CSE 112: Networked Life
Problem Set 1

Due Thursday, February 8, in hardcopy form at the start of class

1. For each of the following general categories of networks discussed in class, give a real-world example of such a network other than those given in class. You should be as clear as possible about what the vertices (nodes) are and about what relationship between two vertices determines whether or not there is an edge (link) between them. For each of your networks, comment in a sentence or two on its static structure, its dynamics, and its formation process:

   (a) Social Networks

   (b) Content Networks

   (c) Business or Economic Networks

   (d) Physical Networks

   (e) Biological Networks

2. Suppose you are given the task of designing a network with the smallest possible worst-case diameter. (Recall that the worst-case diameter of a network is the *longest* shortest path between any pair of vertices.) The network is to have exactly 7 vertices and exactly k edges. For each value of k below, draw a network of 7 vertices and k edges that minimizes the worst-case diameter. In each case, label the two vertices that are furthest apart.

   (a) $k = 6$

   (b) $k = 7$

   (c) $k = 10$

3. In class we mentioned that random connectivity (e.g. in which each possible edge appears randomly and independently with some probability) is not a good fit for the social networks that arise in many natural social networks, such as those of the Facebook, mathematical collaboration, and real-world acquaintanceship. In what ways do such random connectivity models fail to explain real social networks? (You may cite both properties mentioned in class, as well as any others you think apply.) What model of network generation might be a better fit? You can explain your model mathematically or in English, but be precise either way.

4. Think about a fad or trend you have observed that occurred *approximately in the last year*. This could be a large-scale, nationwide trend or just

something that was popular in your circle of friends or colleagues, but should not be a trend that was mentioned specifically in "The Tipping Point" or in class.

(a) Briefly describe this trend.

(b) In the language of Gladwell, Who were the connectors, mavens, and salesmen who caused this trend to spread?

(c) What made this trend stick?

(d) Was the spread of this trend gradual and steady, or did it exhibit tipping phenomena?. Why do you think this is the case?

5. Consider a network in which person A has ten friends, the ten friends of person A each have ten friends, and those ten friends each have ten friends. There are no other people in this network besides the ones that have been mentioned. We assume that people cannot be their own friends.

   (a) What is largest number of people who could possibly be in such a network? Describe the structure of this network.

   (b) What is the smallest number? Describe the structure of this network.

6. For this problem you will need to have an account on the Facebook. For those of you who do not already have one, you can obtain one by going to `www.thefacebook.com` and following the sign-up instructions. Note that in order to complete this problem, there is no need for you to create a publicly viewable profile, nor to have any connections (friends).

   (a) Find the individual in the Penn community Facebook with the largest number of friends (highest degree) that you can. You can use any method that you like, including those that use information external to the Facebook. Write the name of the individual and the value of their degree, and briefly describe your method of finding them. Try to make sure you find an actual person, as opposed to a group or organization.

   (b) Repeat part (a), but instead find the individual with the lowest degree (other than yourself) . Again write the name of the individual and their degree, and describe your method of finding them.

   (c) By using the Facebook's search facility, and using the search within "All Networks" option, you can find all users who have a chosen word, name, phrase, etc. in their profiles. Pick any phrase such that the resulting search produces at least 30 and at most 100 individuals. Then generate a histogram of the degrees of these individuals; you may use Excel or any other plotting software that you like, or you can do it carefully and clearly by hand. Compute and provide the

minimum, maximum, and average degrees of this population, and provide them with your histogram.

(d) Discuss the extent to which your degree histogram from part (c) does or does not seem to exhibit the properties discussed in class for such distributions in natural social networks.

(e) (Extra Credit) A cycle in a network is a set of vertices that are connected in a ring; for instance:

A connected to B
B connected to C
C connected to D
D connected to A

where A, B, C and D are all different individuals, is a cycle of length 4. Note that there may or may not be "chords" that cut across this cycle (e.g. B and D might be directly connected). Find the longest cycle you can in the Facebook of length at least 3, and write down the names of the individuals in the order they appear in this cycle. Describe your method for discovering this cycle.

7. Consider the undirected graph shown in the figure below. Answer the following questions with respect to that graph:

(a) What are the minimum and maximum degrees in the graph?

(b) What is the worst-case diameter? Identify the vertices that are farthest apart (if more than one such pair is possible, then list all of them).

(c) What is the size of the largest clique in the graph? List the vertices in each clique of this size in the graph.

(d) What is the minimum number of edges that could be removed to make the graph unconnected? List these edges.