot \vdash \mathtt{e}_1 : \mathtt{t}}}$$ qvty1

(qvty1-true)

1:	$\mathtt{true} \sim_{\ell} \mathtt{true} : \mathtt{bool}$	*given
2:	$\cdot \vdash \mathtt{true} : \mathtt{bool}$	ty-true

(qvty1-false)

1:	$\mathtt{false} \sim_{\ell} \mathtt{false} : \mathtt{bool}$	*given
2:	$\cdot \vdash \mathtt{false} : \mathtt{bool}$	ty-false

(qvty1-fun)

1:	$\mathtt{e}_1 \sim_{\ell} \mathtt{e}_2 : (\mathtt{t}_1 \rightarrow \mathtt{t}_2)$	*given
2:	$\mathtt{val}\ \mathtt{e}_1$	↓qv-fun: 1
3:	$\mathtt{val}\ \mathtt{e}_2$	↓qv-fun: 1
4:	$\cdot \vdash \mathtt{e}_2 : (\mathtt{t}_1 \rightarrow \mathtt{t}_2)$	↓qv-fun: 1
5:	$\forall (\mathtt{e}_3 \sim_{\ell} \mathtt{e}_4 : \mathtt{t}_1).\, (\mathtt{e}_1\ \mathtt{e}_3) \approx_{\ell} (\mathtt{e}_2\ \mathtt{e}_4) : \mathtt{t}_2$	↓qv-fun: 1
6:	$\cdot \vdash \mathtt{e}_1 : (\mathtt{t}_1 \rightarrow \mathtt{t}_2)$	↓qv-fun: 1

(qv-labl1)

1:	$(\mathtt{e}_1[\ell_2]) \sim_{\ell_1} (\mathtt{e}_2[\ell_2]) : (\mathtt{t}[\ell_2])$	*given
2:	$\mathtt{e}_1 \sim_{\ell_1} \mathtt{e}_2 : \mathtt{t}$	↓qv-lab1: 1
3:	$\cdot \vdash \mathtt{e}_1 : \mathtt{t}$	qvty1: 2
4:	$\cdot \vdash (\mathtt{e}_1[\ell_2]) : (\mathtt{t}[\ell_2])$	ty-lab: 3

(qv-labl2)

1:	$\mathtt{e}_1 \sim_{\ell_1} \mathtt{e}_2 : (\mathtt{t}[\ell_2])$	*given
2:	$\mathtt{val}\ \mathtt{e}_1$	↓qv-lab2: 1
3:	$\mathtt{val}\ \mathtt{e}_2$	↓qv-lab2: 1
4:	$\cdot \vdash \mathtt{e}_2 : (\mathtt{t}[\ell_2])$	↓qv-lab2: 1
5:	$\ell_2 \not\sqsubseteq \ell_1$	↓qv-lab2: 1
6:	$\cdot \vdash \mathtt{e}_1 : (\mathtt{t}[\ell_2])$	↓qv-lab2: 1

$$\boxed{\frac{\mathtt{e}_1 \sim_{\zeta} \mathtt{e}_2 : \mathtt{t}}{\cdot \vdash \mathtt{e}_2 : \mathtt{t}}}$$ qvty2

(qvty2-z)

1:	$\mathtt{e}_2 \sim_{\ell} \mathtt{e}_1 : \mathtt{t}$	*given
2:	$\mathtt{e}_1 \sim_{\ell} \mathtt{e}_2 : \mathtt{t}$	qvsym: 1
3:	$\cdot \vdash \mathtt{e}_1 : \mathtt{t}$	qvty1: 2

$$\boxed{\Gamma - \Gamma = \Gamma}$$ gsub

$$\Gamma - \cdot = \Gamma \qquad \text{(gsub-z1)}$$

$$\Gamma - \Gamma = \cdot \qquad \text{(gsub-z2)}$$

$$\frac{\Gamma_1 - \Gamma_2 = \Gamma_3}{(\Gamma_1, \mathtt{x}{:}\mathtt{t}) - (\Gamma_2, \mathtt{x}{:}\mathtt{t}) = \Gamma_3} \quad \text{(gsub-x1)}$$

$$\frac{\Gamma_1 - (\Gamma_2, \mathtt{x}_2{:}\mathtt{t}) = \Gamma_3}{(\Gamma_1, \mathtt{x}_1{:}\mathtt{t}) - (\Gamma_2, \mathtt{x}_2{:}\mathtt{t}) = (\Gamma_3, \mathtt{x}_1{:}\mathtt{t})} \quad \text{(gsub-x2)}$$

$$\boxed{\frac{\Gamma_1 \vdash \mathtt{e}_1 : \mathtt{t} \quad \gamma \models \Gamma_2 \quad \Gamma_1 - \Gamma_2 = \Gamma_3 \quad \gamma(\mathtt{e}_1) = \mathtt{e}_2}{\Gamma_3 \vdash \mathtt{e}_2 : \mathtt{t}}} \quad \boxed{\text{tysubh}}$$

(tysubh-z)

1:	$\cdot \models \cdot$	*given
2:	$\Gamma - \cdot = \Gamma$	*given
3:	$\cdot(\mathtt{e}) = \mathtt{e}$	*given
4:	$\Gamma \vdash \mathtt{e} : \mathtt{t}$	*given

(tysubh-true)

1:	$\Gamma_1 \vdash \mathtt{true} : \mathtt{bool}$	*given
2:	$\gamma \models \Gamma_2$	*given
3:	$\Gamma_1 - \Gamma_2 = \Gamma_3$	*given
4:	$\gamma(\mathtt{true}) = \mathtt{true}$	*given
5:	$\Gamma_3 \vdash \mathtt{true} : \mathtt{bool}$	ty-true

(tysubh-false)

1:	$\Gamma_1 \vdash \mathtt{false} : \mathtt{bool}$	*given
2:	$\gamma \models \Gamma_2$	*given
3:	$\Gamma_1 - \Gamma_2 = \Gamma_3$	*given
4:	$\gamma(\mathtt{false}) = \mathtt{false}$	*given
5:	$\Gamma_3 \vdash \mathtt{false} : \mathtt{bool}$	ty-false

(tysubh-if)

1:	$\gamma((\mathtt{if}\ \mathtt{e}_3\ \mathtt{e}_1\ \mathtt{e}_1)) = (\mathtt{if}\ \mathtt{e}_4\ \mathtt{e}_2\ \mathtt{e}_2)$	*given
2:	$\gamma(\mathtt{e}_1) = \mathtt{e}_2$	↓subh-if: 1
3:	$\Gamma_1 \vdash (\mathtt{if}\ \mathtt{e}_3\ \mathtt{e}_1\ \mathtt{e}_1) : \mathtt{t}$	*given
4:	$\Gamma_1 \vdash \mathtt{e}_3 : \mathtt{bool}$	↓ty-if: 3
5:	$\gamma \models \Gamma_2$	*given
6:	$\Gamma_1 - \Gamma_2 = \Gamma_3$	*given
7:	$\gamma(\mathtt{e}_3) = \mathtt{e}_4$	↓subh-if: 1
8:	$\Gamma_3 \vdash \mathtt{e}_4 : \mathtt{bool}$	tysubh: 4,5,6,7
9:	$\Gamma_1 \vdash \mathtt{e}_1 : \mathtt{t}$	↓ty-if: 3
10:	$\gamma(\mathtt{e}_1) = \mathtt{e}_2$	↓subh-if: 1
11:	$\Gamma_3 \vdash \mathtt{e}_2 : \mathtt{t}$	tysubh: 9,5,6,10
12:	$\Gamma_1 \vdash \mathtt{e}_1 : \mathtt{t}$	↓ty-if: 3
13:	$\Gamma_3 \vdash \mathtt{e}_2 : \mathtt{t}$	tysubh: 12,5,6,10
14:	$\Gamma_3 \vdash (\mathtt{if}\ \mathtt{e}_4\ \mathtt{e}_2\ \mathtt{e}_2) : \mathtt{t}$	ty-if: 8,11,13

(tysubh-var1)

1:	$(\Gamma_1, \mathtt{x}{:}\mathtt{t}) \vdash (\mathtt{x}) : \mathtt{t}$	*given
2:	$((\gamma, \mathtt{x} \mapsto \mathtt{e})) \models (\Gamma_2, \mathtt{x}{:}\mathtt{t})$	*given
3:	$\gamma \models \Gamma_2$	↓mod-x: 2
4:	$(\Gamma_1, \mathtt{x}{:}\mathtt{t}) - (\Gamma_2, \mathtt{x}{:}\mathtt{t}) = \cdot$	*given
5:	$((\gamma, \mathtt{x} \mapsto \mathtt{e}))((\mathtt{x})) = \mathtt{e}$	*given
6:	$\cdot \vdash \mathtt{e} : \mathtt{t}$	↓mod-x: 2

(tysubh-var2)

1:	$((\gamma, \mathtt{x}_2 \mapsto \mathtt{e}_2)) \models (\Gamma_2, \mathtt{x}_2{:}\mathtt{t}_2)$	*given
2:	$\cdot \vdash \mathtt{e}_2 : \mathtt{t}_2$	↓mod-x: 1
3:	$(\Gamma_1, \mathtt{x}_2{:}\mathtt{t}_2) \vdash (\mathtt{x}_1) : \mathtt{t}_1$	*given
4:	$\Gamma_1 \vdash (\mathtt{x}_1) : \mathtt{t}_1$	↓ty-var2: 3

5 :	$\gamma \models \Gamma_2$	↓mod-x: 1
6 :	$(\Gamma_1, x_2 : t_2) - (\Gamma_2, x_2 : t_2) = \Gamma_3$	*given
7 :	$\Gamma_1 - \Gamma_2 = \Gamma_3$	↓gsub-x1: 6
8 :	$((\gamma, x_2 \mapsto e_2))((x_1)) = e_1$	*given
9 :	$\gamma((x_1)) = e_1$	↓subh-var2: 8
10 :	$\Gamma_3 \vdash e_1 : t_1$	tysubh: 4,5,7,9

(tysubh-fun)

1 :	$\Gamma_1 \vdash (\lambda x_1 : t_1 . e_1) : (t_1 \rightarrow t_2)$	*given
2 :	$(\Gamma_1, x_1 : t_1) \vdash e_1 : t_2$	↓ty-fun: 1
3 :	$\gamma \models (\Gamma_2, x_2 : t_1)$	*given
4 :	$\Gamma_1 - (\Gamma_2, x_2 : t_1) = \Gamma_3$	*given
5 :	$(\Gamma_1, x_1 : t_1) - (\Gamma_2, x_2 : t_1) = (\Gamma_3, x_1 : t_1)$	gsub-x2: 4
6 :	$\gamma((\lambda x_1 : t_1 . e_1)) = (\lambda x_1 : t_1 . e_2)$	*given
7 :	$\gamma(e_1) = e_2$	↓subh-fun: 6
8 :	$(\Gamma_3, x_1 : t_1) \vdash e_2 : t_2$	tysubh: 2,3,5,7
9 :	$\Gamma_3 \vdash (\lambda x_1 : t_1 . e_2) : (t_1 \rightarrow t_2)$	ty-fun: 8

(tysubh-app)

1 :	$\Gamma_1 \vdash (e_3\ e_1) : t_2$	*given
2 :	$\Gamma_1 \vdash e_3 : (t_1 \rightarrow t_2)$	↓ty-app: 1
3 :	$\gamma \models \Gamma_2$	*given
4 :	$\Gamma_1 - \Gamma_2 = \Gamma_3$	*given
5 :	$\gamma((e_3\ e_1)) = (e_4\ e_2)$	*given
6 :	$\gamma(e_3) = e_4$	↓subh-app: 5
7 :	$\Gamma_3 \vdash e_4 : (t_1 \rightarrow t_2)$	tysubh: 2,3,4,6
8 :	$\Gamma_1 \vdash e_1 : t_1$	↓ty-app: 1
9 :	$\gamma(e_1) = e_2$	↓subh-app: 5
10 :	$\Gamma_3 \vdash e_2 : t_1$	tysubh: 8,3,4,9
11 :	$\Gamma_3 \vdash (e_4\ e_2) : t_2$	ty-app: 7,10

(tysubh-lab)

1 :	$\Gamma_1 \vdash (e_1[\ell]) : (t[\ell])$	*given
2 :	$\Gamma_1 \vdash e_1 : t$	↓ty-lab: 1
3 :	$\gamma \models \Gamma_2$	*given
4 :	$\Gamma_1 - \Gamma_2 = \Gamma_3$	*given
5 :	$\gamma((e_1[\ell])) = (e_2[\ell])$	*given
6 :	$\gamma(e_1) = e_2$	↓subh-lab: 5
7 :	$\Gamma_3 \vdash e_2 : t$	tysubh: 2,3,4,6
8 :	$\Gamma_3 \vdash (e_2[\ell]) : (t[\ell])$	ty-lab: 7

(tysubh-bind)

1 :	$\Gamma_1 \vdash (\texttt{bind}\ x_1 = e_3\ \texttt{in}\ e_1) : t_2$	*given
2 :	$\Gamma_1 \vdash e_3 : (t_1[\ell])$	↓ty-bind: 1
3 :	$\gamma \models (\Gamma_2, x_2 : t_1)$	*given
4 :	$\Gamma_1 - (\Gamma_2, x_2 : t_1) = \Gamma_3$	*given
5 :	$\gamma((\texttt{bind}\ x_1 = e_3\ \texttt{in}\ e_1)) = (\texttt{bind}\ x_1 = e_4\ \texttt{in}\ e_2)$	*given
6 :	$\gamma(e_3) = e_4$	↓subh-bind: 5
7 :	$\Gamma_3 \vdash e_4 : (t_1[\ell])$	tysubh: 2,3,4,6
8 :	$(\Gamma_1, x_1 : t_1) \vdash e_1 : t_2$	↓ty-bind: 1
9 :	$(\Gamma_1, x_1 : t_1) - (\Gamma_2, x_2 : t_1) = (\Gamma_3, x_1 : t_1)$	gsub-x2: 4
10 :	$\gamma(e_1) = e_2$	↓subh-bind: 5
11 :	$(\Gamma_3, x_1 : t_1) \vdash e_2 : t_2$	tysubh: 8,3,9,10
12 :	$\ell \preceq t_2$	↓ty-bind: 1
13 :	$\Gamma_3 \vdash (\texttt{bind}\ x_1 = e_4\ \texttt{in}\ e_2) : t_2$	ty-bind: 7,11,12

$$\frac{\gamma_1 \sim_\zeta \gamma_2 : \Gamma}{\gamma_1 \models \Gamma}$$ qhmod1

(qhmod1-z)

1: $\cdot \sim_\ell \cdot : \cdot$ — *given
2: $\cdot \models \cdot$ — mod-z

(qhmod1-x)

1: $((\gamma_1, \mathtt{x} \mapsto \mathtt{e}_1)) \sim_\ell ((\gamma_2, \mathtt{x} \mapsto \mathtt{e}_2)) : (\Gamma, \mathtt{x}{:}\mathtt{t})$ — *given
2: $\gamma_1 \sim_\ell \gamma_2 : \Gamma$ — ↓qhx: 1
3: $\gamma_1 \models \Gamma$ — qhmod1: 2
4: $\mathtt{e}_1 \sim_\ell \mathtt{e}_2 : \mathtt{t}$ — ↓qhx: 1
5: $\cdot \vdash \mathtt{e}_1 : \mathtt{t}$ — qvty1: 4
6: $((\gamma_1, \mathtt{x} \mapsto \mathtt{e}_1)) \models (\Gamma, \mathtt{x}{:}\mathtt{t})$ — mod-x: 3,5

$$\frac{\gamma_1 \sim_\zeta \gamma_2 : \Gamma}{\gamma_2 \models \Gamma} \quad \boxed{\text{qhmod2}}$$

(qhmod2-z)

1: $\cdot \sim_\ell \cdot : \cdot$ — *given
2: $\cdot \models \cdot$ — mod-z

(qhmod2-x)

1: $((\gamma_1, \mathtt{x} \mapsto \mathtt{e}_1)) \sim_\ell ((\gamma_2, \mathtt{x} \mapsto \mathtt{e}_2)) : (\Gamma, \mathtt{x}{:}\mathtt{t})$ — *given
2: $\gamma_1 \sim_\ell \gamma_2 : \Gamma$ — ↓qhx: 1
3: $\gamma_2 \models \Gamma$ — qhmod2: 2
4: $\mathtt{e}_1 \sim_\ell \mathtt{e}_2 : \mathtt{t}$ — ↓qhx: 1
5: $\cdot \vdash \mathtt{e}_2 : \mathtt{t}$ — qvty2: 4
6: $((\gamma_2, \mathtt{x} \mapsto \mathtt{e}_2)) \models (\Gamma, \mathtt{x}{:}\mathtt{t})$ — mod-x: 3,5

$$\frac{\mathtt{e}_1 \Downarrow \mathtt{e}_2}{\mathtt{val}\ \mathtt{e}_2} \quad \boxed{\text{evval}}$$

(evval-true)

1: $\mathtt{true} \Downarrow \mathtt{true}$ — *given
2: $\mathtt{val}\ \mathtt{true}$ — val-true

(evval-false)

1: $\mathtt{false} \Downarrow \mathtt{false}$ — *given
2: $\mathtt{val}\ \mathtt{false}$ — val-false

(evval-fun)

1: $(\lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e}) \Downarrow (\lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e})$ — *given
2: $\mathtt{val}\ (\lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e})$ — val-fun

(evval-if1)

1: $(\mathtt{if}\ \mathtt{e}_3\ \mathtt{e}_1\ \mathtt{e}_4) \Downarrow \mathtt{e}_2$ — *given
2: $\mathtt{e}_3 \Downarrow \mathtt{true}$ — ↓ev-if1: 1
3: $\mathtt{e}_1 \Downarrow \mathtt{e}_2$ — ↓ev-if1: 1
4: $\mathtt{val}\ \mathtt{e}_2$ — evval: 3

(evval-if2)

1: $(\mathtt{if}\ \mathtt{e}_3\ \mathtt{e}_4\ \mathtt{e}_1) \Downarrow \mathtt{e}_2$ — *given
2: $\mathtt{e}_3 \Downarrow \mathtt{false}$ — ↓ev-if2: 1
3: $\mathtt{e}_1 \Downarrow \mathtt{e}_2$ — ↓ev-if2: 1
4: $\mathtt{val}\ \mathtt{e}_2$ — evval: 3

(evval-fun)

1 :	$(\lambda x{:}t.e) \Downarrow (\lambda x{:}t.e)$	*given
2 :	$\mathtt{val}\ (\lambda x{:}t.e)$	val-fun

(evval-app)

1 :	$(e_3\ e_4) \Downarrow e_2$	*given
2 :	$e_3 \Downarrow (\lambda x{:}t.e_5)$	↓ev-app: 1
3 :	$e_4 \Downarrow e_6$	↓ev-app: 1
4 :	$e_5[e_6/x] = e_1$	↓ev-app: 1
5 :	$e_1 \Downarrow e_2$	↓ev-app: 1
6 :	$\mathtt{val}\ e_2$	evval: 5

(evval-lab)

1 :	$(e_1[\ell]) \Downarrow (e_2[\ell])$	*given
2 :	$e_1 \Downarrow e_2$	↓ev-lab: 1
3 :	$\mathtt{val}\ e_2$	evval: 2
4 :	$\mathtt{val}\ (e_2[\ell])$	val-lab: 3

(evval-bind)

1 :	$(\mathtt{bind}\ x = e_3\ \mathtt{in}\ e_4) \Downarrow e_2$	*given
2 :	$e_3 \Downarrow (e_5[\ell])$	↓ev-bind: 1
3 :	$e_4[e_5/x] = e_1$	↓ev-bind: 1
4 :	$e_1 \Downarrow e_2$	↓ev-bind: 1
5 :	$\mathtt{val}\ e_2$	evval: 4

$$\frac{\mathtt{val}\ e}{e \Downarrow e} \qquad \boxed{\text{valev}}$$

(valev-true)

1 :	$\mathtt{val}\ \mathtt{true}$	*given
2 :	$\mathtt{true} \Downarrow \mathtt{true}$	ev-true

(valev-false)

1 :	$\mathtt{val}\ \mathtt{false}$	*given
2 :	$\mathtt{false} \Downarrow \mathtt{false}$	ev-false

(valev-fun)

1 :	$\mathtt{val}\ (\lambda x{:}t.e)$	*given
2 :	$(\lambda x{:}t.e) \Downarrow (\lambda x{:}t.e)$	ev-fun

(valev-lab)

1 :	$\mathtt{val}\ (e[\ell])$	*given
2 :	$\mathtt{val}\ e$	↓val-lab: 1
3 :	$e \Downarrow e$	valev: 2
4 :	$(e[\ell]) \Downarrow (e[\ell])$	ev-lab: 3

$$\frac{\cdot \vdash e_1 : t}{e_1 \Downarrow e_2} \qquad \boxed{\text{tyev}}$$

$$\frac{\cdot \vdash e_1 : t \quad e_1 \Downarrow e_2}{\cdot \vdash e_2 : t} \qquad \boxed{\text{evty}}$$

$$\frac{\ell_1 \preceq \ell_2 \quad \ell_1 \not\preceq \ell_3}{\ell_2 \not\preceq \ell_3} \qquad \boxed{\text{stnst}}$$

$$\boxed{\frac{\texttt{val}\ \texttt{e}_1 \quad \texttt{val}\ \texttt{e}_2 \quad \cdot \vdash \texttt{e}_1 : \texttt{t} \quad \cdot \vdash \texttt{e}_2 : \texttt{t} \quad \ell \preceq \texttt{t} \quad \ell \not\preceq \zeta}{\texttt{e}_1 \sim_\zeta \texttt{e}_2 : \texttt{t}}} \qquad \boxed{\text{nipt}}$$

(nipt-fun)

1 :	$\texttt{val}\ (\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3)$	*given
2 :	$\texttt{val}\ (\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4)$	*given
3 :	$\cdot \vdash (\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3) : (\texttt{t}_1 \to \texttt{t}_4)$	*given
4 :	$\cdot \vdash (\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4) : (\texttt{t}_1 \to \texttt{t}_4)$	*given
5 :	$\texttt{assume}\ \texttt{e}_1 \sim_{\ell_1} \texttt{e}_2 : \texttt{t}_1$	assume-total
6 :	$\texttt{e}_1 \sim_{\ell_1} \texttt{e}_2 : \texttt{t}_1$	↓assume: 5
7 :	$\cdot \vdash \texttt{e}_1 : \texttt{t}_1$	qvty1: 6
8 :	$\cdot \vdash ((\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3)\ \texttt{e}_1) : \texttt{t}_4$	ty-app: 3,7
9 :	$((\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3)\ \texttt{e}_1) \Downarrow \texttt{e}_5$	tyev: 8
10 :	$\cdot \vdash \texttt{e}_2 : \texttt{t}_1$	qvty2: 6
11 :	$\cdot \vdash ((\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4)\ \texttt{e}_2) : \texttt{t}_4$	ty-app: 4,10
12 :	$((\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4)\ \texttt{e}_2) \Downarrow \texttt{e}_6$	tyev: 11
13 :	$\texttt{val}\ \texttt{e}_5$	evval: 9
14 :	$\texttt{val}\ \texttt{e}_6$	evval: 12
15 :	$\cdot \vdash \texttt{e}_5 : \texttt{t}_4$	evty: 8,9
16 :	$\cdot \vdash \texttt{e}_6 : \texttt{t}_4$	evty: 11,12
17 :	$\ell_2 \preceq (\texttt{t}_1 \to \texttt{t}_4)$	*given
18 :	$\ell_2 \preceq \texttt{t}_4$	↓pt-fun: 17
19 :	$\ell_2 \not\preceq \ell_1$	*given
20 :	$\texttt{e}_5 \sim_{\ell_1} \texttt{e}_6 : \texttt{t}_4$	nipt: 13,14,15,16,18,19
21 :	$((\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3)\ \texttt{e}_1) \approx_{\ell_1} ((\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4)\ \texttt{e}_2) : \texttt{t}_4$	qe-ev: 9,12,20
22 :	$\forall (\texttt{e}_1 \sim_{\ell_1} \texttt{e}_2 : \texttt{t}_1).\ ((\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3)\ \texttt{e}_1) \approx_{\ell_1} ((\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4)\ \texttt{e}_2) : \texttt{t}_4$	discharge: 5,21
23 :	$(\lambda \texttt{x}_1 : \texttt{t}_2 . \texttt{e}_3) \sim_{\ell_1} (\lambda \texttt{x}_2 : \texttt{t}_3 . \texttt{e}_4) : (\texttt{t}_1 \to \texttt{t}_4)$	qv-fun: 1,2,3,4,22

(nipt-lab1)

1 :	$\texttt{val}\ (\texttt{e}_1[\ell_3])$	*given
2 :	$\texttt{val}\ \texttt{e}_1$	↓val-lab: 1
3 :	$\texttt{val}\ (\texttt{e}_2[\ell_3])$	*given
4 :	$\texttt{val}\ \texttt{e}_2$	↓val-lab: 3
5 :	$\cdot \vdash (\texttt{e}_1[\ell_3]) : (\texttt{t}[\ell_3])$	*given
6 :	$\cdot \vdash \texttt{e}_1 : \texttt{t}$	↓ty-lab: 5
7 :	$\cdot \vdash (\texttt{e}_2[\ell_3]) : (\texttt{t}[\ell_3])$	*given
8 :	$\cdot \vdash \texttt{e}_2 : \texttt{t}$	↓ty-lab: 7
9 :	$\ell_1 \preceq (\texttt{t}[\ell_3])$	*given
10 :	$\ell_1 \preceq \texttt{t}$	↓pt-lab1: 9
11 :	$\ell_1 \not\preceq \ell_2$	*given
12 :	$\texttt{e}_1 \sim_{\ell_2} \texttt{e}_2 : \texttt{t}$	nipt: 2,4,6,8,10,11
13 :	$(\texttt{e}_1[\ell_3]) \sim_{\ell_2} (\texttt{e}_2[\ell_3]) : (\texttt{t}[\ell_3])$	qv-lab1: 12

(nipt-lab2)

1 :	$\texttt{val}\ (\texttt{e}[\ell_4])$	*given
2 :	$\texttt{val}\ \texttt{e}$	↓val-lab: 1
3 :	$\texttt{val}\ (\texttt{e}[\ell_5])$	*given
4 :	$\texttt{val}\ \texttt{e}$	↓val-lab: 3
5 :	$\cdot \vdash (\texttt{e}[\ell_4]) : (\texttt{t}[\ell_2])$	*given
6 :	$\cdot \vdash (\texttt{e}[\ell_5]) : (\texttt{t}[\ell_2])$	*given
7 :	$\ell_1 \preceq (\texttt{t}[\ell_2])$	*given
8 :	$\ell_1 \preceq \ell_2$	↓pt-lab2: 7
9 :	$\ell_1 \not\preceq \ell_3$	*given
10 :	$\ell_2 \not\preceq \ell_3$	stnst: 8,9
11 :	$(\texttt{e}[\ell_4]) \sim_{\ell_3} (\texttt{e}[\ell_5]) : (\texttt{t}[\ell_2])$	qv-lab2: 1,3,5,6,10

$$\boxed{\frac{\Gamma \vdash \texttt{e} : \texttt{t} \quad \gamma_1 \sim_\zeta \gamma_2 : \Gamma \quad \gamma_1(\texttt{e}) = \texttt{e}_1 \quad \gamma_2(\texttt{e}) = \texttt{e}_2}{\texttt{e}_1 \approx_\zeta \texttt{e}_2 : \texttt{t}}} \qquad \boxed{\text{ni}}$$

(ni-true)

1 :	$\Gamma \vdash \texttt{true} : \texttt{bool}$	*given
2 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	*given
3 :	$\gamma_1(\texttt{true}) = \texttt{true}$	*given
4 :	$\gamma_2(\texttt{true}) = \texttt{true}$	*given
5 :	$\texttt{true} \Downarrow \texttt{true}$	ev-true
6 :	$\texttt{true} \sim_\ell \texttt{true} : \texttt{bool}$	qv-true
7 :	$\texttt{true} \approx_\ell \texttt{true} : \texttt{bool}$	qe-ev: 5,5,6

(ni-false)

1 :	$\Gamma \vdash \texttt{false} : \texttt{bool}$	*given
2 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	*given
3 :	$\gamma_1(\texttt{false}) = \texttt{false}$	*given
4 :	$\gamma_2(\texttt{false}) = \texttt{false}$	*given
5 :	$\texttt{false} \Downarrow \texttt{false}$	ev-false
6 :	$\texttt{false} \sim_\ell \texttt{false} : \texttt{bool}$	qv-false
7 :	$\texttt{false} \approx_\ell \texttt{false} : \texttt{bool}$	qe-ev: 5,5,6

(ni-var1)

1 :	$(\Gamma, \texttt{x}:\texttt{t}) \vdash (\texttt{x}) : \texttt{t}$	*given
2 :	$((\gamma_1, \texttt{x} \mapsto \texttt{e}_2)) \sim_\ell ((\gamma_2, \texttt{x} \mapsto \texttt{e}_1)) : (\Gamma, \texttt{x}:\texttt{t})$	*given
3 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	↓qhx: 2
4 :	$((\gamma_1, \texttt{x} \mapsto \texttt{e}_2))((\texttt{x})) = \texttt{e}_2$	*given
5 :	$((\gamma_2, \texttt{x} \mapsto \texttt{e}_1))((\texttt{x})) = \texttt{e}_1$	*given
6 :	$\texttt{e}_2 \sim_\ell \texttt{e}_1 : \texttt{t}$	↓qhx: 2
7 :	$\texttt{val}\ \texttt{e}_2$	qvval1: 6
8 :	$\texttt{e}_2 \Downarrow \texttt{e}_2$	valev: 7
9 :	$\texttt{val}\ \texttt{e}_1$	qvval2: 6
10 :	$\texttt{e}_1 \Downarrow \texttt{e}_1$	valev: 9
11 :	$\texttt{e}_2 \approx_\ell \texttt{e}_1 : \texttt{t}$	qe-ev: 8,10,6

(ni-var2)

1 :	$((\gamma_1, \texttt{x}_2 \mapsto \texttt{e}_3)) \sim_\ell ((\gamma_2, \texttt{x}_2 \mapsto \texttt{e}_4)) : (\Gamma, \texttt{x}_2:\texttt{t}_2)$	*given
2 :	$\texttt{e}_3 \sim_\ell \texttt{e}_4 : \texttt{t}_2$	↓qhx: 1
3 :	$(\Gamma, \texttt{x}_2:\texttt{t}_2) \vdash (\texttt{x}_1) : \texttt{t}_1$	*given
4 :	$\Gamma \vdash (\texttt{x}_1) : \texttt{t}_1$	↓ty-var2: 3
5 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	↓qhx: 1
6 :	$((\gamma_1, \texttt{x}_2 \mapsto \texttt{e}_3))((\texttt{x}_1)) = \texttt{e}_1$	*given
7 :	$\gamma_1((\texttt{x}_1)) = \texttt{e}_1$	↓subh-var2: 6
8 :	$((\gamma_2, \texttt{x}_2 \mapsto \texttt{e}_4))((\texttt{x}_1)) = \texttt{e}_2$	*given
9 :	$\gamma_2((\texttt{x}_1)) = \texttt{e}_2$	↓subh-var2: 8
10 :	$\texttt{e}_1 \approx_\ell \texttt{e}_2 : \texttt{t}_1$	ni: 4,5,7,9

(ni-if1)

1 :	$\Gamma \vdash (\texttt{if}\ \texttt{e}_6\ \texttt{e}_1\ \texttt{e}_9) : \texttt{t}$	*given
2 :	$\Gamma \vdash \texttt{e}_9 : \texttt{t}$	↓ty-if: 1
3 :	$\gamma_1((\texttt{if}\ \texttt{e}_6\ \texttt{e}_1\ \texttt{e}_9)) = (\texttt{if}\ \texttt{e}_7\ \texttt{e}_2\ \texttt{e}_{10})$	*given
4 :	$\gamma_1(\texttt{e}_9) = \texttt{e}_{10}$	↓subh-if: 3
5 :	$\gamma_2((\texttt{if}\ \texttt{e}_6\ \texttt{e}_1\ \texttt{e}_9)) = (\texttt{if}\ \texttt{e}_8\ \texttt{e}_3\ \texttt{e}_{11})$	*given
6 :	$\gamma_2(\texttt{e}_9) = \texttt{e}_{11}$	↓subh-if: 5
7 :	$\Gamma \vdash \texttt{e}_6 : \texttt{bool}$	↓ty-if: 1
8 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	*given
9 :	$\gamma_1(\texttt{e}_6) = \texttt{e}_7$	↓subh-if: 3
10 :	$\gamma_2(\texttt{e}_6) = \texttt{e}_8$	↓subh-if: 5
11 :	$\texttt{e}_7 \approx_\ell \texttt{e}_8 : \texttt{bool}$	ni: 7,8,9,10
12 :	$\texttt{true} \sim_\ell \texttt{true} : \texttt{bool}$	↓qe-ev: 11
13 :	$\texttt{e}_7 \Downarrow \texttt{true}$	↓qe-ev: 11
14 :	$\Gamma \vdash \texttt{e}_1 : \texttt{t}$	↓ty-if: 1
15 :	$\gamma_1(\texttt{e}_1) = \texttt{e}_2$	↓subh-if: 3
16 :	$\gamma_2(\texttt{e}_1) = \texttt{e}_3$	↓subh-if: 5
17 :	$\texttt{e}_2 \approx_\ell \texttt{e}_3 : \texttt{t}$	ni: 14,8,15,16

18 :	$e_2 \Downarrow e_4$	↓qe-ev: 17
19 :	$(\mathtt{if}\ e_7\ e_2\ e_{10}) \Downarrow e_4$	ev-if1: 13,18
20 :	$e_8 \Downarrow \mathtt{true}$	↓qe-ev: 11
21 :	$e_3 \Downarrow e_5$	↓qe-ev: 17
22 :	$(\mathtt{if}\ e_8\ e_3\ e_{11}) \Downarrow e_5$	ev-if1: 20,21
23 :	$e_4 \sim_\ell e_5 : t$	↓qe-ev: 17
24 :	$(\mathtt{if}\ e_7\ e_2\ e_{10}) \approx_\ell (\mathtt{if}\ e_8\ e_3\ e_{11}) : t$	qe-ev: 19,22,23

(ni-if2)

1 :	$\Gamma \vdash (\mathtt{if}\ e_6\ e_9\ e_1) : t$	*given
2 :	$\Gamma \vdash e_9 : t$	↓ty-if: 1
3 :	$\gamma_1((\mathtt{if}\ e_6\ e_9\ e_1)) = (\mathtt{if}\ e_7\ e_{10}\ e_2)$	*given
4 :	$\gamma_1(e_9) = e_{10}$	↓subh-if: 3
5 :	$\gamma_2((\mathtt{if}\ e_6\ e_9\ e_1)) = (\mathtt{if}\ e_8\ e_{11}\ e_3)$	*given
6 :	$\gamma_2(e_9) = e_{11}$	↓subh-if: 5
7 :	$\Gamma \vdash e_6 : \mathtt{bool}$	↓ty-if: 1
8 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	*given
9 :	$\gamma_1(e_6) = e_7$	↓subh-if: 3
10 :	$\gamma_2(e_6) = e_8$	↓subh-if: 5
11 :	$e_7 \approx_\ell e_8 : \mathtt{bool}$	ni: 7,8,9,10
12 :	$\mathtt{false} \sim_\ell \mathtt{false} : \mathtt{bool}$	↓qe-ev: 11
13 :	$e_7 \Downarrow \mathtt{false}$	↓qe-ev: 11
14 :	$\Gamma \vdash e_1 : t$	↓ty-if: 1
15 :	$\gamma_1(e_1) = e_2$	↓subh-if: 3
16 :	$\gamma_2(e_1) = e_3$	↓subh-if: 5
17 :	$e_2 \approx_\ell e_3 : t$	ni: 14,8,15,16
18 :	$e_2 \Downarrow e_4$	↓qe-ev: 17
19 :	$(\mathtt{if}\ e_7\ e_{10}\ e_2) \Downarrow e_4$	ev-if2: 13,18
20 :	$e_8 \Downarrow \mathtt{false}$	↓qe-ev: 11
21 :	$e_3 \Downarrow e_5$	↓qe-ev: 17
22 :	$(\mathtt{if}\ e_8\ e_{11}\ e_3) \Downarrow e_5$	ev-if2: 20,21
23 :	$e_4 \sim_\ell e_5 : t$	↓qe-ev: 17
24 :	$(\mathtt{if}\ e_7\ e_{10}\ e_2) \approx_\ell (\mathtt{if}\ e_8\ e_{11}\ e_3) : t$	qe-ev: 19,22,23

(ni-fun)

1 :	$(\lambda x{:}t_1.e_3) \Downarrow (\lambda x{:}t_1.e_3)$	ev-fun
2 :	$(\lambda x{:}t_1.e_4) \Downarrow (\lambda x{:}t_1.e_4)$	ev-fun
3 :	$\mathtt{val}\ (\lambda x{:}t_1.e_3)$	val-fun
4 :	$\mathtt{val}\ (\lambda x{:}t_1.e_4)$	val-fun
5 :	$\Gamma \vdash (\lambda x{:}t_1.e_9) : (t_1 \rightarrow t_2)$	*given
6 :	$\gamma_2 \sim_\ell \gamma_1 : \Gamma$	*given
7 :	$\gamma_2 \models \Gamma$	qhmod1: 6
8 :	$\Gamma - \Gamma = \cdot$	gsub-z2
9 :	$\gamma_2((\lambda x{:}t_1.e_9)) = (\lambda x{:}t_1.e_3)$	*given
10 :	$\cdot \vdash (\lambda x{:}t_1.e_3) : (t_1 \rightarrow t_2)$	tysubh: 5,7,8,9
11 :	$\gamma_1 \models \Gamma$	qhmod2: 6
12 :	$\gamma_1((\lambda x{:}t_1.e_9)) = (\lambda x{:}t_1.e_4)$	*given
13 :	$\cdot \vdash (\lambda x{:}t_1.e_4) : (t_1 \rightarrow t_2)$	tysubh: 5,11,8,12
14 :	$\mathtt{assume}\ e_1 \sim_\ell e_2 : t_1$	assume-total
15 :	$e_1 \sim_\ell e_2 : t_1$	↓assume: 14
16 :	$\mathtt{val}\ e_1$	qvval1: 15
17 :	$e_1 \Downarrow e_1$	valev: 16
18 :	$((\gamma_2, x \mapsto e_1))(e_9) = e_8$	subh-total
19 :	$\gamma_2(e_9) = e_3$	↓subh-fun: 9
20 :	$e_3[e_1/x] = e_8$	subhsub: 18,19
21 :	$(\Gamma, x{:}t_1) \vdash e_9 : t_2$	↓ty-fun: 5
22 :	$((\gamma_2, x \mapsto e_1)) \sim_\ell ((\gamma_1, x \mapsto e_2)) : (\Gamma, x{:}t_1)$	qhx: 6,15
23 :	$((\gamma_1, x \mapsto e_2))(e_9) = e_7$	subh-total
24 :	$e_8 \approx_\ell e_7 : t_2$	ni: 21,22,18,23
25 :	$e_8 \Downarrow e_5$	↓qe-ev: 24
26 :	$((\lambda x{:}t_1.e_3)\ e_1) \Downarrow e_5$	ev-app: 1,17,20,25
27 :	$\mathtt{val}\ e_2$	qvval2: 15
28 :	$e_2 \Downarrow e_2$	valev: 27

29 :	$\gamma_1(e_9) = e_4$	↓subh-fun: 12
30 :	$e_4[e_2/x] = e_7$	subhsub: 23,29
31 :	$e_7 \Downarrow e_6$	↓qe-ev: 24
32 :	$((\lambda x{:}t_1.e_4)\ e_2) \Downarrow e_6$	ev-app: 2,28,30,31
33 :	$e_5 \sim_\ell e_6 : t_2$	↓qe-ev: 24
34 :	$((\lambda x{:}t_1.e_3)\ e_1) \approx_\ell ((\lambda x{:}t_1.e_4)\ e_2) : t_2$	qe-ev: 26,32,33
35 :	$\forall(e_1 \sim_\ell e_2 : t_1).\ ((\lambda x{:}t_1.e_3)\ e_1) \approx_\ell ((\lambda x{:}t_1.e_4)\ e_2) : t_2$	discharge: 14,34
36 :	$(\lambda x{:}t_1.e_3) \sim_\ell (\lambda x{:}t_1.e_4) : (t_1 \to t_2)$	qv-fun: 3,4,10,13,35
37 :	$(\lambda x{:}t_1.e_3) \approx_\ell (\lambda x{:}t_1.e_4) : (t_1 \to t_2)$	qe-ev: 1,2,36

(ni-app)

1 :	$\Gamma \vdash (e_{12}\ e_9) : t_4$	*given
2 :	$\Gamma \vdash e_{12} : (t_3 \to t_4)$	↓ty-app: 1
3 :	$\gamma_1 \sim_\ell \gamma_2 : \Gamma$	*given
4 :	$\gamma_1((e_{12}\ e_9)) = (e_{13}\ e_{10})$	*given
5 :	$\gamma_1(e_{12}) = e_{13}$	↓subh-app: 4
6 :	$\gamma_2((e_{12}\ e_9)) = (e_{14}\ e_{11})$	*given
7 :	$\gamma_2(e_{12}) = e_{14}$	↓subh-app: 6
8 :	$e_{13} \approx_\ell e_{14} : (t_3 \to t_4)$	ni: 2,3,5,7
9 :	$(\lambda x_1{:}t_1.e_1) \sim_\ell (\lambda x_2{:}t_2.e_2) : (t_3 \to t_4)$	↓qe-ev: 8
10 :	$\forall(e_3 \sim_\ell e_4 : t_3).\ ((\lambda x_1{:}t_1.e_1)\ e_3) \approx_\ell ((\lambda x_2{:}t_2.e_2)\ e_4) : t_4$	↓qv-fun: 9
11 :	$\Gamma \vdash e_9 : t_3$	↓ty-app: 1
12 :	$\gamma_1(e_9) = e_{10}$	↓subh-app: 4
13 :	$\gamma_2(e_9) = e_{11}$	↓subh-app: 6
14 :	$e_{10} \approx_\ell e_{11} : t_3$	ni: 11,3,12,13
15 :	$e_3 \sim_\ell e_4 : t_3$	↓qe-ev: 14
16 :	$((\lambda x_1{:}t_1.e_1)\ e_3) \approx_\ell ((\lambda x_2{:}t_2.e_2)\ e_4) : t_4$	apply: 10,15
17 :	$((\lambda x_1{:}t_1.e_1)\ e_3) \Downarrow e_5$	↓qe-ev: 16
18 :	$(\lambda x_1{:}t_1.e_1) \Downarrow (\lambda x_1{:}t_6.e_1)$	↓ev-app: 17
19 :	$e_3 \Downarrow e_3$	↓ev-app: 17
20 :	$((\lambda x_2{:}t_2.e_2)\ e_4) \Downarrow e_6$	↓qe-ev: 16
21 :	$(\lambda x_2{:}t_2.e_2) \Downarrow (\lambda x_2{:}t_5.e_2)$	↓ev-app: 20
22 :	$e_4 \Downarrow e_4$	↓ev-app: 20
23 :	$\mathtt{val}\ (\lambda x_1{:}t_1.e_1)$	↓qv-fun: 9
24 :	$\mathtt{val}\ (\lambda x_2{:}t_2.e_2)$	↓qv-fun: 9
25 :	$\cdot \vdash (\lambda x_1{:}t_1.e_1) : (t_3 \to t_4)$	↓qv-fun: 9
26 :	$\cdot \vdash (\lambda x_2{:}t_2.e_2) : (t_3 \to t_4)$	↓qv-fun: 9
27 :	$e_{13} \Downarrow (\lambda x_1{:}t_1.e_1)$	↓qe-ev: 8
28 :	$e_{10} \Downarrow e_3$	↓qe-ev: 14
29 :	$e_1[e_3/x_1] = e_8$	↓ev-app: 17
30 :	$e_8 \Downarrow e_5$	↓ev-app: 17
31 :	$(e_{13}\ e_{10}) \Downarrow e_5$	ev-app: 27,28,29,30
32 :	$e_{14} \Downarrow (\lambda x_2{:}t_2.e_2)$	↓qe-ev: 8
33 :	$e_{11} \Downarrow e_4$	↓qe-ev: 14
34 :	$e_2[e_4/x_2] = e_7$	↓ev-app: 20
35 :	$e_7 \Downarrow e_6$	↓ev-app: 20
36 :	$(e_{14}\ e_{11}) \Downarrow e_6$	ev-app: 32,33,34,35
37 :	$e_5 \sim_\ell e_6 : t_4$	↓qe-ev: 16
38 :	$(e_{13}\ e_{10}) \approx_\ell (e_{14}\ e_{11}) : t_4$	qe-ev: 31,36,37

(ni-lab)

1 :	$\Gamma \vdash (e_1[\ell_2]) : (t[\ell_2])$	*given
2 :	$\Gamma \vdash e_1 : t$	↓ty-lab: 1
3 :	$\gamma_1 \sim_{\ell_1} \gamma_2 : \Gamma$	*given
4 :	$\gamma_1((e_1[\ell_2])) = (e_2[\ell_2])$	*given
5 :	$\gamma_1(e_1) = e_2$	↓subh-lab: 4
6 :	$\gamma_2((e_1[\ell_2])) = (e_3[\ell_2])$	*given
7 :	$\gamma_2(e_1) = e_3$	↓subh-lab: 6
8 :	$e_2 \approx_{\ell_1} e_3 : t$	ni: 2,3,5,7
9 :	$e_2 \Downarrow e_4$	↓qe-ev: 8
10 :	$(e_2[\ell_2]) \Downarrow (e_4[\ell_2])$	ev-lab: 9
11 :	$e_3 \Downarrow e_5$	↓qe-ev: 8
12 :	$(e_3[\ell_2]) \Downarrow (e_5[\ell_2])$	ev-lab: 11

13 :	$e_4 \sim_{\ell_1} e_5 : t$	↓qe-ev: 8
14 :	$(e_4[\ell_2]) \sim_{\ell_1} (e_5[\ell_2]) : (t[\ell_2])$	qv-lab1: 13
15 :	$(e_2[\ell_2]) \approx_{\ell_1} (e_3[\ell_2]) : (t[\ell_2])$	qe-ev: 10,12,14

(ni-bind1)

1 :	$\Gamma \vdash (\mathtt{bind}\ x = e_{10}\ \mathtt{in}\ e_1) : t_2$	*given
2 :	$\ell_2 \preceq t_2$	↓ty-bind: 1
3 :	$\Gamma \vdash e_{10} : (t_1[\ell_2])$	↓ty-bind: 1
4 :	$\gamma_1 \sim_{\ell_1} \gamma_2 : \Gamma$	*given
5 :	$\gamma_1((\mathtt{bind}\ x = e_{10}\ \mathtt{in}\ e_1)) = (\mathtt{bind}\ x = e_{11}\ \mathtt{in}\ e_9)$	*given
6 :	$\gamma_1(e_{10}) = e_{11}$	↓subh-bind: 5
7 :	$\gamma_2((\mathtt{bind}\ x = e_{10}\ \mathtt{in}\ e_1)) = (\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_8)$	*given
8 :	$\gamma_2(e_{10}) = e_{12}$	↓subh-bind: 7
9 :	$e_{11} \approx_{\ell_1} e_{12} : (t_1[\ell_2])$	ni: 3,4,6,8
10 :	$e_{11} \Downarrow (e_2[\ell_2])$	↓qe-ev: 9
11 :	$((\gamma_1, x \mapsto e_2))(e_1) = e_4$	subh-total
12 :	$\gamma_1(e_1) = e_9$	↓subh-bind: 5
13 :	$e_9[e_2/x] = e_4$	subhsub: 11,12
14 :	$(\Gamma, x{:}t_1) \vdash e_1 : t_2$	↓ty-bind: 1
15 :	$(e_2[\ell_2]) \sim_{\ell_1} (e_3[\ell_2]) : (t_1[\ell_2])$	↓qe-ev: 9
16 :	$e_2 \sim_{\ell_1} e_3 : t_1$	↓qv-lab1: 15
17 :	$((\gamma_1, x \mapsto e_2)) \sim_{\ell_1} ((\gamma_2, x \mapsto e_3)) : (\Gamma, x{:}t_1)$	qhx: 4,16
18 :	$((\gamma_2, x \mapsto e_3))(e_1) = e_5$	subh-total
19 :	$e_4 \approx_{\ell_1} e_5 : t_2$	ni: 14,17,11,18
20 :	$e_4 \Downarrow e_6$	↓qe-ev: 19
21 :	$(\mathtt{bind}\ x = e_{11}\ \mathtt{in}\ e_9) \Downarrow e_6$	ev-bind: 10,13,20
22 :	$e_{12} \Downarrow (e_3[\ell_2])$	↓qe-ev: 9
23 :	$\gamma_2(e_1) = e_8$	↓subh-bind: 7
24 :	$e_8[e_3/x] = e_5$	subhsub: 18,23
25 :	$e_5 \Downarrow e_7$	↓qe-ev: 19
26 :	$(\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_8) \Downarrow e_7$	ev-bind: 22,24,25
27 :	$e_6 \sim_{\ell_1} e_7 : t_2$	↓qe-ev: 19
28 :	$(\mathtt{bind}\ x = e_{11}\ \mathtt{in}\ e_9) \approx_{\ell_1} (\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_8) : t_2$	qe-ev: 21,26,27

(ni-bind2)

1 :	$\Gamma \vdash (\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_5) : t_1$	*given
2 :	$\Gamma \vdash e_{12} : (t_2[\ell_1])$	↓ty-bind: 1
3 :	$\gamma_2 \sim_{\ell_2} \gamma_1 : \Gamma$	*given
4 :	$\gamma_2((\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_5)) = (\mathtt{bind}\ x = e_{11}\ \mathtt{in}\ e_9)$	*given
5 :	$\gamma_2(e_{12}) = e_{11}$	↓subh-bind: 4
6 :	$\gamma_1((\mathtt{bind}\ x = e_{12}\ \mathtt{in}\ e_5)) = (\mathtt{bind}\ x = e_{10}\ \mathtt{in}\ e_8)$	*given
7 :	$\gamma_1(e_{12}) = e_{10}$	↓subh-bind: 6
8 :	$e_{11} \approx_{\ell_2} e_{10} : (t_2[\ell_1])$	ni: 2,3,5,7
9 :	$(e_7[\ell_1]) \sim_{\ell_2} (e_6[\ell_1]) : (t_2[\ell_1])$	↓qe-ev: 8
10 :	$\mathtt{val}\ (e_7[\ell_1])$	↓qv-lab2: 9
11 :	$\mathtt{val}\ (e_6[\ell_1])$	↓qv-lab2: 9
12 :	$e_{11} \Downarrow (e_7[\ell_1])$	↓qe-ev: 8
13 :	$e_9[e_7/x] = e_4$	sub-total
14 :	$(\Gamma, x{:}t_2) \vdash e_5 : t_1$	↓ty-bind: 1
15 :	$\gamma_2 \models \Gamma$	qhmod1: 3
16 :	$\Gamma - \Gamma = \cdot$	gsub-z2
17 :	$\cdot \vdash e_{11} : (t_2[\ell_1])$	tysubh: 2,15,16,5
18 :	$\cdot \vdash (e_7[\ell_1]) : (t_2[\ell_1])$	evty: 17,12
19 :	$\cdot \vdash e_7 : t_2$	↓ty-lab: 18
20 :	$((\gamma_2, x \mapsto e_7)) \models (\Gamma, x{:}t_2)$	mod-x: 15,19
21 :	$(\Gamma, x{:}t_2) - (\Gamma, x{:}t_2) = \cdot$	gsub-z2
22 :	$\gamma_2(e_5) = e_9$	↓subh-bind: 4
23 :	$((\gamma_2, x \mapsto e_7))(e_5) = e_4$	subsubh: 22,13
24 :	$\cdot \vdash e_4 : t_1$	tysubh: 14,20,21,23
25 :	$e_4 \Downarrow e_1$	tyev: 24
26 :	$(\mathtt{bind}\ x = e_{11}\ \mathtt{in}\ e_9) \Downarrow e_1$	ev-bind: 12,13,25
27 :	$e_{10} \Downarrow (e_6[\ell_1])$	↓qe-ev: 8
28 :	$e_8[e_6/x] = e_3$	sub-total

29 :	$\gamma_1 \models \Gamma$	qhmod2: 3
30 :	$\cdot \vdash \mathtt{e}_{10} : (\mathtt{t}_2[\ell_1])$	tysubh: 2,29,16,7
31 :	$\cdot \vdash (\mathtt{e}_6[\ell_1]) : (\mathtt{t}_2[\ell_1])$	evty: 30,27
32 :	$\cdot \vdash \mathtt{e}_6 : \mathtt{t}_2$	↓ty-lab: 31
33 :	$((\gamma_1, \mathtt{x} \mapsto \mathtt{e}_6)) \models (\Gamma, \mathtt{x}{:}\mathtt{t}_2)$	mod-x: 29,32
34 :	$\gamma_1(\mathtt{e}_5) = \mathtt{e}_8$	↓subh-bind: 6
35 :	$((\gamma_1, \mathtt{x} \mapsto \mathtt{e}_6))(\mathtt{e}_5) = \mathtt{e}_3$	subsubh: 34,28
36 :	$\cdot \vdash \mathtt{e}_3 : \mathtt{t}_1$	tysubh: 14,33,21,35
37 :	$\mathtt{e}_3 \Downarrow \mathtt{e}_2$	tyev: 36
38 :	$(\mathtt{bind}\ \mathtt{x} = \mathtt{e}_{10}\ \mathtt{in}\ \mathtt{e}_8) \Downarrow \mathtt{e}_2$	ev-bind: 27,28,37
39 :	$\mathtt{val}\ \mathtt{e}_1$	evval: 25
40 :	$\mathtt{val}\ \mathtt{e}_2$	evval: 37
41 :	$\cdot \vdash \mathtt{e}_1 : \mathtt{t}_1$	evty: 24,25
42 :	$\cdot \vdash \mathtt{e}_2 : \mathtt{t}_1$	evty: 36,37
43 :	$\ell_1 \preceq \mathtt{t}_1$	↓ty-bind: 1
44 :	$\ell_1 \not\preceq \ell_2$	↓qv-lab2: 9
45 :	$\mathtt{e}_1 \sim_{\ell_2} \mathtt{e}_2 : \mathtt{t}_1$	nipt: 39,40,41,42,43,44
46 :	$(\mathtt{bind}\ \mathtt{x} = \mathtt{e}_{11}\ \mathtt{in}\ \mathtt{e}_9) \approx_{\ell_2} (\mathtt{bind}\ \mathtt{x} = \mathtt{e}_{10}\ \mathtt{in}\ \mathtt{e}_8) : \mathtt{t}_1$	qe-ev: 26,38,45