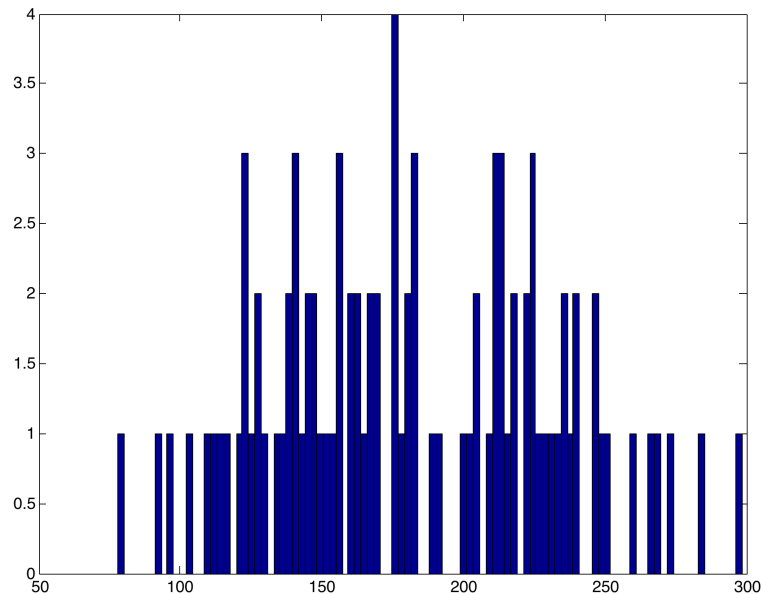


Graph Coloring Update

Best Times by Graph

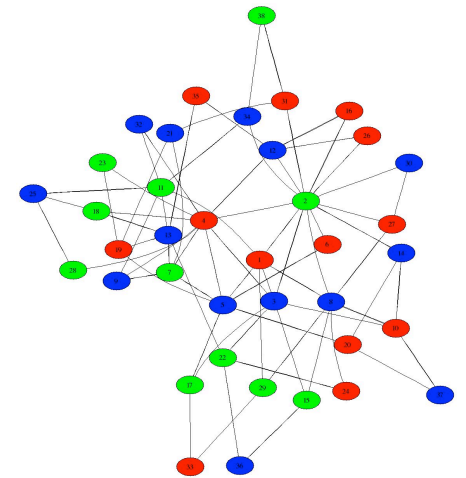
Buenos Aires 26.952
Toronto 29.95
Tehran 32.203
Moscow 35.249
Tokyo 35.384
Taipei 36.166
Santiago 37.453
Madrid 39.495
Delhi 39.579
New York City 43.921
Istanbul 45.353
Jakarta 45.578
Mexico City 46.305
Sao Paulo 46.682
Shanghai 46.977
Seoul 49.793
Paris 52.029
Milan 52.581
London 53.132
Lima 53.282
Manila 54.073
Mumbai 55.002
Cairo 56.125
Kyoto 57.242
Los Angeles 58.311
Hong Kong 64.48
Johannesburg 65.608
Baghdad 72.03
Bangkok 73.315
Beijing 74.776

Player Averages Across Graphs



Talking About Coloring

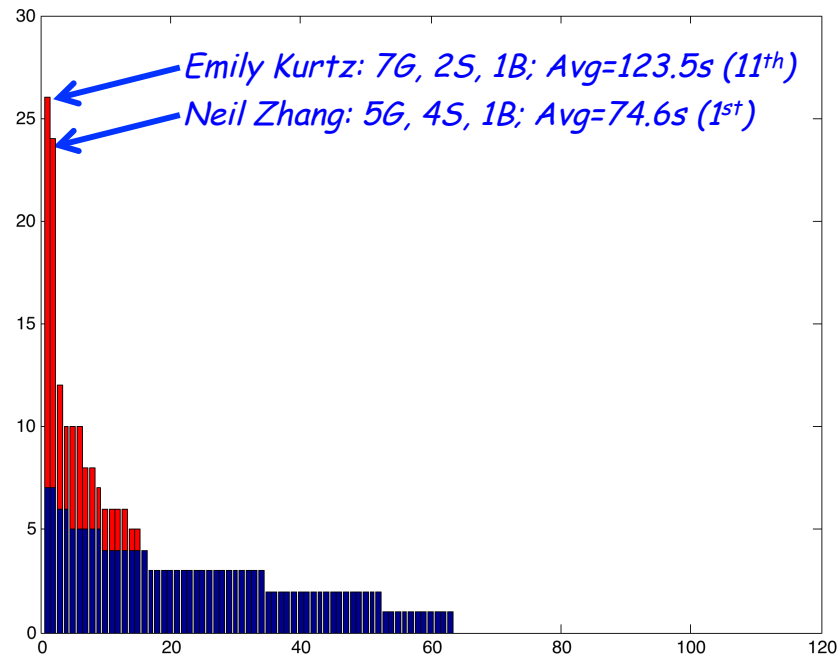
- What strategies did you use? Did they change over time?
- How did you resolve conflicts?
- How would you design an algorithm for the problem?
- How do you think the graphs were generated?
- How would you design a graph that is hard for people to color?
- What if a group of you had to color graphs in distributed fashion?



Graph Coloring Olympics



Ranked Point Totals



Navigation in Networks

Networked Life

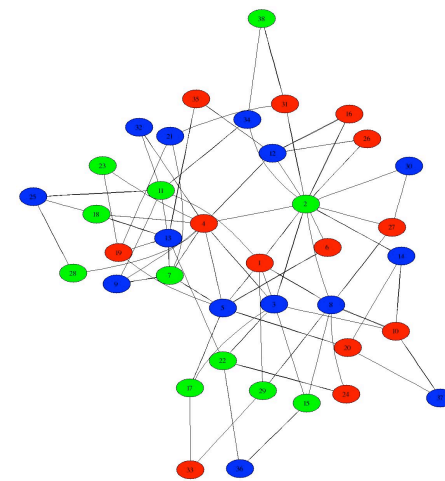
NETS 112

Fall 2014

Prof. Michael Kearns

The Navigation Problem

- You are an individual (vertex) in a very large social network
- You want to find a (short) chain of friendships to another individual
- You don't have huge computers and a global/bird's-eye view
- All you (hopefully) know is who your neighbors/friends are
 - ...and perhaps information about them (age, interests, religion, address, job,...)
- You can ask your friends to make introductions, which lead to more
- How would you do it?
- Also known as search in networks and the "small world problem"
- Small diameter is necessary but not sufficient!
 - ...navigation is an algorithmic problem
- Related to the problem of routing data packets in the Internet



Small Worlds and the Law of the Few

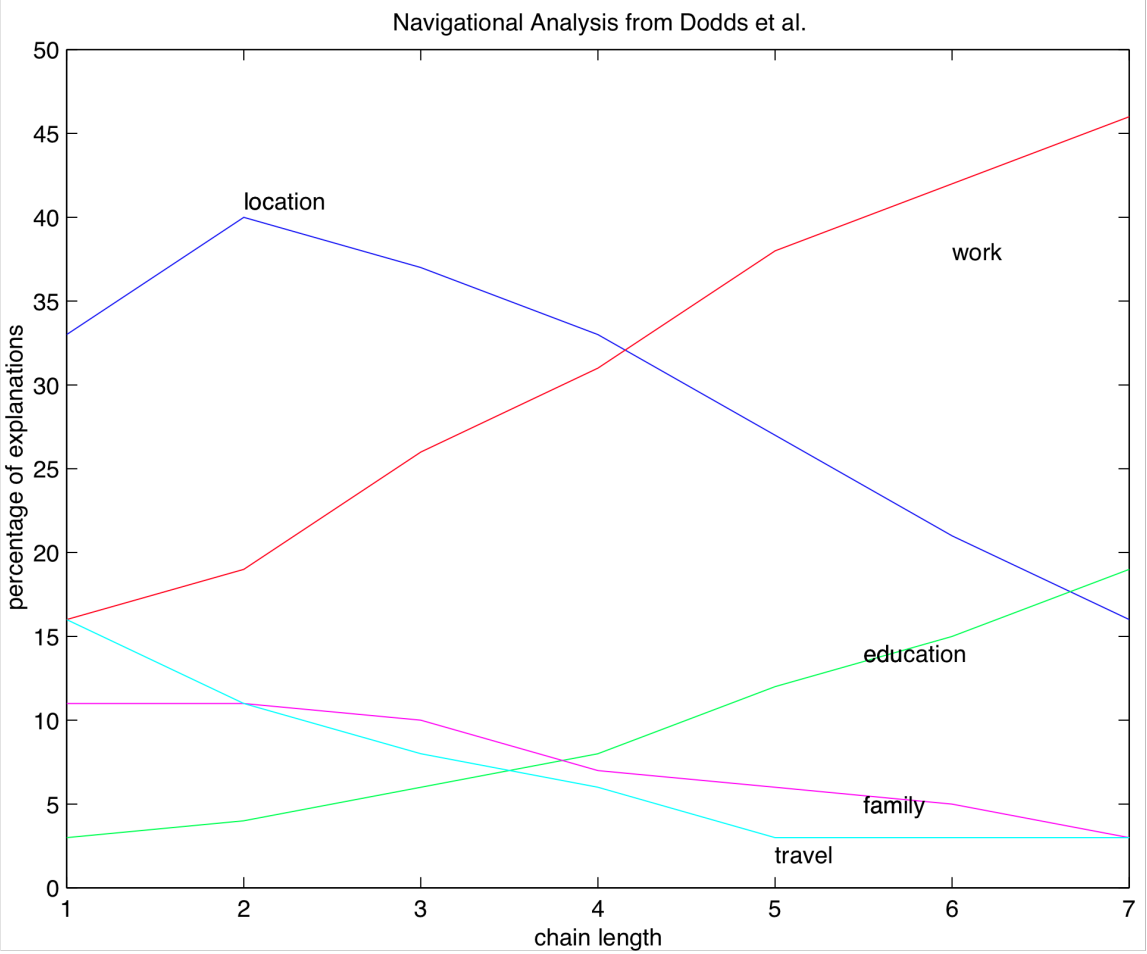
- Travers & Milgram 1969: classic early social network study
 - destination: a Boston stockbroker; lived in Sharon, MA
 - sources: Nebraska stockowners; Nebraska and Boston "randoms"
 - forward letter to a first-name acquaintance "closer" to target
 - target information provided:
 - name, address, occupation, firm, college, wife's name and hometown
 - navigational value?
- Basic findings:
 - 64 of 296 chains reached the target
 - average length of *completed* chains: 5.2
 - interaction of chain length and navigational difficulties
 - main approach routes: home (6.1) and work (4.6)
 - Boston sources (4.4) faster than Nebraska (5.5)
 - no advantage for Nebraska stockowners

The Connectors to the Target

- T & M found that many of the completed chains passed through a very small number of penultimate individuals
 - Mr. G, Sharon merchant: 16/64 chains
 - Mr. D and Mr. P: 10 and 5 chains
- Connectors are individuals with extremely high degree
 - why should connectors exist?
 - how common are they?
 - how do they get that way? (see Gladwell for anecdotes)
- Connectors can be viewed as the "hubs" of social traffic
- Note: no reason *target* must be a connector for small worlds
- Two ways of getting small worlds (low diameter):
 - truly random connection pattern → dense network
 - a small number of well-placed connectors in a sparse network

Small Worlds: A Modern Experiment

- The Columbia Small Worlds Project:
 - considerably larger subject pool, uses email
 - subject of Dodds et al. assigned paper
- Basic methodology:
 - 18 targets from 13 countries
 - on-line registration of initial participants, all tracking electronic
 - 99K registered, 24K initiated chains, 384 reached targets
- Some findings:
 - < 5% of messages through any penultimate individual
 - large "friend degree" rarely (< 10%) cited
 - Dodds et al: → no evidence of connectors!
 - (but could be that connectors are not cited for this reason...)
 - interesting analysis of reasons for forwarding
 - interesting analysis of navigation method vs. chain length



The Strength of Weak Ties

- Not all links are of equal importance
- Granovetter 1974: study of job searches
 - 56% found current job via a personal connection
 - of these, 16.7% saw their contact "often"
 - the rest saw their contact "occasionally" or "rarely"
- Your "closest" contacts might not be the most useful
 - similar backgrounds and experience
 - they may not know much more than you do
 - connectors derive power from a large fraction of weak ties
- Further evidence in Dodds et al. paper
- T&M, Granovetter, Gladwell: multiple "spaces" & "distances"
 - geographic, professional, social, recreational, political,...
 - we can reason about general principles without precise measurement

The Magic Number 150

- Social channel capacity
 - correlation between neocortex size and group size
 - Dunbar's equation: neocortex ratio \rightarrow group size
- Clear implications for many kinds of social networks
- Again, a *topological* constraint on typical degree
- From primates to military units to Gore-Tex

Neocortex size and group size in primates

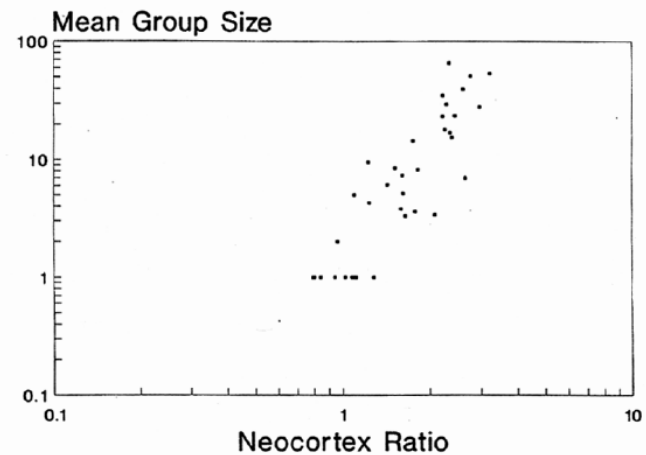


Figure 1. Group size plotted against neocortex ratio for nonhuman primates (redrawn from Dunbar 1992a).

Summary, and a Mathematical Digression

- So far:
 - large-scale social networks reliably have high-degree vertices
 - large-scale social networks have small diameter
 - furthermore, people can find or navigate the short paths from only local, distributed knowledge
 - these properties are true of other types of networks, too
- But there must be some limits to degrees
 - can't be "close friends" with too many people (150? 1000?)
- Large N , small diameter and limited degrees are in tension
 - not all combinations are possible
- Let N be population size, Δ be the maximum degree, and D be the diameter
- If $\Delta = 2$ then must have $D \sim N/4$ ($\gg 6$, $\gg \log(N)$)

Summary, and a Mathematical Digression

- The relationship between Δ , D and N has been studied mathematically
- For fixed Δ and D , largest N can be is

$$N \leq \Delta^D$$

- For example: if $N = 300\text{M}$ (U.S. population) and $\Delta = 150$, get constraint on D :

$$300,000,000 \leq (150)^D$$

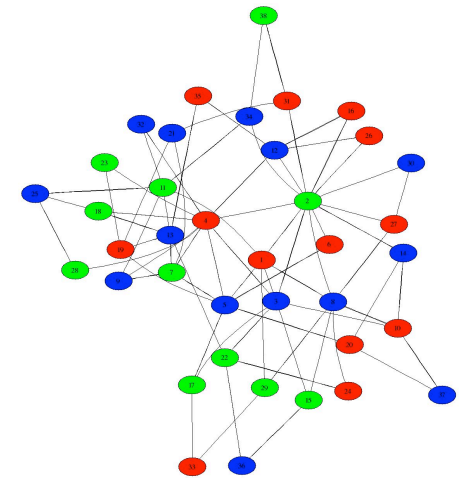
$$\log(300,000,000) \leq D \log(150)$$

$$D \geq 3.9$$

- So calculation consistent with reality (whew!)
- More generally: multiple structural properties may be *competing*

Two Aspects of Navigation

- In order for people (or machines) to find short paths in networks:
 - short paths must exist (structural; small diameter)
 - people must be able to find the short paths via only local forwarding (algorithmic)
- The algorithmic constraints are strong (Travers & Milgram)
 - only know your neighbors in the network
 - limited information about the target/destination (physical location, some background)
- Look at a model incorporating structural and algorithmic constraints

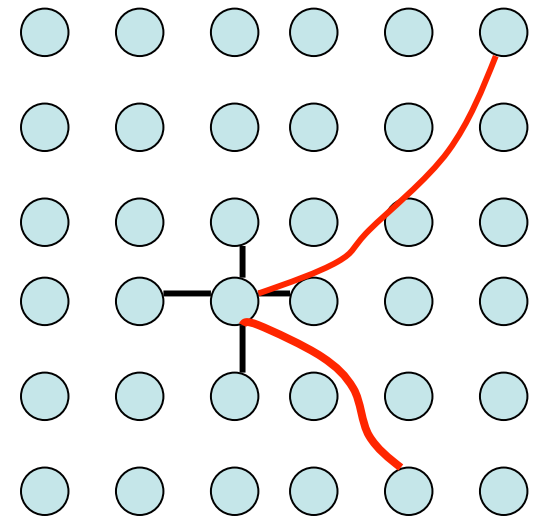
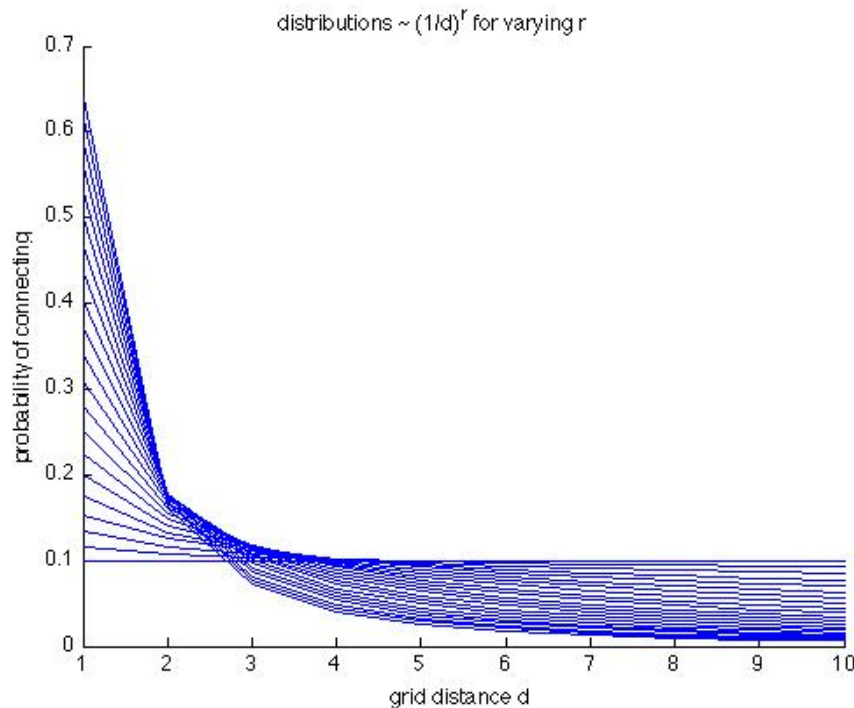


Kleinberg's Model

- Start with an k by k *grid* of vertices (so $N = k^2$)
 - each vertex connected to compass neighbors
 - add a few random "long-distance" connections to each vertex
 - probability $p(d)$ of connecting to a vertex at grid distance d :

$$p(d) \propto (1/d)^r, r \geq 0$$

- large r : heavy bias towards "more local" long-distance connections
- small r : approach uniformly random



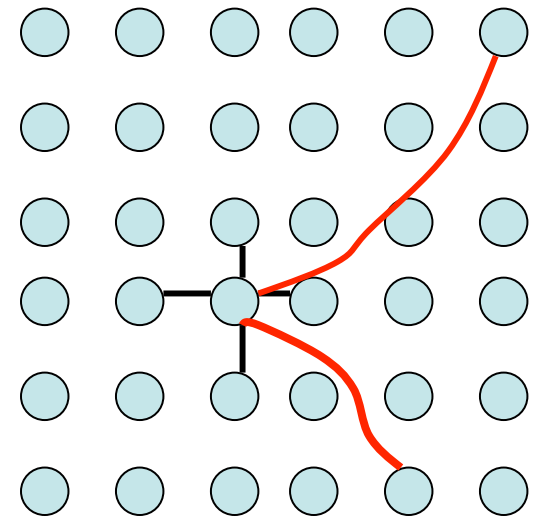
Kleinberg's Question

- Which values of r :

$$p(d) \propto (1/d)^r, r \geq 0$$

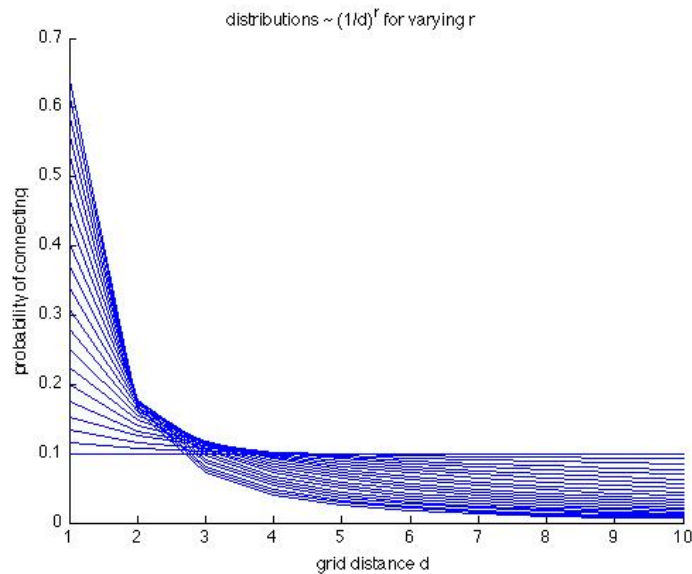
permit efficient navigation?

- Efficient: number of hops $\ll N$, e.g. $\log(N)$
- Algorithmic assumption:
 - vertices know the grid addresses of their neighbors
 - vertices know the grid address of the target (Sharon, MA)
 - vertices always forward the message to neighbor closest to the target in grid distance
 - no "backwards" steps, even if helpful
 - purely geographic information



Kleinberg's Result

- Intuition:
 - if r is too *large* (strong local bias), then "long-distance" connections never help much; short paths may not even *exist*
 - if r is too *small* (no local bias), we may quickly get close to the target; but then we'll have to use grid links to finish
 - effective search requires a delicate *mixture* of link distances
- The result (informally): as N becomes large:
 - $r = 2$ is the *only value* that permits rapid navigation ($\sim \log(N)$ steps)
 - a "knife's edge" result; very sensitive
- Note: *locality of information* crucial to this argument
 - At $r \leq 2$, will have small diameter, but local algorithms can't find the short paths



Where's George? Bill Tracking Report

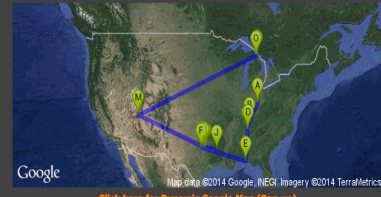
[Printer Friendly Version](#)

One Dollar Bill, Serial# K24-----1 Series: 1999

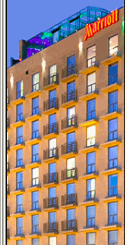
This bill has traveled at least 4,183 Miles in 3 Yrs, 11 Days, 17 Hrs, 26 Mins at an average of 3.8 Miles per day. The bill is now 452 Miles from its starting location. This travel history below is in reverse-chronological order

Entry Time (Local Time of Zip)	Location, State/Province (Green=USA, Blue=Canada, Purple=International)	Travel Time (from previous entry)	Distance (Miles) ¹	Average Speed Miles Per Day	Social Networking
26-Mar-2005 08:34 PM	Rudyard, MI	212 Days, 14 Hrs, 37 Mins	8.6	0.04	
User's Note: this bill is getting pretty old looking					
26-Aug-2004 06:57 AM	Kincheloe, MI	112 Days, 7 Hrs, 10 Mins	1,539	14	
User's Note: Motel					
05-May-2004 09:48 PM	Pangulitch, UT	104 Days, 3 Hrs, 51 Mins	937	9.0	
User's Note: I FOUND THIS BILL AT THE FLYING M					
22-Jan-2004 05:57 PM	Irving, TX	25 Days, 23 Hrs, 57 Mins	30	1.1	
User's Note: Mr K's Food Mart, Irving Tx					
27-Dec-2003 06:00 PM	Rockwall, TX	3 Days, 3 Hrs, 57 Mins	152	48	
User's Note: its condition is good got it at Jack in the box in rockwall tx.					
24-Dec-2003 02:03 PM	Shreveport, LA	8 Days, 1 Hr, 52 Mins	160	20	
16-Dec-2003 12:11 PM	Garland, TX	13 Days, 18 Hrs, 13 Mins	17	1.2	
02-Dec-2003 05:59 PM	Dallas, TX	22 Days, 20 Hrs, 8 Mins	15	0.67	
User's Note: Found on the floor at the Penthouse Key Club.					
09-Nov-2003 09:51 PM	Grapevine, TX	11 Days, 15 Hrs, 11 Mins	25	2.1	
User's Note: [Hit #326, 346, 359, 362, 419, 640, 667][Hit #740 dit'd 20 July 04][Hit #803, #858]. Rec'd marked bill at the restaurant , My 72nd link .					
29-Oct-2003 06:41 AM	Fort Worth, TX	292 Days, 15 Hrs, 52 Mins	632	2.2	
User's Note: Bill is still in good shape. I got it as change at a McDonalds in Keller TX.					
09-Jan-2003 02:49 PM	Milton, FL	177 Days, 6 Hrs, 45 Mins	348	2.0	
User's Note: Not sure where I received it, in change from somewhere. The condition of the bill is good.					
16-Jul-2002 09:04 AM	Unionville, TN	13 Days, 20 Hrs, 16 Mins	7.3	0.53	
User's Note: This bill was at a country store at Halls Mill community in Unionville, Tennessee USA					
02-Jul-2002 12:48 PM	Chapel Hill, TN	48 Days, 51 Mins	84	1.7	
User's Note: Came into my possession at the Shell Food Mart in Chapel Hill, TN.					
15-May-2002 11:57 AM	Scottsville, KY	61 Days, 8 Hrs, 49 Mins	229	3.7	
User's Note: I work at Sonic and received it as a tip. It's in pretty good condition.					
15-Mar-2002 03:08 AM	Dayton, OH	Initial Entry	n/a	n/a	
User's Note: Thanks for entering this bill, and welcome to Where's George! [wb]					

Size: [Small](#) [Large](#) [Hide](#) Type: [Roadmap](#) [Terrain](#) [Satellite](#) [Hybrid](#)



A 4 Diamond Experience



BOOK NOW

Real Time Hit Feed

One:8388--4J
1st hit
I got this bill from a Chinese restaurant in Roswell GA. The bill is in great condition and is very crisp.

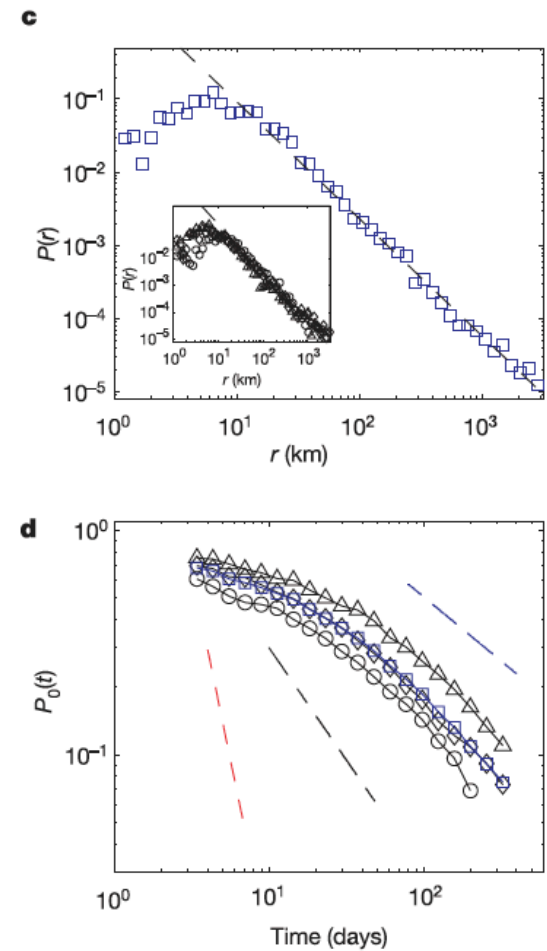
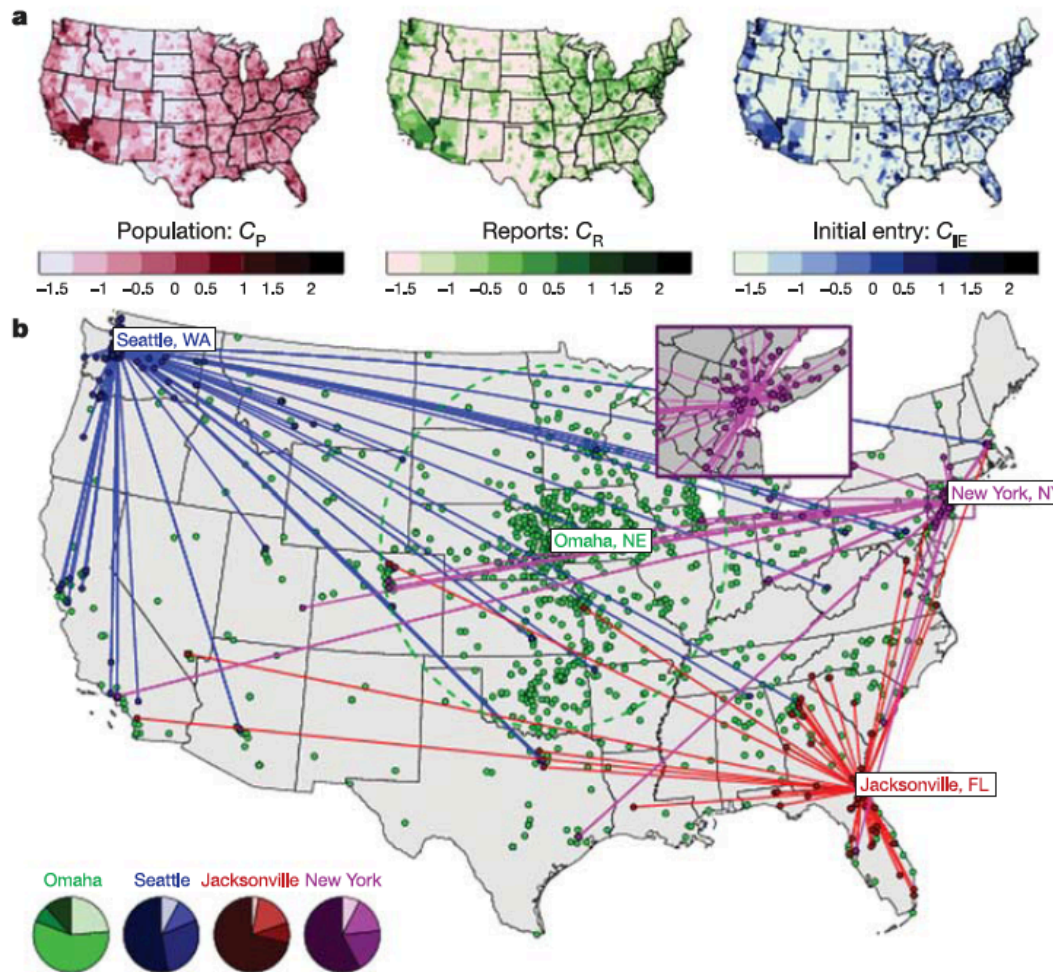
One:8201--6G
1st hit
I received from it a retail store in Columbia - still in great condition.

One:85274--4H
1st hit
wrinkly

Where's George? 4.0!

Announcements

Welcome to the all new Where's George? website. To see all the new changes at Where's George? 4.0, please click to see -> [this forum](#)

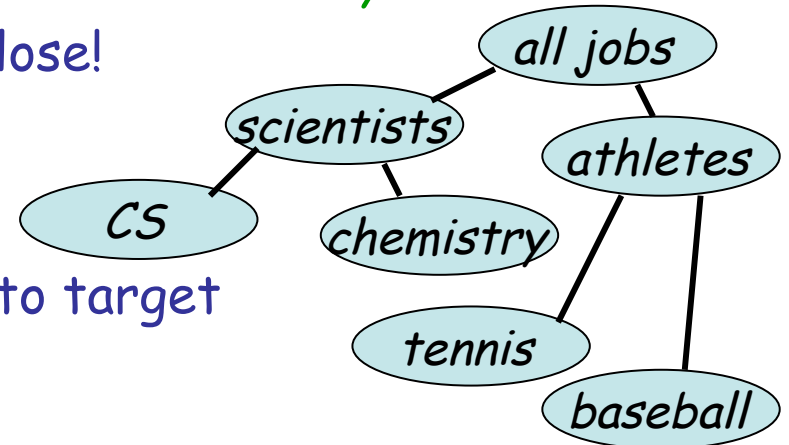


From Brockmann, Hufnagel, Geisel (2006)

Best-fit value of $r = 1.59$

Navigation via Identity

- Watts et al.:
 - we don't navigate social networks by purely "geographic" information
 - we don't use any *single* criterion; recall Dodds et al. on Columbia SW
 - different criteria used at different points in the chain
- Represent individuals by a *vector* of attributes
 - profession, religion, hobbies, education, background, etc...
 - attribute values have distances between them (tree-structured)
 - distance between individuals: minimum distance in *any* attribute
 - only need *one thing in common* to be close!
- Algorithm:
 - given attribute vector of target
 - forward message to neighbor closest to target
- Let's look a bit at the [paper](#)
- Permits fast navigation under broad conditions
 - not as sensitive as Kleinberg's model



Summary

- Efficient navigation has both structural and algorithmic requirements
- Kleinberg's model and question captures both
- Result predicts delicate mixture of connectivity for success
- Not too far from reality? (Where's George? data)
- Watts et al. provide more "sociological" model
- More complex, but less sensitive

