Incentives and Pricing in Communication Networks

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Problem statement

- **Network resource allocation**
  - Competing users in communications networks
  - Quality of Service (QoS) requirements
  - Multiple self-interested agents may require tools from game theory
  - Difficulty to formulate and implement centralized control protocols
  - Scalable as the growth of networks and newly interactions between administrative domains and end users
  - Without centralized control, the interaction of multiple selfish agents may lead to suboptimal resource allocation

Roadmap

- Two strategic settings of ***pricing***:
  - **Achieve socially optimal objective for the network**
  - Multiple competing service providers set prices to maximize their revenues using game-theoretic techniques

Additional: emerging applications of game theory to communication networks, and future directions

Network model

- Network is shared by **many users**, and network resources are **link bandwidths**, i.e. maximum data transmitting rate
- Each end user is interested in transferring data between a source and a destination along a fixed route
- Links has finite capabilities c_l, which are shared by a set of sources, and one-to-one mapping between users and routes
- **Utility function**: $U_r(x_r)$ is the utility of source r as a function of its rate x_r (packets per unit time), assumed to be ***strictly increasing, strictly concave***

Network model

- Goal: **socially optimal**, i.e. maximize the total utilities in the network by **pricing** scheme. Nonlinear optimization problem (Kelly, 1997):

$$\max_{x \geq 0} \sum_r U_r(x_r), \qquad Rx \leq c.$$

- *x* is the vector of source rates; *c* is the vector of link capabilities; *R* is the routing matrix, i.e. (*l, r*): 1 if route *r* includes link *l* and 0 otherwise
- Constraint: source rates can not exceed link capabilities
- **If the utility functions are strictly concave, there exists a unique optimal solution**

Karush-Kuhn-Tucker (KKT) conditions

The nonlinear optimization to maximize $f(x_1, x_2, \ldots, x_n)$ subjecting to $g_i(x_1, x_2, \ldots, x_n) \leq 0$ and $h_j(x_1, x_2, \ldots, x_n) = 0$ can be equalized to solve:

$$\lambda \nabla f(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{m} \mu_i \nabla g_i(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{j=1}^{l} \upsilon_j \nabla h_j(x_1^*, x_2^*, \ldots, x_n^*) = 0 \qquad (1)$$

$$\mu_i g_i(x_1^*, x_2^*, \ldots, x_n^*) = 0 \quad (i = 1, 2, \ldots, m) \qquad (2)$$

Each $(x_1^*, x_2^*, \ldots, x_n^*)$ satisfying (1) and (2) is a local solution for the maximization of $f(x_1, x_2, \ldots, x_n)$.

There $\lambda \leq 0$, and $\mu_i \geq 0 (i = 1, 2 \ldots m)$. $g_i(x_1, x_2, \ldots, x_n) \leq 0 \quad (1 \leq i \leq m)$ are the inequality constraints and $h_j(x_1, x_2, \ldots, x_n) = 0 \quad (0 \leq j \leq l)$ are the equality constraints, while m and l are the numbers of these constraints, respectively.

Optimal solution

- By applying KKT conditions, we have:

$$U_r'(\hat{x}_r) = \sum_{l:l \in r} \hat{p}_l, \qquad \forall\, r,$$

$$\hat{p}_l \left(\sum_{r:l \in r} \hat{x}_r - c_l \right) = 0, \qquad \forall\, l,$$

$$\sum_{r:l \in r} \hat{x}_r \leq c_l, \qquad \forall\, l,$$

$$\hat{p}, \hat{x} \geq 0.$$

- $\hat{x}$: a vector of optimal rates
- $\hat{p}$: Lagrange multipliers

Optimal solution

- Assume the price per bit for each user is:

$$\hat{q}_r = \sum_{r:l \in r} \hat{p}_l = R^T \hat{p}.$$

- For each user, the target is to:

$$\max_{x_r \geq 0} U_r(x_r) - \hat{q}_r x_r .$$

- The equilibrium under this pricing scheme coincides with socially optimum outcome

We will discuss how the situation is different when prices are set by profit-maximizing service providers in next section

Optimal solution

- Problems
  - The network is unaware of the utility function of users
  - Centralized authority is required to solve optimization
  - Computationally complex
  - Not realistic for the Internet
- Simpler mechanism
  - Achieve the optimal allocation of resources in the presence of selfish users
  - Kelly 1997, Kelly et. al. 1998, Low and Lapsley 1999, etc

Weighted proportional fair rate allocation

- Each user r announces a ***bid*** w_r , i.e. the price per unit time that it is willing to pay, and the goal is to:

$$\max_{x \geq 0} \sum_r w_r \log(x_r), \qquad Rx \leq c.$$

- Apply KKT conditions:

$$\frac{w_r}{x_r^*} = \sum_{r:l \in r} p_l^*, \qquad \forall\, r,$$

$$p_l^* \left(\sum_{r:l \in r} x_r^* - c_l \right) = 0, \qquad \forall\, l,$$

$$\sum_{r:l \in r} x_r^* \leq c_l, \qquad \forall\, l,$$

$$p^*, x^* \geq 0,$$

Adjustment of price bid

- A dynamic algorithm where each link computes a price as a function of time according to a differential equation
- In steady state, the price of each link converges to the Lagrange multiplier
- An example differential equation: $\dot{p}_l = (y_l - c_l)^+_{p_l}$
  - $p_l(t)$ is the instantaneous link prices at time t, y_l is the total arrival rate at link l, and $(a)_b+$ is equal to *max*(a, 0) when $b=0$ and is equal to a if $b>0$
  - Equilibrium: $y_l=c_l$ or $p_l=0$, which satisfy previous KKT conditions

Implementation

- Congestion control algorithm
  - Each user is equipped with a protocol to collect q_r, i.e. the price of its path from the network (price data can be piggybacked by packets between source and destination
  - Users reacts to congestion indication in the form of q_r
  - Each user is hardwired with a program that computes rates according to the equation:

$$x_r = \frac{w_r}{q_r}$$

    where q_r is the price of route r and is given by $q_r = \sum_{l:l\in r} p_l$.

  - More analysis using Lyapunov function indicates that the congestion control algorithm is stable if w_r is fixed.

Implementation

- The user's optimization problem: $\max_{w_r} U_r\left(\frac{w_r}{q_r}\right) - w_r$
- Thus the user choose w_r to satisfy: $U_r'\left(\frac{w_r}{q_r}\right) = q_r$
- The equilibrium point of the differential equation is given by KKT conditions with w_r replaced by $x_r^* U_r'(x^*)$
- If the user is price-taking and myopic (i.e. they ignore strategic aspects and simply maximize instantaneous net utility), then users' selfish objectives *coincide* with the socially optimal objective of the system

Extend from wired to wireless

- Difference
  - wireless interference, i.e. simultaneous link transmissions not possible within interference range
- Similarity
  - using KKT conditions and congestion control algorithms to get optimal solution, i.e. network traffic scheduling
- Details omitted

Summary

- Problem of network resource allocation
- Socially optimal objective for the network
- Summation of utility function and apply KKT conditions to get optimal solution
- Unrealistic centralized control protocol
- Weighted proportional fair rate allocation
- Extend from wired to wireless networks