Soundness Theorem for Simply-Typed Lambda Calculus in Abstract Variable Representation (tree format)

Stephen Tse

Aug 1, 2004*

This report is automatically generated by the tool `lf2tex` from the semantics specification `lambda.elf` (Twelf source) and the syntactic specification `lambda.tex` (LaTeX source).

Some notes about the typesetting used in this report:

- A syntax class is displayed boxed like $\boxed{\mathtt{e} ::= \cdots}$ (*terms*).
- A syntax entity is displayed unboxed like $\mathtt{e} ::= \lambda \mathtt{x{:}t.\ e}$ (*functions*).
- A judgement form is displayed boxed like $\boxed{\Gamma \vdash \mathtt{e} : \mathtt{t}}$ (*typings*).
- A judgement rule is displayed unboxed like $\dfrac{\Gamma, \mathtt{x{:}t_1} \vdash \mathtt{e} : \mathtt{t_2}}{\Gamma \vdash \lambda \mathtt{x{:}t.e} : \mathtt{t_1} \rightarrow \mathtt{t_2}}$ (*typing for functions*).
- A theorem is displayed boxed like $\boxed{\dfrac{\Gamma \vdash \mathtt{e_1{:}t} \quad \mathtt{e_1} \longrightarrow \mathtt{e_2}}{\Gamma \vdash \mathtt{e_2} : \mathtt{t}}}$ (*preservations*).
- A proof is displayed as a numbered list with the last one of the list being the conclusion. (A rule name with downarrow ($\downarrow$ ty-fun) means by the inversion of such rule.)

Contents

*Last update: April 7, 2005

1 Syntax

1.1 Variables

$$\boxed{\mathtt{x} ::= \cdots} \qquad \boxed{\mathrm{x}}$$

1.2 Types

$$\boxed{\mathtt{t} ::= \cdots} \qquad \boxed{\mathrm{t}}$$

$$\mathtt{t} ::= \mathtt{<>} \qquad \text{(tunit)}$$

$$\mathtt{t} ::= \mathtt{t} \to \mathtt{t} \qquad \text{(tfun)}$$

1.3 Terms

$$\boxed{\mathtt{e} ::= \cdots} \qquad \boxed{\mathrm{e}}$$

$$\mathtt{e} ::= \mathtt{<>} \qquad \text{(unit)}$$

$$\mathtt{e} ::= \mathtt{x} \qquad \text{(var)}$$

$$\mathtt{e} ::= \lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e} \qquad \text{(fun)}$$

$$\mathtt{e} ::= \mathtt{e}\ \mathtt{e} \qquad \text{(app)}$$

1.4 Contexts

$$\boxed{\Gamma ::= \cdots} \qquad \boxed{\mathrm{g}}$$

$$\Gamma ::= \cdot \qquad \text{(gn)}$$

$$\Gamma ::= \Gamma, \mathtt{x}{:}\mathtt{t} \qquad \text{(gt)}$$

2 Semantics

2.1 Operations

$$\boxed{\Gamma\ [\mathtt{x}] = \mathtt{t}} \qquad \boxed{\text{map}}$$

$$\boxed{\mathtt{e}\{\mathtt{e}/\mathtt{x}\} = \mathtt{e}} \qquad \boxed{\text{sub}}$$

2.2 Typings

$$\boxed{\Gamma \vdash \mathtt{e} : \mathtt{t}} \qquad \boxed{\text{ty}}$$

$$\Gamma \vdash \langle\rangle : \langle\rangle \qquad \text{(ty-unit)}$$

$$\frac{\Gamma\ [\mathtt{x}] = \mathtt{t}}{\Gamma \vdash \mathtt{x} : \mathtt{t}} \qquad \text{(ty-var)}$$

$$\frac{\Gamma, \mathtt{x}{:}\mathtt{t}_1 \vdash \mathtt{e} : \mathtt{t}_2}{\Gamma \vdash \lambda \mathtt{x}{:}\mathtt{t}_1 . \mathtt{e} : \mathtt{t}_1 \to \mathtt{t}_2} \qquad \text{(ty-fun)}$$

$$\frac{\Gamma \vdash \mathtt{e}_1 : \mathtt{t}_1 \to \mathtt{t}_2 \quad \Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1}{\Gamma \vdash \mathtt{e}_1\ \mathtt{e}_2 : \mathtt{t}_2} \qquad \text{(ty-app)}$$

2.3 Values

$$\boxed{\mathtt{val}\ \mathtt{e}} \qquad \boxed{\text{val}}$$

$$\mathtt{val}\ \langle\rangle \qquad \text{(val-unit)}$$

$$\mathtt{val}\ (\lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e}) \qquad \text{(val-fun)}$$

2.4 Evaluation

$$\boxed{\mathtt{e} \longrightarrow \mathtt{e}} \qquad \boxed{\text{ev}}$$

$$\frac{\mathtt{e}_1 \longrightarrow \mathtt{e}_3}{\mathtt{e}_1\ \mathtt{e}_2 \longrightarrow \mathtt{e}_3\ \mathtt{e}_2} \qquad \text{(ev-app1)}$$

$$\frac{\mathtt{val}\ \mathtt{e}_1 \quad \mathtt{e}_2 \longrightarrow \mathtt{e}_3}{\mathtt{e}_1\ \mathtt{e}_2 \longrightarrow \mathtt{e}_1\ \mathtt{e}_3} \qquad \text{(ev-app2)}$$

$$\frac{\mathtt{val}\ \mathtt{e}_2 \quad \mathtt{e}_1\{\mathtt{e}_2/\mathtt{x}\} = \mathtt{e}_3}{(\lambda \mathtt{x}{:}\mathtt{t}.\mathtt{e}_1)\ \mathtt{e}_2 \longrightarrow \mathtt{e}_3} \qquad \text{(ev-app3)}$$

3 Theorems

3.1 Preservation

$$\boxed{\frac{\Gamma, \mathtt{x}{:}\mathtt{t}_1 \vdash \mathtt{e}_1 : \mathtt{t}_2 \quad \Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1 \quad \mathtt{e}_1\{\mathtt{e}_2/\mathtt{x}\} = \mathtt{e}_3}{\Gamma \vdash \mathtt{e}_3 : \mathtt{t}_2}} \quad \boxed{\text{pssub}}$$

$$\boxed{\frac{\Gamma \vdash \mathtt{e}_1 : \mathtt{t} \quad \mathtt{e}_1 \longrightarrow \mathtt{e}_2}{\Gamma \vdash \mathtt{e}_2 : \mathtt{t}}} \quad \boxed{\text{ps}}$$

$$\dfrac{\dfrac{\dfrac{\Gamma \vdash \mathtt{e}_1\ \mathtt{e}_3 : \mathtt{t}_2}{\Gamma \vdash \mathtt{e}_1 : \mathtt{t}_1 \to \mathtt{t}_2}\downarrow\text{ty-app} \quad \dfrac{\mathtt{e}_1\ \mathtt{e}_3 \longrightarrow \mathtt{e}_2\ \mathtt{e}_3}{\mathtt{e}_1 \longrightarrow \mathtt{e}_2}\text{ ps}\downarrow\text{ev-app1}}{\Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1 \to \mathtt{t}_2} \quad \dfrac{\Gamma \vdash \mathtt{e}_1\ \mathtt{e}_3 : \mathtt{t}_2}{\Gamma \vdash \mathtt{e}_3 : \mathtt{t}_1}\downarrow\text{ty-app}}{\Gamma \vdash \mathtt{e}_2\ \mathtt{e}_3 : \mathtt{t}_2}\text{ ty-app} \qquad \text{(ps-app1)}$$

$$\dfrac{\dfrac{\dfrac{\mathtt{e}_3\ \mathtt{e}_1 \longrightarrow \mathtt{e}_3\ \mathtt{e}_2}{\mathtt{val}\ \mathtt{e}_3}\downarrow\text{ev-app2}}{\vdots}}{\dfrac{\Gamma \vdash \mathtt{e}_3 : \mathtt{t}_1 \to \mathtt{t}_2}{\Gamma \vdash \mathtt{e}_3 : \mathtt{t}_1 \to \mathtt{t}_2}\downarrow\text{ty-app} \quad \dfrac{\dfrac{\Gamma \vdash \mathtt{e}_3\ \mathtt{e}_1 : \mathtt{t}_2}{\Gamma \vdash \mathtt{e}_1 : \mathtt{t}_1}\downarrow\text{ty-app} \quad \dfrac{\mathtt{e}_3\ \mathtt{e}_1 \longrightarrow \mathtt{e}_3\ \mathtt{e}_2}{\mathtt{e}_1 \longrightarrow \mathtt{e}_2}\text{ ps}\downarrow\text{ev-app2}}{\Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1}}{\Gamma \vdash \mathtt{e}_3\ \mathtt{e}_2 : \mathtt{t}_2}\text{ ty-app} \qquad \text{(ps-app2)}$$

$$\dfrac{\dfrac{\dfrac{(\lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}_1)\ \mathtt{e}_2 \longrightarrow \mathtt{e}_3}{\mathtt{val}\ \mathtt{e}_2}\downarrow\text{ev-app3}}{\vdots}}{\dfrac{\dfrac{\Gamma \vdash (\lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}_1)\ \mathtt{e}_2 : \mathtt{t}_2}{\Gamma \vdash \lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}_1 : \mathtt{t}_1 \to \mathtt{t}_2}\downarrow\text{ty-app}}{\Gamma, \mathtt{x}{:}\mathtt{t}_1 \vdash \mathtt{e}_1 : \mathtt{t}_2}\downarrow\text{ty-fun} \quad \dfrac{\Gamma \vdash (\lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}_1)\ \mathtt{e}_2 : \mathtt{t}_2}{\Gamma \vdash \mathtt{e}_2 : \mathtt{t}_1}\downarrow\text{ty-app} \quad \dfrac{(\lambda \mathtt{x}{:}\mathtt{t}_1.\mathtt{e}_1)\ \mathtt{e}_2 \longrightarrow \mathtt{e}_3}{\mathtt{e}_1\{\mathtt{e}_2/\mathtt{x}\} = \mathtt{e}_3}\downarrow\text{ev-app3}}{\Gamma \vdash \mathtt{e}_3 : \mathtt{t}_2}\text{ pssub} \qquad \text{(ps-app3)}$$

3.2 Progress

$$\boxed{\mathtt{evval}\ \mathtt{e}} \quad \boxed{\text{evval}}$$

$$\frac{\mathtt{e}_1 \longrightarrow \mathtt{e}_2}{\mathtt{evval}\ \mathtt{e}_1} \qquad \text{(evval-ev)}$$

$$\frac{\mathtt{val}\ \mathtt{e}}{\mathtt{evval}\ \mathtt{e}} \qquad \text{(evval-val)}$$

$$\boxed{\frac{}{\mathtt{e}_1\{\mathtt{e}_2/\mathtt{x}\} = \mathtt{e}_3}} \quad \boxed{\text{sub-total}}$$

$$\boxed{\frac{\cdot \vdash \mathtt{e} : \mathtt{t}}{\mathtt{evval}\ \mathtt{e}}} \quad \boxed{\text{pg}}$$

$$\dfrac{\dfrac{\dfrac{\cdot \vdash \langle\rangle : \langle\rangle}{\vdots}}{\dfrac{}{\mathtt{val}\ \langle\rangle}\text{ val-unit}}}{\mathtt{evval}\ \langle\rangle}\text{ evval-val} \qquad \text{(pg-unit)}$$

$$\dfrac{\dfrac{\dfrac{\cdot \vdash \lambda x{:}t_1.e : t_1 \to t_2}{\cdot \vdash \lambda x{:}t_1.e : t_1 \to t_2}\ \vdots\ \dfrac{\cdot \vdash \lambda x{:}t_1.e : t_1 \to t_2}{\cdot, x{:}t_1 \vdash e : t_2}\ \downarrow\text{ty-fun}}{\texttt{val}\ (\lambda x{:}t_1.e)}\ \text{val-fun}}{\texttt{evval}\ (\lambda x{:}t_1.e)}\ \text{evval-val} \qquad \text{(pg-fun)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cdot \vdash e_1\ e_3 : t_2}{\cdot \vdash e_3 : t_1}\ \downarrow\text{ty-app}}{\vdots}}{\cdot \vdash e_1\ e_3 : t_2}\ \downarrow\text{ty-app}}{\cdot \vdash e_1 : t_1 \to t_2}\ \text{pg}}{\dfrac{\texttt{evval}\ e_1}{e_1 \longrightarrow e_2}\ \downarrow\text{evval-ev}}\ \text{ev-app1}}{\texttt{evval}\ (e_1\ e_3)}\ \text{evval-ev} \qquad \text{(pg-app1)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\cdot \vdash e_3\ e_1 : t_2}{\cdot \vdash e_3 : t_1 \to t_2}\ \downarrow\text{ty-app}}{\texttt{evval}\ e_3}\ \text{pg}}{\texttt{val}\ e_3}\ \downarrow\text{evval-val} \qquad \dfrac{\dfrac{\dfrac{\cdot \vdash e_3\ e_1 : t_2}{\cdot \vdash e_1 : t_1}\ \downarrow\text{ty-app}}{\texttt{evval}\ e_1}\ \text{pg}}{e_1 \longrightarrow e_2}\ \downarrow\text{evval-ev}}{e_3\ e_1 \longrightarrow e_3\ e_2}\ \text{ev-app2}}{\texttt{evval}\ (e_3\ e_1)}\ \text{evval-ev} \qquad \text{(pg-app2)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cdot \vdash (\lambda x{:}t_2.e_2)\ e_1 : t_3}{\cdot \vdash \lambda x{:}t_2.e_2 : t_1 \to t_3}\ \downarrow\text{ty-app}}{\vdots}}{\cdot \vdash (\lambda x{:}t_2.e_2)\ e_1 : t_3}\ \downarrow\text{ty-app}}{\dfrac{\cdot \vdash e_1 : t_1}{\texttt{evval}\ e_1}\ \text{pg}}{\texttt{val}\ e_1}\ \downarrow\text{evval-val} \qquad \dfrac{\dfrac{\cdot \vdash (\lambda x{:}t_2.e_2)\ e_1 : t_3}{\cdot \vdash \lambda x{:}t_2.e_2 : t_1 \to t_3}\ \downarrow\text{ty-app}}{\texttt{evval}\ (\lambda x{:}t_2.e_2)}\ \text{pg}}{e_2\{e_1/x\} = e_3}\ \text{sub-total}}{(\lambda x{:}t_2.e_2)\ e_1 \longrightarrow e_3}\ \text{ev-app3}}{\texttt{evval}\ ((\lambda x{:}t_2.e_2)\ e_1)}\ \text{evval-ev} \qquad \text{(pg-app3)}$$