Structural Properties of Networks: Introduction

Networked Life

NETS 112

Fall 2015

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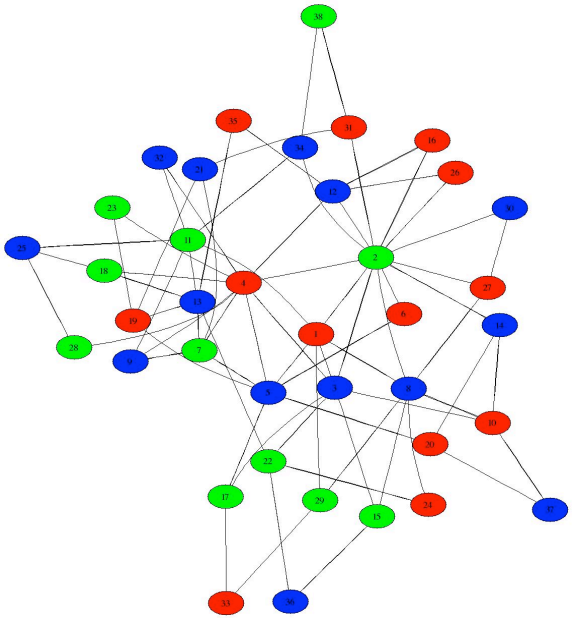
Networks: Basic Definitions

- A network (or graph) is:
  - a collection of individuals or entities, each called a vertex or node
  - a list of pairs of vertices that are neighbors, representing edges or links
- Examples:
  - vertices are mathematicians, edges represent coauthorship relationships
  - vertices are Facebook users, edges represent Facebook friendships
  - vertices are news articles, edges represent word overlap
- Networks can represent any binary relationship over individuals
- Often helpful to visualize networks with a diagram
- But to us, the network is the list of edges, not the visualization
  - same network has many different visualizations

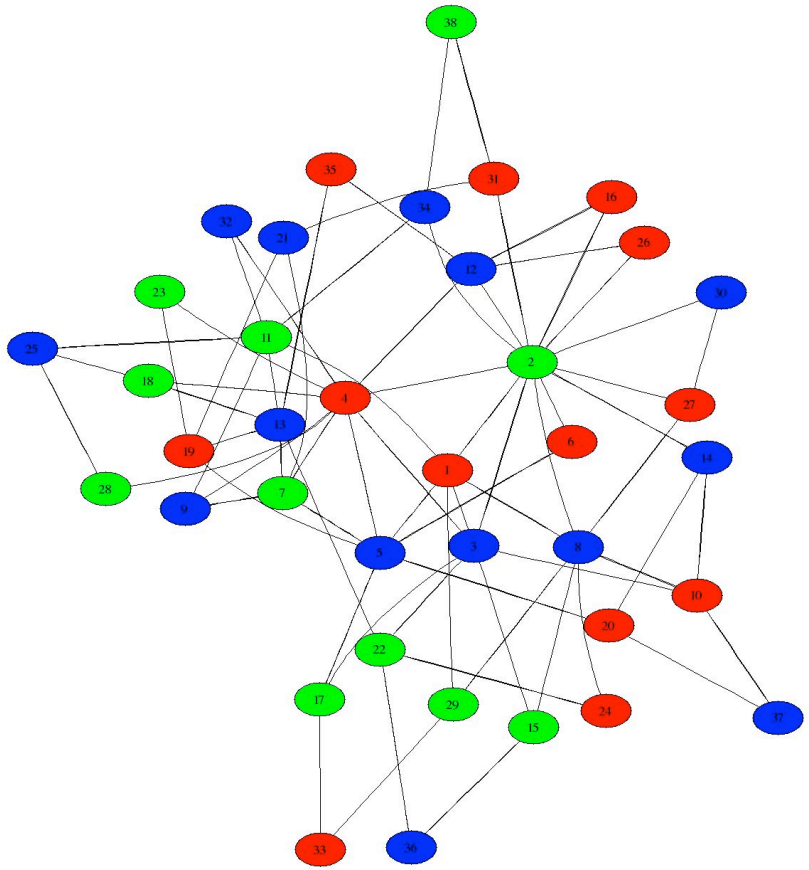


Networks: Basic Definitions

- We will use N to denote the number of vertices in a network
- Number of possible edges:

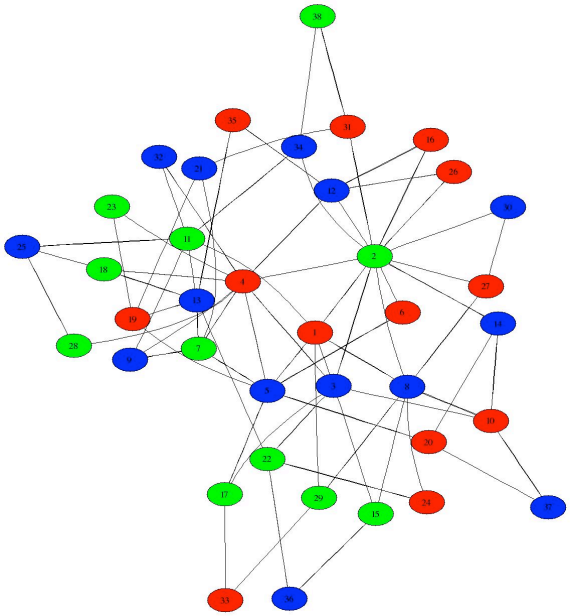
$$N(N-1)/2 \approx N^2/2$$

- The degree of a vertex is its number of neighbors



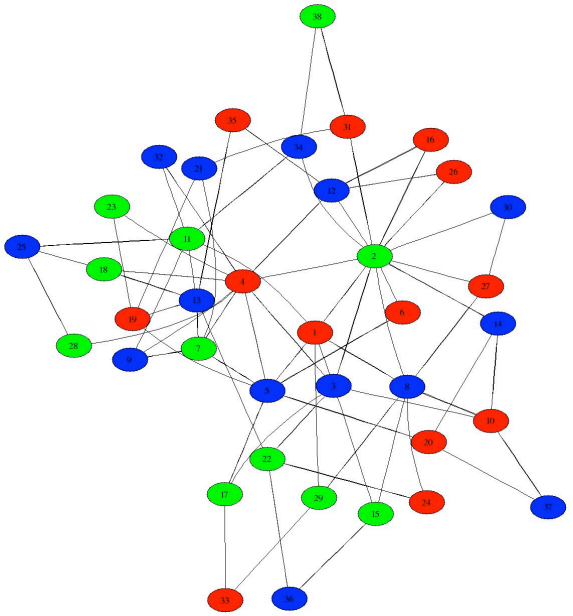
Networks: Basic Definitions

- The distance between two vertices is the length of the shortest path connecting them
- This assumes the network has only a single component or “piece”
- If two vertices are in different components, their distance is undefined or infinite
- The diameter of a network is the average distance between pairs
- It measures how near or far typical individuals are from each other



Networks: Basic Definitions

- So far, we have been discussing undirected networks
- Connection relationship is symmetric:
  - if vertex u is connected to vertex v, then v is also connected to u
  - Facebook friendship is symmetric/reciprocal
- Sometimes we'll want to discuss directed networks
  - I can follow you on Twitter without you following me
  - web page A may link to page B, but not vice-versa
- In such cases, directionality matters and edges are annotated by arrows



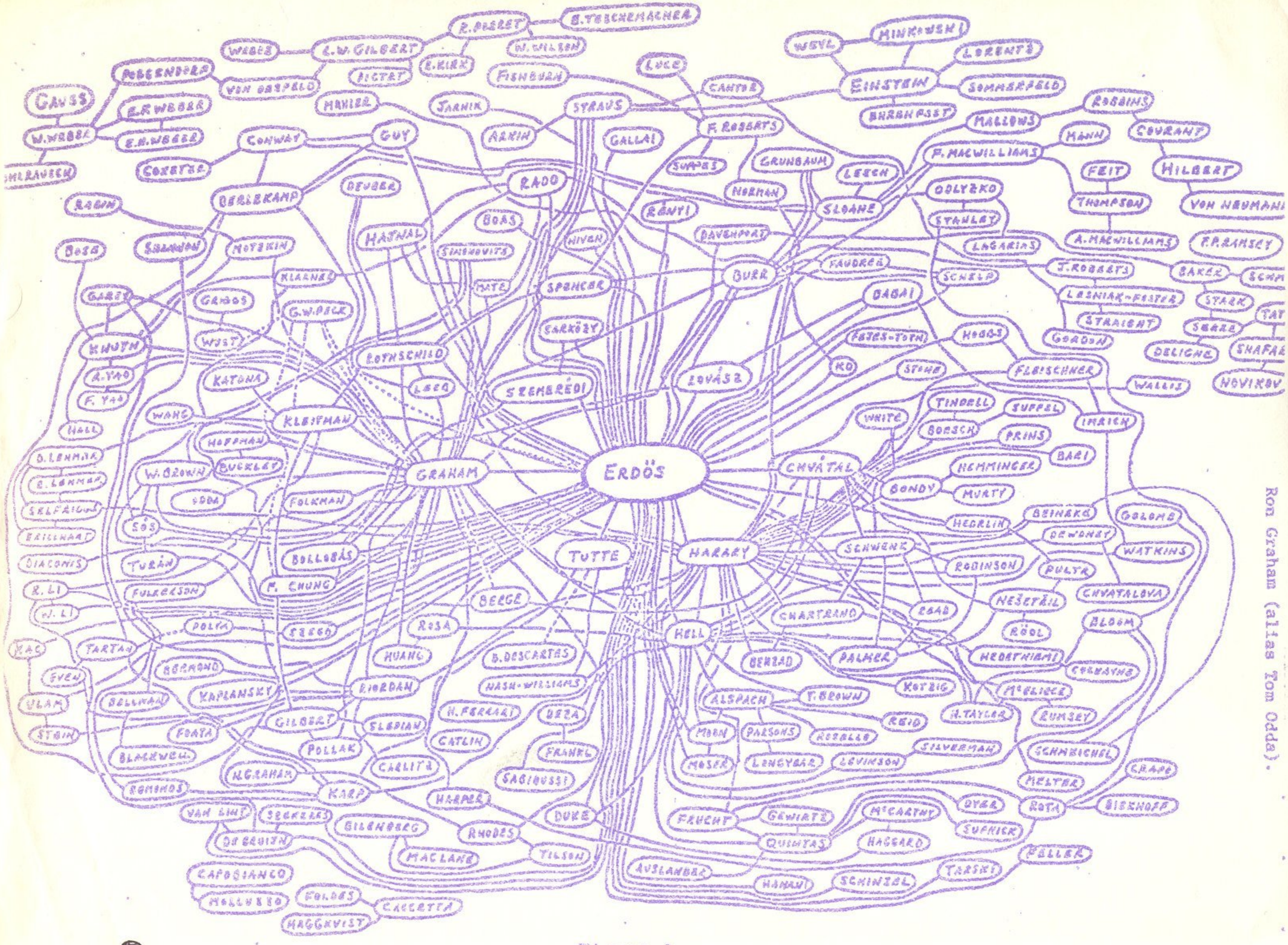


Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.) New York Academy of Sciences (1979).

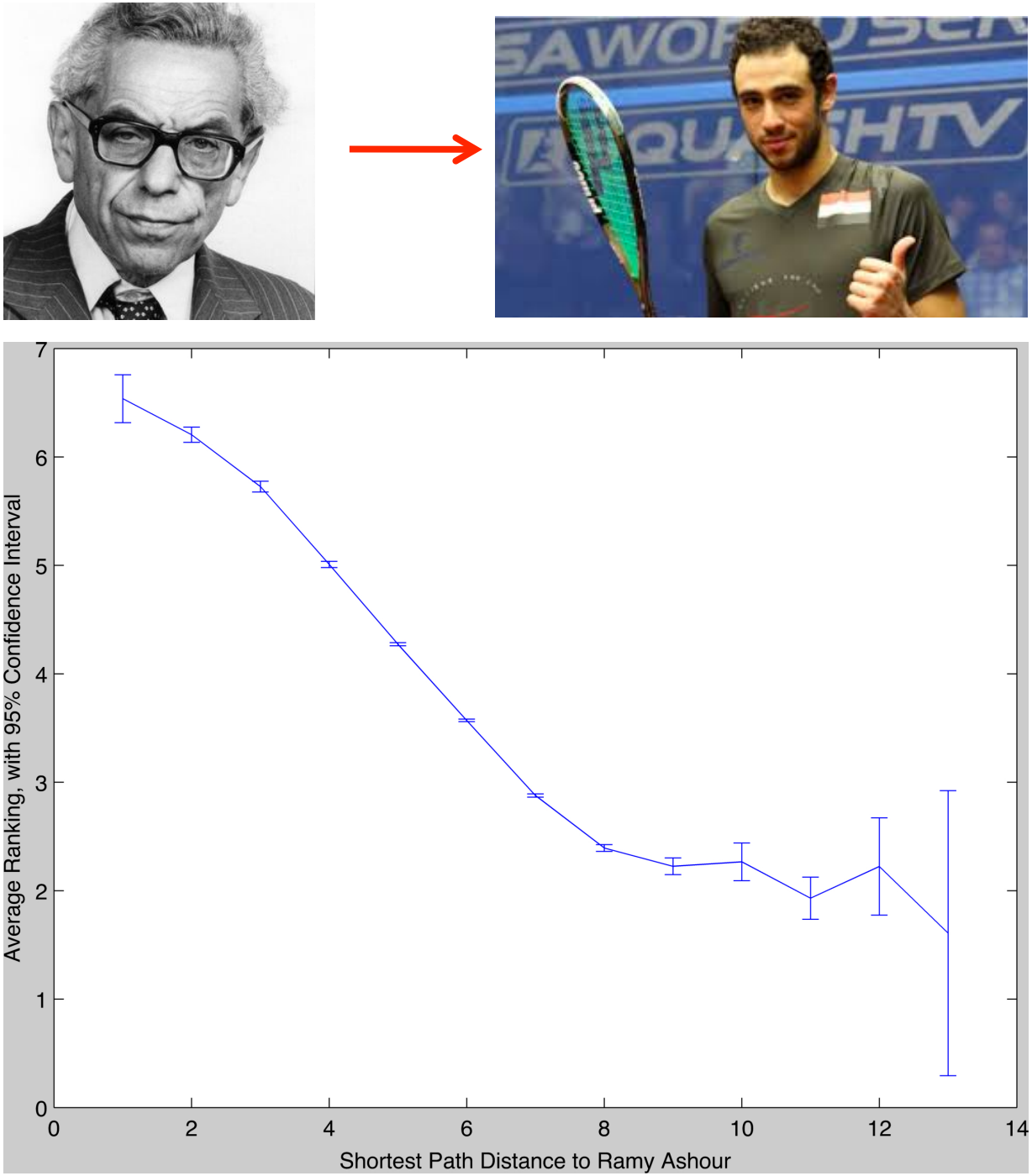
Ron Graham (alias Tom Odda).

Erdös number 0 --- 1 person
Erdös number 1 --- 504 people
Erdös number 2 --- 6593 people
Erdös number 3 --- 33605 people
Erdös number 4 --- 83642 people
Erdös number 5 --- 87760 people
Erdös number 6 --- 40014 people
Erdös number 7 --- 11591 people
Erdös number 8 --- 3146 people
Erdös number 9 --- 819 people
Erdös number 10 --- 244 people
Erdös number 11 --- 68 people
Erdös number 12 --- 23 people
Erdös number 13 --- 5 people

The median Erdös number is 5; the mean is 4.65, and the standard deviation is 1.21.

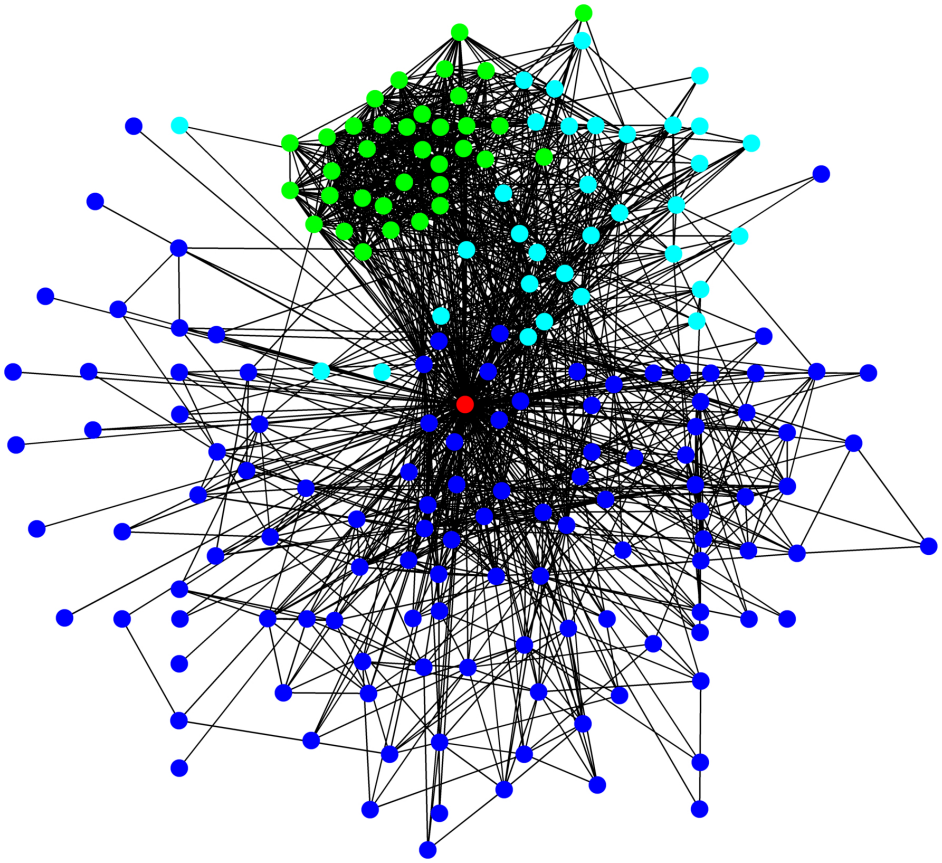
Illustrating the Concepts

- Example: scientific collaboration
  - vertices: math and computer science researchers
  - links: between coauthors on a published paper
  - *Erdos numbers* : distance to Paul Erdos
  - Erdos was definitely a *hub* or *connector;* had 507 coauthors
  - MK's Erdos number is 3, via Kearns → Mansour → Alon → Erdos
  - how do we *navigate* in such networks?
  - how does network distance relate to the real world?


Average Ranking, with 95% Confidence Interval
Shortest Path Distance to Ramy Ashour
0
1
2
3
4
5
6
7
0
2
4
6
8
10
12
14

Measures of Vertex “Importance”

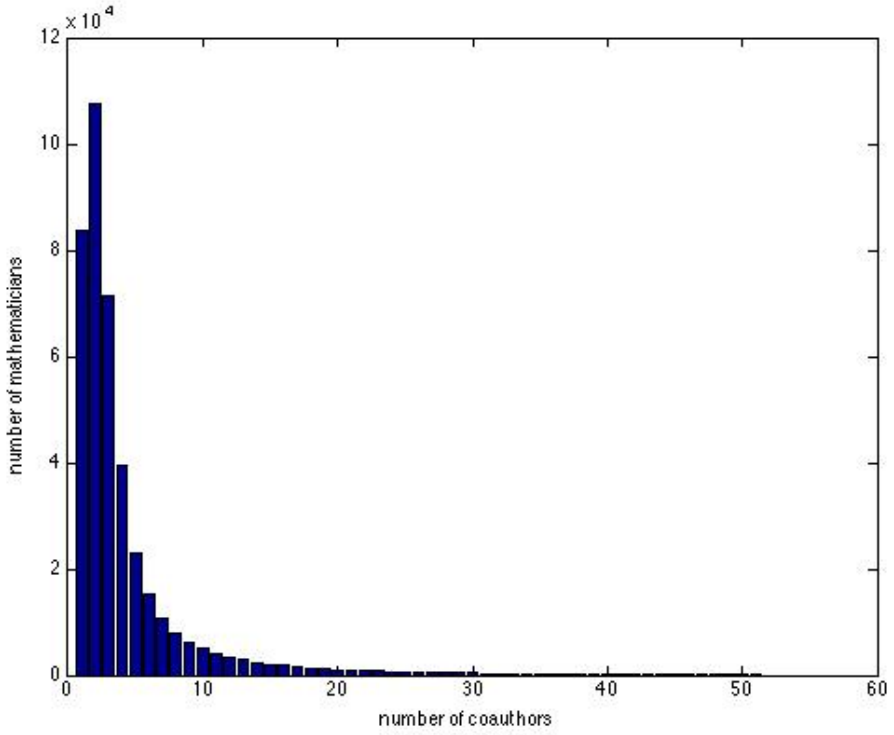
- Exogenous: famous/accomplished/influential/etc individuals
- “Hubs”: high-degree individuals
- Centrality: individuals in the “middle” of the network
- How are these related?



most central squash player, local network

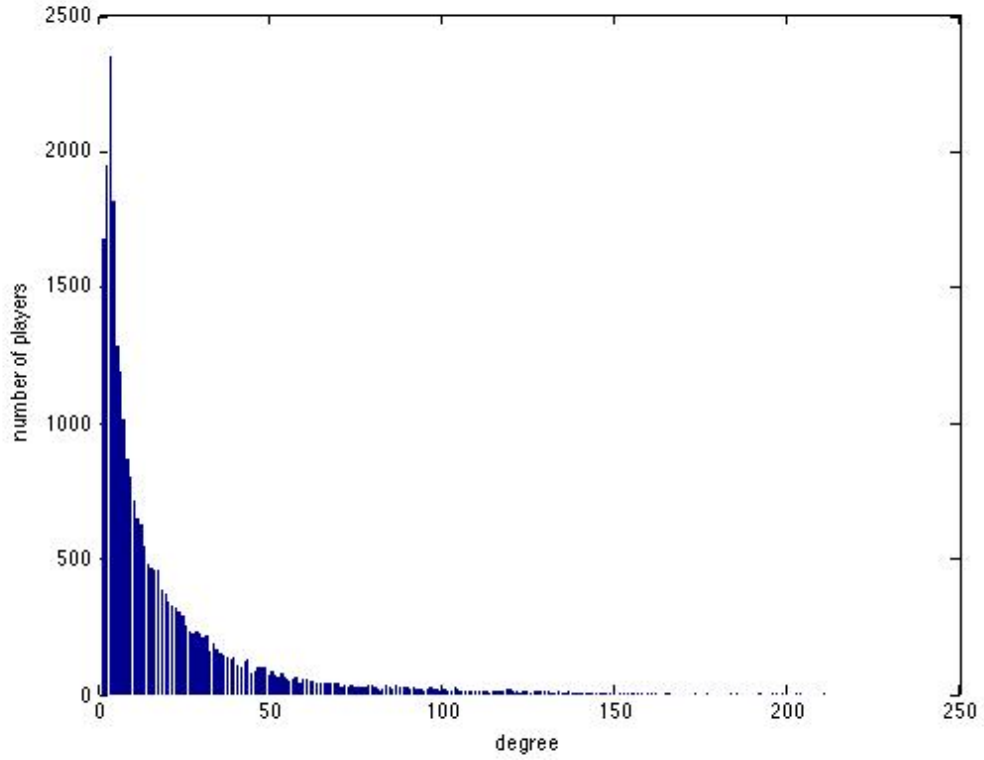
Math Collaboration Degree Distribution

- x axis: number of neighbors/coauthors (degree)
- y axis: number of mathematicians with that degree



Squash Network Degree Distribution

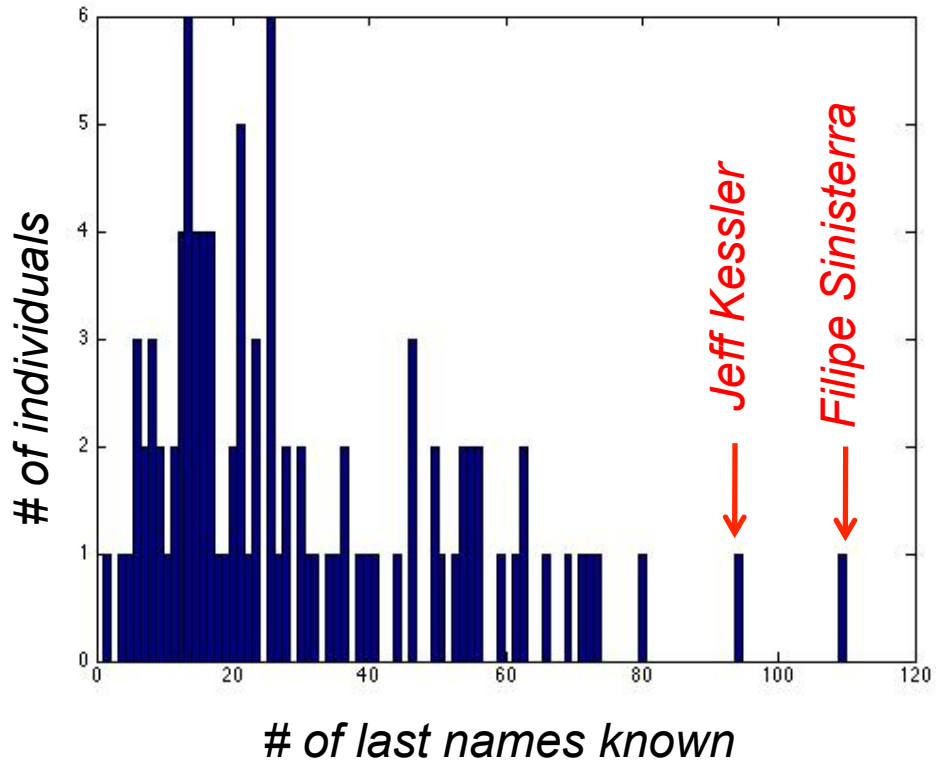
- x axis: number of opponents (degree)
- y axis: number of players with that degree



Illustrating the Concepts

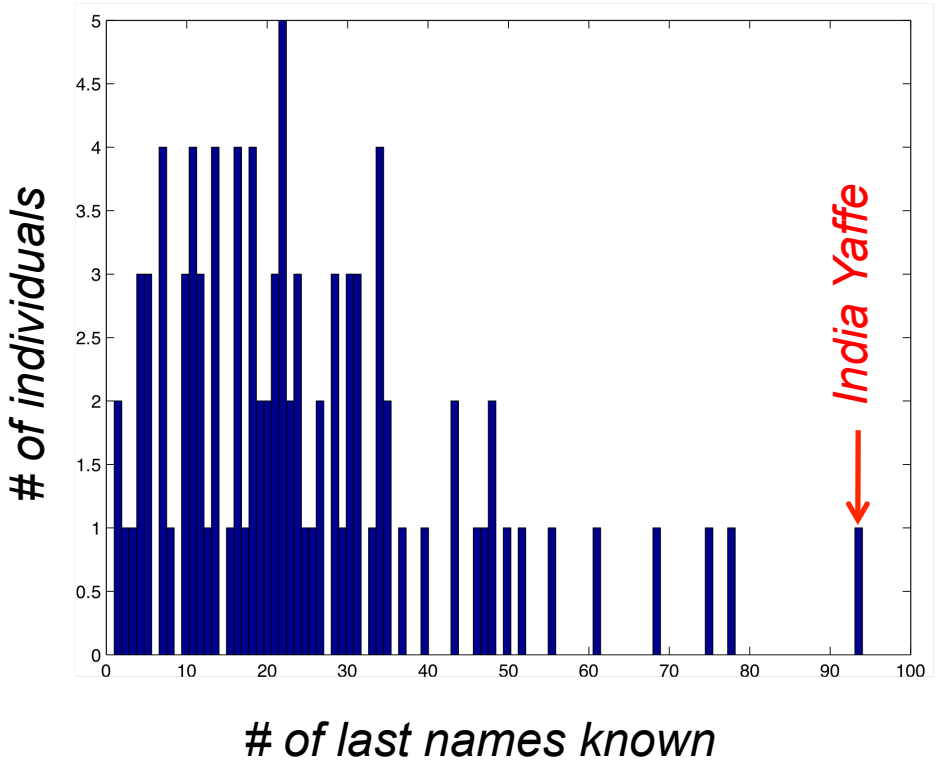
- Example: “real-world” acquaintanceship networks
  - vertices: people in the world
  - links: have met in person and know last names
  - hard to measure
  - let’s examine the results of our own *last-names exercise*

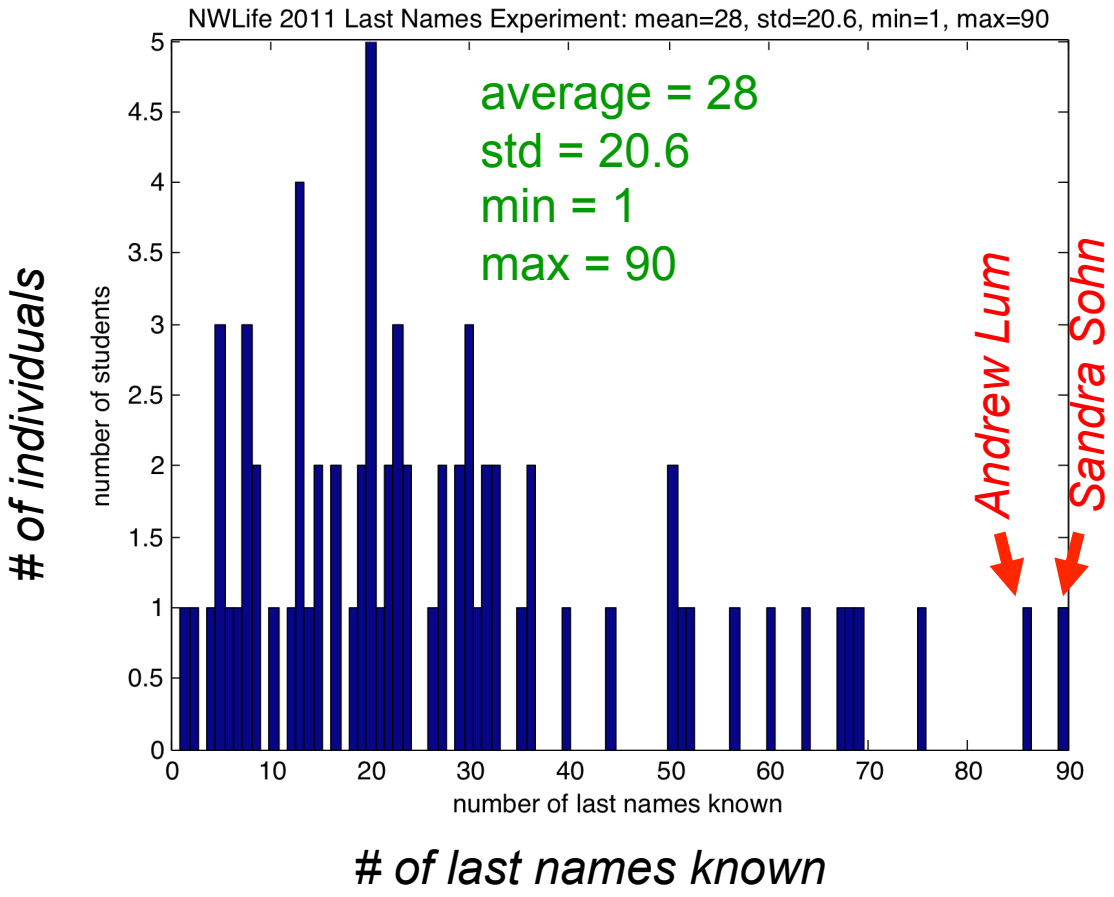
Algazi, Alvarez, Alpern, Ametrano, Andrews, Aran, Arnstein, Ashford, Bailey Ballout, Bamberger, Baptista, Barr, Barrows, Baskerville, Bassiri, Bell, Bokgese, Brandao, Bravo, Brooke, Brightman, Billy, Blau, Bohen, Bohn, Borsuk, Brendle, Butler, Calle, Cantwell, Carrell, Chinlund, Cirker, Cohen, Collas, Couch, Callegher, Calcaterra, Cook, Carey, Cassell, Chen, Chung, Clarke, Cohn, Carton, Crowley, Curbelo, Dellamanna, Diaz, Dirar, Duncan, Dagostino, Delakas, Dillon, Donaghey, Daly, Dawson, Edery, Ellis, Elliott, Eastman, Easton, Famous, Fermin, Fialco, Finklestein, Farber, Falkin, Feinman, Friedman, Gardner, Gelpi, Glascock, Grandfield, Greenbaum Greenwood, Gruber, Garil, Goff, Gladwell, Greenup, Gannon, Ganshaw, Garcia, Gennis, Gerard, Gericke, Gilbert, Glassman, Glazer, Gomendio, Gonzalez, Greenstein, Guglielmo, Gurman, Haberkorn, Hoskins, Hussein, Hamm, Hardwick, Harrell, Hauptman, Hawkins, Henderson, Hayman, Hibara, Hehmann, Herbst, Hedges, Hogan, Hoffman, Horowitz, Hsu, Huber, Ikiz, Jaroschy, Johann, Jacobs, Jara, Johnson, Kassel, Keegan, Kuroda, Kavanau, Keller, Kevill, Kiew, Kimbrough, Kline, Kossoff, Kotzitzky, Kahn, Kiesler, Kosser, Korte, Leibowitz, Lin, Liu, Lowrance, Lundh, Laux, Leifer, Leung, Levine, Leiw, Lockwood, Logrono, Lohnes, Lowet, Laber, Leonardi, Marten, McLean, Michaels, Miranda, Moy, Marin, Muir, Murphy, Marodon, Matos, Mendoza, Muraki, Neck, Needham, Noboa, Null, O'Flynn, O'Neill, Orlowski, Perkins, Pieper, Pierre, Pons, Pruska, Paulino, Popper, Potter, Purpura, Palma, Perez, Portocarrero, Punwasi, Rader, Rankin, Ray, Reyes, Richardson, Ritter, Roos, Rose, Rosenfeld, Roth, Rutherford, Rustin, Ramos, Regan, Reisman, Renkert, Roberts, Rowan, Rene, Rosario, Rothbart, Saperstein, Schoenbrod, Schwed, Sears, Statosky, Sutphen, Sheehy, Silverton, Silverman, Silverstein, Sklar, Slotkin, Speros, Stollman, Sadowski, Schles, Shapiro, Sigdel, Snow, Spencer, Steinkol, Stewart, Stires, Stopnik, Stonehill, Tayss, Tilney, Temple, Torfield, Townsend, Trimpin, Turchin, Villa, Vasillov, Voda, Waring, Weber, Weinstein, Wang, Wegimont, Weed, Weishaus.

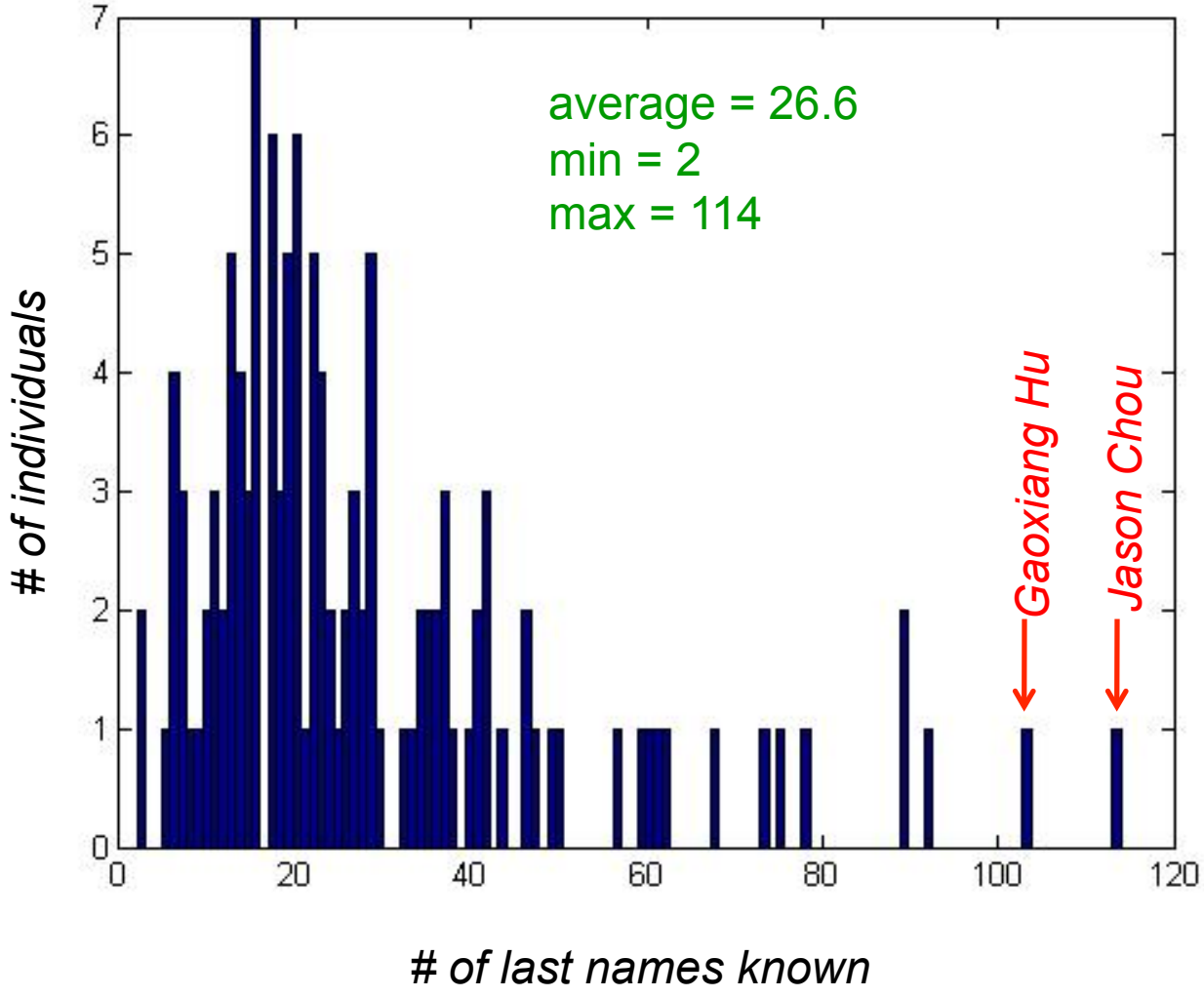


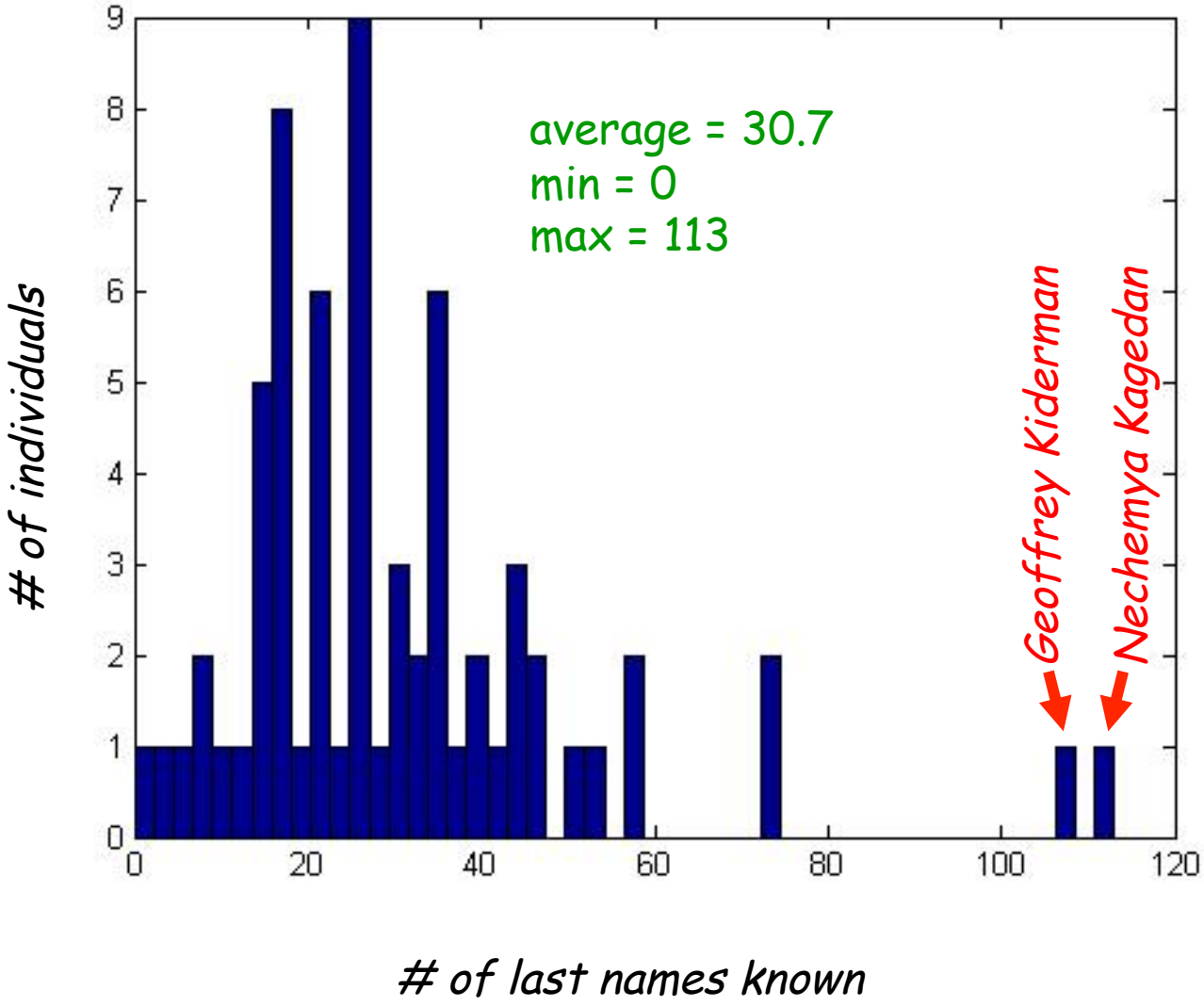
average = 30.2
std = 22.1
min = 1
max = 110

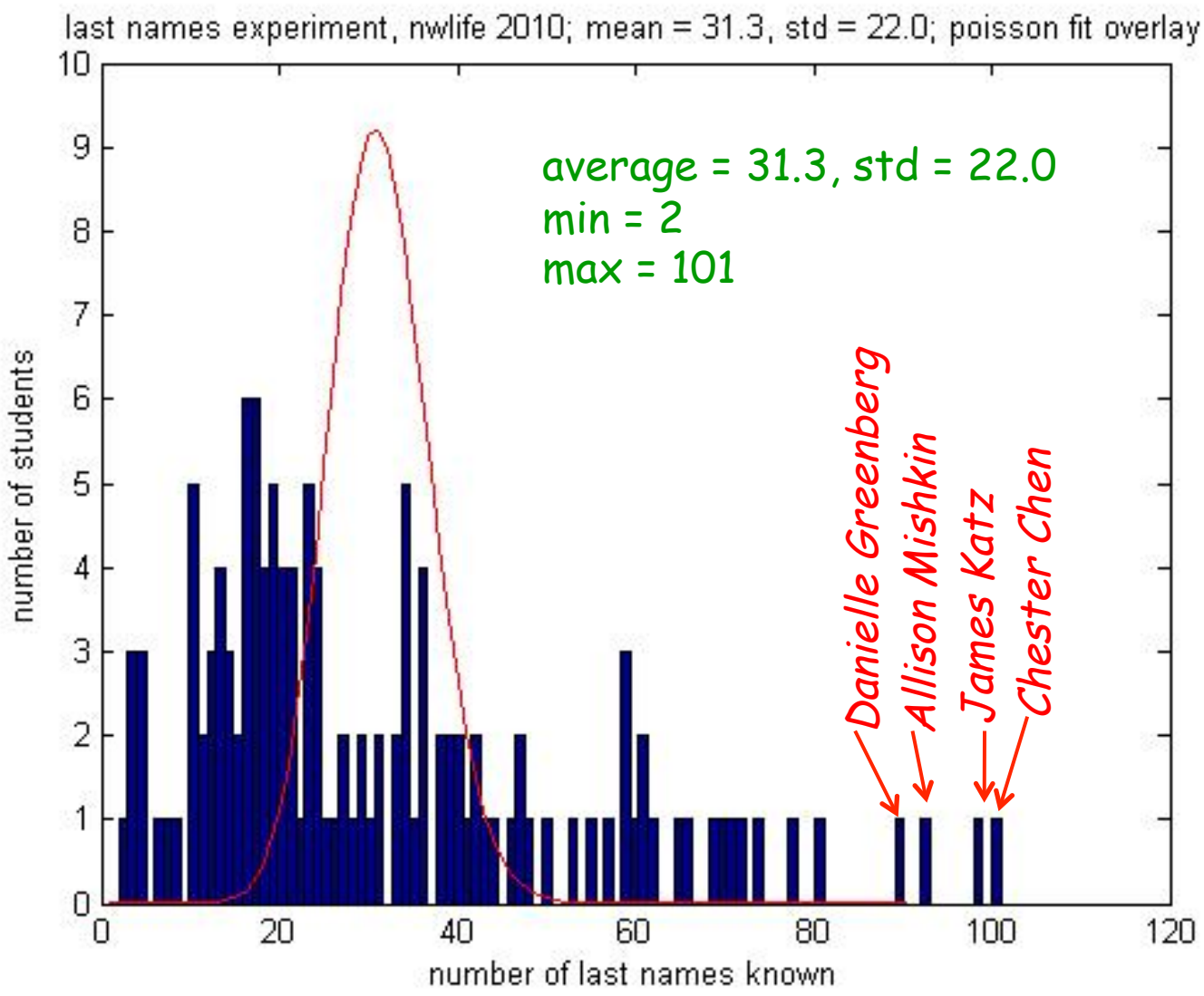
average = 24.6
std = 17.7
min = 1
max = 94




NWLife 2011 Last Names Experiment: mean=28, std=20.6, min=1, max=90
average = 28
std = 20.6
min = 1
max = 90
Andrew Lum
Sandra Sohn
number of students
number of last names known
# of individuals
# of last names known


average = 26.6
min = 2
max = 114
Gaoxiang Hu
Jason Chou
# of individuals
# of last names known
0
1
2
3
4
5
6
7
0
20
40
60
80
100
120


average = 30.7
min = 0
max = 113
Geoffrey Kiderman
Nechemya Kagedan
# of individuals
# of last names known
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120


last names experiment, nwlife 2010; mean = 31.3, std = 22.0; poisson fit overlay
number of students
number of last names known
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
average = 31.3, std = 22.0
min = 2
max = 101
Danielle Greenberg
Allison Mishkin
James Katz
Chester Chen

Structure, Dynamics, and Formation

Network *Structure (Statics)*

- Emphasize purely *structural* properties
  - size, diameter, connectivity, degree distribution, etc.
  - may examine statistics across many networks
  - will also use the term *topology* to refer to structure
- Structure can reveal:
  - community
  - “important” vertices, centrality, etc.
  - robustness and vulnerabilities
  - can also impose *constraints* on dynamics
- Less emphasis on what actually occurs *on* network
  - web pages are linked… but people surf the web
  - buyers and sellers exchange goods and cash
  - friends are connected… but have specific interactions

Network *Dynamics*

- Emphasis on what *happens* on networks
- Examples:
  - spread of disease/meme/fad in a social network
  - computation of a proper coloring
  - computation in the brain
  - spread of wealth in an economic network
- Statics and dynamics often closely linked
  - rate of disease spread (dynamic) depends critically on network connectivity (static)
  - distribution of wealth depends on network topology
- Dynamics of *transmission* most often studied
- What about dynamics with self-interest, deliberation, rationality?

Network *Formation*

- Why does a particular structure emerge?
- Plausible processes for network formation?
- Generally interested in processes that are
  - decentralized
  - distributed
  - limited to local communication and interaction
  - “organic” and growing
  - consistent with (some) measurement
- The Internet versus traditional telephony