Networks: Basic Definitions

• A network (or graph) is:
  - a collection of individuals or entities, each called a vertex or node
  - a list of pairs of vertices that are neighbors, representing edges or links

• Examples:
  - vertices are mathematicians, edges represent coauthorship relationships
  - vertices are Facebook users, edges represent Facebook friendships
  - vertices are news articles, edges represent word overlap

• Networks can represent any binary relationship over individuals
• Often helpful to visualize networks with a diagram
• But to us, the network is the list of edges, not the visualization
  - same network has many different visualizations
Networks: Basic Definitions

- We will use $N$ to denote the number of vertices in a network.
- Number of possible edges:

$$N(N - 1)/2 \approx N^2/2$$

- The degree of a vertex is its number of neighbors.
Networks: Basic Definitions

• The distance between two vertices is the length of the shortest path connecting them.
• This assumes the network has only a single component or “piece”.
• If two vertices are in different components, their distance is undefined or infinite.
• The diameter of a network is the average distance between pairs.
• It measures how near or far typical individuals are from each other.
Networks: Basic Definitions

- So far, we have been discussing undirected networks
- Connection relationship is symmetric:
  - if vertex u is connected to vertex v, then v is also connected to u
  - Facebook friendship is symmetric/reciprocal
- Sometimes we’ll want to discuss directed networks
  - I can follow you on Twitter without you following me
  - web page A may link to page B, but not vice-versa
- In such cases, directionality matters and edges are annotated by arrows
Illustrating the Concepts

• Example: scientific collaboration
  - vertices: math and computer science researchers
  - links: between coauthors on a published paper
  - Erdos numbers: distance to Paul Erdos
    - Erdos was definitely a hub or connector; had 507 coauthors
    - MK’s Erdos number is 3, via Kearns → Mansour → Alon → Erdos
    - how do we navigate in such networks?

• Example: “real-world” acquaintanceship networks
  - vertices: people in the world
  - links: have met in person and know last names
  - hard to measure
  - let’s examine the results of our own last-names exercise
# of last names known

average = 24.6  
std = 17.7  
min = 1  
max = 94

# of individuals

India Yaffe
# of last names known

- Average: 28
- Std: 20.6
- Min: 1
- Max: 90

Andrew Lum

Sandra Sohn
average = 26.6
min = 2
max = 114
The histogram shows the distribution of the number of last names known among individuals. The average is 30.7, with a minimum of 0 and a maximum of 113. Notable individuals mentioned are Geoffrey Kiderman and Nechemya Kagedan.
average = 31.3, std = 22.0
min = 2
max = 101
Structure, Dynamics, and Formation
Network \textit{Structure (Statics)}

- Emphasize purely \textit{structural} properties
  - size, diameter, connectivity, degree distribution, etc.
  - may examine statistics across many networks
  - will also use the term \textit{topology} to refer to structure

- Structure can reveal:
  - community
  - “important” vertices, centrality, etc.
  - robustness and vulnerabilities
  - can also impose \textit{constraints} on dynamics

- Less emphasis on what actually occurs \textit{on} network
  - web pages are linked... but people surf the web
  - buyers and sellers exchange goods and cash
  - friends are connected... but have specific interactions
Network Dynamics

• Emphasis on what *happens* on networks
• Examples:
  - spread of disease/meme/fad in a social network
  - computation of a proper coloring
  - computation in the brain
  - spread of wealth in an economic network
• Statics and dynamics often closely linked
  - rate of disease spread (dynamic) depends critically on network connectivity (static)
  - distribution of wealth depends on network topology
• Dynamics of *transmission* most often studied
• What about dynamics with self-interest, deliberation, rationality?
Network Formation

• Why does a particular structure emerge?
• Plausible processes for network formation?
• Generally interested in processes that are
  - decentralized
  - distributed
  - limited to local communication and interaction
  - “organic” and growing
  - consistent with (some) measurement
• The Internet versus traditional telephony