Incentives and Pricing in Communication Networks

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Problem statement

• **Network resource allocation**
  – Competing users in communications networks
  – Quality of Service (QoS) requirements
  – Multiple self-interested agents may require tools from game theory
  – Difficulty to formulate and implement centralized control protocols
  – Scalable as the growth of networks and newly interactions between administrative domains and end users
  – Without centralized control, the interaction of multiple selfish agents may lead to suboptimal resource allocation
Roadmap

• Two strategic settings of *pricing*:
  – Achieve socially optimal objective for the network
  – Multiple competing service providers set prices to maximize their revenues using game-theoretic techniques

Additional: emerging applications of game theory to communication networks, and future directions
Network model

- Network is shared by **many users**, and network resources are **link bandwidths**, i.e. maximum data transmitting rate.
- Each end user is interested in transferring data between a source and a destination along a fixed route.
- Links have finite capabilities $c_l$, which are shared by a set of sources, and one-to-one mapping between users and routes.
- **Utility function**: $U_r(x_r)$ is the utility of source $r$ as a function of its rate $x_r$ (packets per unit time), assumed to be strictly increasing, strictly concave.
Network model

• Goal: **socially optimal**, i.e. maximize the total utilities in the network by **pricing** scheme. Nonlinear optimization problem (Kelly, 1997):

\[
\max_{x \geq 0} \sum_r U_r(x_r), \quad Rx \leq c.
\]

• \(x\) is the vector of source rates; \(c\) is the vector of link capabilities; \(R\) is the routing matrix, i.e. \((l, r)\): 1 if route \(r\) includes link \(l\) and 0 otherwise

• Constraint: source rates can not exceed link capabilities

• **If the utility functions are strictly concave, there exists a unique optimal solution**
Karush-Kuhn-Tucker (KKT) conditions

The nonlinear optimization to maximize $f(x_1, x_2, \ldots, x_n)$ subjecting to $g_i(x_1, x_2, \ldots, x_n) \leq 0$ and $h_j(x_1, x_2, \ldots, x_n) = 0$ can be equalized to solve:

$$\lambda \nabla f(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{m} \mu_i \nabla g_i(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{j=1}^{i} \nu_j \nabla h_j(x_1^*, x_2^*, \ldots, x_n^*) = 0$$

(1)

$$\mu_i g_i(x_1^*, x_2^*, \ldots, x_n^*) = 0 \quad (i = 1, 2, \ldots, m)$$

(2)

Each $(x_1^*, x_2^*, \ldots, x_n^*)$ satisfying (1) and (2) is a local solution for the maximization of $f(x_1, x_2, \ldots, x_n)$.

There $\lambda \leq 0$, and $\mu_i \geq 0 (i = 1, 2, \ldots m)$. $g_i(x_1, x_2, \ldots, x_n) \leq 0$ $(1 \leq i \leq m)$ are the inequality constraints and $h_j(x_1, x_2, \ldots, x_n) = 0$ $(0 \leq j \leq l)$ are the equality constraints, while $m$ and $l$ are the numbers of these constraints, respectively.
Optimal solution

• By applying KKT conditions, we have:

\[ U'_r(\hat{x}_r) = \sum_{l : l \in r} \hat{p}_l, \quad \forall r, \]

\[ \hat{p}_l \left( \sum_{r : l \in r} \hat{x}_r - c_l \right) = 0, \quad \forall l, \]

\[ \sum_{r : l \in r} \hat{x}_r \leq c_l, \quad \forall l, \]

\[ \hat{p}, \hat{x} \geq 0. \]

• \( \hat{x} \): a vector of optimal rates
• \( \hat{p} \): Lagrange multipliers
Optimal solution

- Assume the price per bit for each user is:
  \[ \hat{q}_r = \sum_{l \in r} \hat{p}_l = R^T \hat{p}. \]

- For each user, the target is to:
  \[ \max_{x_r \geq 0} U_r(x_r) - \hat{q}_r x_r. \]

- The equilibrium under this pricing scheme coincides with socially optimum outcome.

We will discuss how the situation is different when prices are set by profit-maximizing service providers in next section.
Optimal solution

• Problems
  – The network is unaware of the utility function of users
  – Centralized authority is required to solve optimization
  – Computationally complex
  – Not realistic for the Internet

• Simpler mechanism
  – Achieve the optimal allocation of resources in the presence of selfish users
  – Kelly 1997, Kelly et. al. 1998, Low and Lapsley 1999, etc
Weighted proportional fair rate allocation

• Each user \( r \) announces a **bid** \( w_r \), i.e. the price per unit time that it is willing to pay, and the goal is to:

\[
\max_{x \geq 0} \sum_r w_r \log(x_r), \quad Rx \leq c.
\]

• Apply KKT conditions:

\[
\frac{w_r}{x_r^*} = \sum_{r:l \in r} p_l^*, \quad \forall r,
\]

\[
p_l^*
\left(\sum_{r:l \in r} x_r^* - c_l\right) = 0, \quad \forall l,
\]

\[
\sum_{r:l \in r} x_r^* \leq c_l, \quad \forall l,
\]

\[
p^*, x^* \geq 0,
\]
Adjustment of price bid

• A dynamic algorithm where each link computes a price as a function of time according to a differential equation

• In steady state, the price of each link converges to the Lagrange multiplier

• An example differential equation:
  \[ \dot{p}_l = (y_l - c_l)_{p_l}^+ \]

  – \( p_l(t) \) is the instantaneous link prices at time \( t \), \( y_l \) is the total arrival rate at link \( l \), and \((a)_b^+ \) is equal to \( \text{max}(a, 0) \) when \( b=0 \) and is equal to \( a \) if \( b>0 \)

  – Equilibrium: \( y_l=c_l \) or \( p_l=0 \), which satisfy previous KKT conditions
Implementation

• Congestion control algorithm
  – Each user is equipped with a protocol to collect $q_r$, i.e. the price of its path from the network (price data can be piggybacked by packets between source and destination
  – Users reacts to congestion indication in the form of $q_r$
  – Each user is hardwired with a program that computes rates according to the equation:
    \[ x_r = \frac{w_r}{q_r} \]
    where $q_r$ is the price of route $r$ and is given by $q_r = \sum_{l:l \in r} p_l$.
  – More analysis using Lyapunov function indicates that the congestion control algorithm is stable if $w_r$ is fixed.
Implementation

• The user’s optimization problem:
  $$\max_{w_r} U_r \left( \frac{w_r}{q_r} \right) - w_r$$

• Thus the user choose $w_r$ to satisfy:
  $$U_r' \left( \frac{w_r}{q_r} \right) = q_r$$

• The equilibrium point of the differential equation is given by KKT conditions with $w_r$ replaced by $x_r^* U_r'(x^*)$

• If the user is price-taking and myopic (i.e. they ignore strategic aspects and simply maximize instantaneous net utility), then users’ selfish objectives coincide with the socially optimal objective of the system
Extend from wired to wireless

- **Difference**
  - wireless interference, i.e. simultaneous link transmissions not possible within interference range

- **Similarity**
  - using KKT conditions and congestion control algorithms to get optimal solution, i.e. network traffic scheduling

- **Details omitted**
Summary

• Problem of network resource allocation
• Socially optimal objective for the network
• Summation of utility function and apply KKT conditions to get optimal solution
• Unrealistic centralized control protocol
• Weighted proportional fair rate allocation
• Extend from wired to wireless networks