Structural Properties of Networks: Introduction

Networked Life
NETS 112
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Prof. Michael Kearns
Networks: Basic Definitions

- A network (or graph) is:
  - a collection of individuals or entities, each called a vertex or node
  - a list of pairs of vertices that are neighbors, representing edges or links

- Examples:
  - vertices are mathematicians, edges represent coauthorship relationships
  - vertices are Facebook users, edges represent Facebook friendships
  - vertices are news articles, edges represent word overlap

- Networks can represent any binary relationship over individuals
- Often helpful to visualize networks with a diagram
- But to us, the network is the list of edges, not the visualization
  - same network has many different visualizations
Networks: Basic Definitions

• We will use $N$ to denote the number of vertices in a network
• Number of possible edges:

\[ N(N - 1)/2 \approx N^2/2 \]

• The degree of a vertex is its number of neighbors
Networks: Basic Definitions

• The distance between two vertices is the length of the shortest path connecting them
• This assumes the network has only a single component or “piece”
• If two vertices are in different components, their distance is undefined or infinite
• The diameter of a network is the average distance between pairs
• It measures how near or far typical individuals are from each other
Networks: Basic Definitions

- So far, we have been discussing undirected networks
- **Connection relationship is symmetric:**
  - if vertex u is connected to vertex v, then v is also connected to u
  - Facebook friendship is symmetric/reciprocal
- **Sometimes we’ll want to discuss directed networks**
  - I can follow you on Twitter without you following me
  - web page A may link to page B, but not vice-versa
- In such cases, directionality matters and edges are annotated by arrows
Illustrating the Concepts

- **Example: scientific collaboration**
  - vertices: math and computer science researchers
  - links: between coauthors on a published paper
  - *Erdos numbers*: distance to Paul Erdos
  - Erdos was definitely a hub or connector; had 507 coauthors
  - MK’s Erdos number is 3, via Kearns → Mansour → Alon → Erdos
  - how do we navigate in such networks?

- **Example: “real-world” acquaintanceship networks**
  - vertices: people in the world
  - links: have met in person and know last names
  - hard to measure
  - let’s examine the results of our own last-names exercise
# of last names known

- average = 28
- std = 20.6
- min = 1
- max = 90

Andrew Lum

Sandra Sohn
average = 31.3, std = 22.0
min = 2
max = 101
# of individuals

Gaoxiang Hu
Jason Chou

average = 26.6
min = 2
max = 114

# of last names known
# of last names known

average = 30.7
min = 0
max = 113

Geoffrey Kiderman
Nechemya Kagedan
Structure, Dynamics, and Formation
Network Structure (Statics)

• Emphasize purely **structural** properties
  - size, diameter, connectivity, degree distribution, etc.
  - may examine statistics across many networks
  - will also use the term **topology** to refer to structure

• Structure can reveal:
  - community
  - “important” vertices, centrality, etc.
  - robustness and vulnerabilities
  - can also impose **constraints** on dynamics

• Less emphasis on what actually occurs *on* network
  - web pages are linked... but people surf the web
  - buyers and sellers exchange goods and cash
  - friends are connected... but have specific interactions
Network Dynamics

- Emphasis on what *happens* on networks
- Examples:
  - mapping spread of disease in a social network
  - mapping spread of a fad
  - computation in the brain
  - spread of wealth in an economic network
- Statics and dynamics often closely linked
  - rate of disease spread (dynamic) depends critically on network connectivity (static)
  - distribution of wealth depends on network topology
- Gladwell emphasizes dynamics
  - but often dynamics of *transmission*
  - what about dynamics involving deliberation, rationality, etc.?
Network Formation

• Why does a particular structure emerge?
• Plausible processes for network formation?
• Generally interested in processes that are
  - decentralized
  - distributed
  - limited to local communication and interaction
  - “organic” and growing
  - consistent with (some) measurement
• The Internet versus traditional telephony
Structure and Dynamics Case Study: A “Contagion” Model of Economic Exchange

- Imagine an undirected, connected network of individuals
  - no model of network formation
- Start each individual off with some amount of currency
- At each time step:
  - each vertex divides their current cash equally among their neighbors
  - (or chooses a random neighbor to give it all to)
  - each vertex thus also receives some cash from its neighbors
  - repeat
- A transmission model of economic exchange --- no “rationality”
- Q: How does network structure influence outcome?
- A: As time goes to infinity:
  - vertex i will have fraction $\text{deg}(i)/D$ of the wealth; $D = \text{sum of deg}(i)$
  - degree distribution entirely determines outcome!
  - “connectors” are the wealthiest
  - not obvious: consider two degree = 2 vertices...
- How does this outcome change when we consider more “realistic” dynamics?
  - e.g. we each have goods available for trade/sale, preferred goods, etc.
- What other processes have similar dynamics?
  - looking ahead: models for web surfing behavior