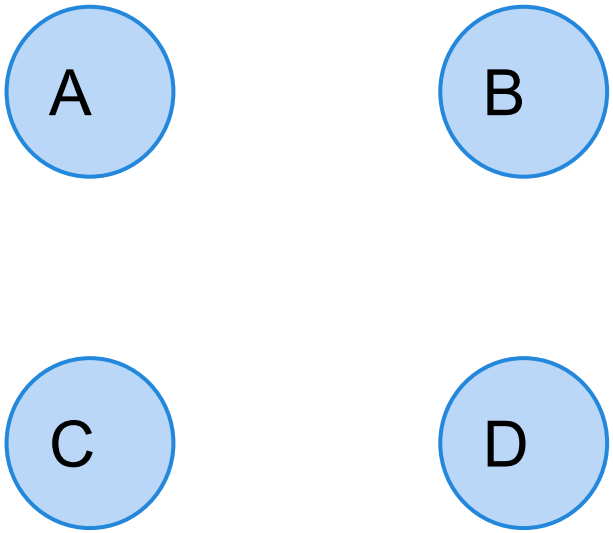
Midterm Review

NETS 112

Part 1: Network Structure

Is this a network?



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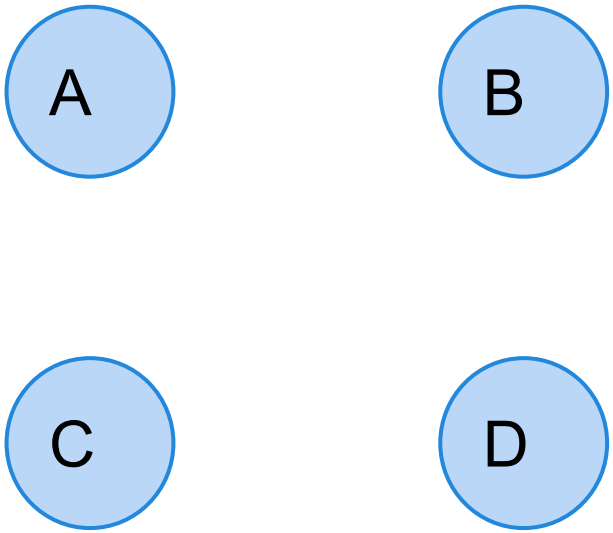
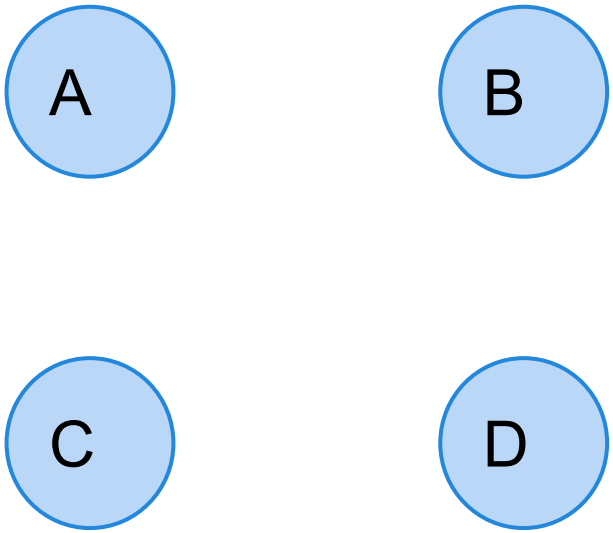


Yes -- G = (V, E), one or both can be empty

Part 1: Network Structure

What is the distance between A and B?



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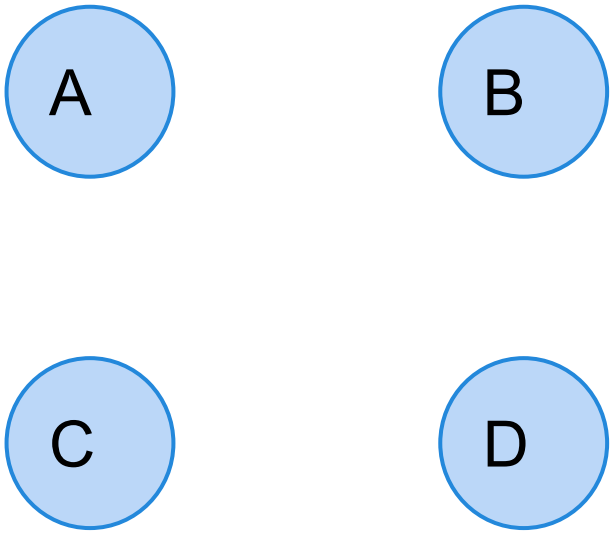
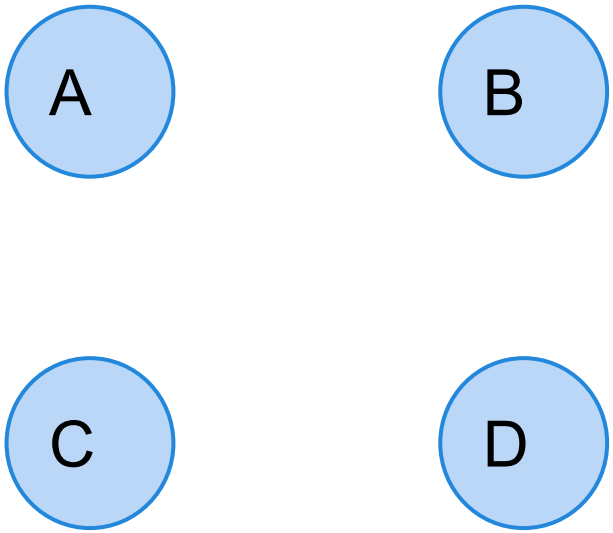


Infinite or undefined

Part 1: Network Structure

What is the diameter?



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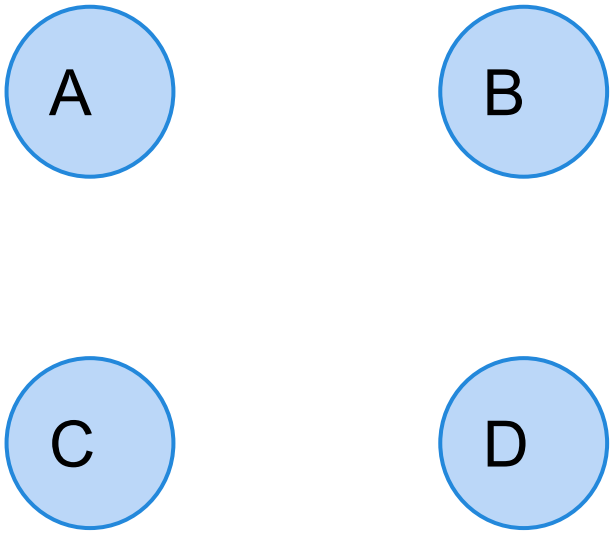
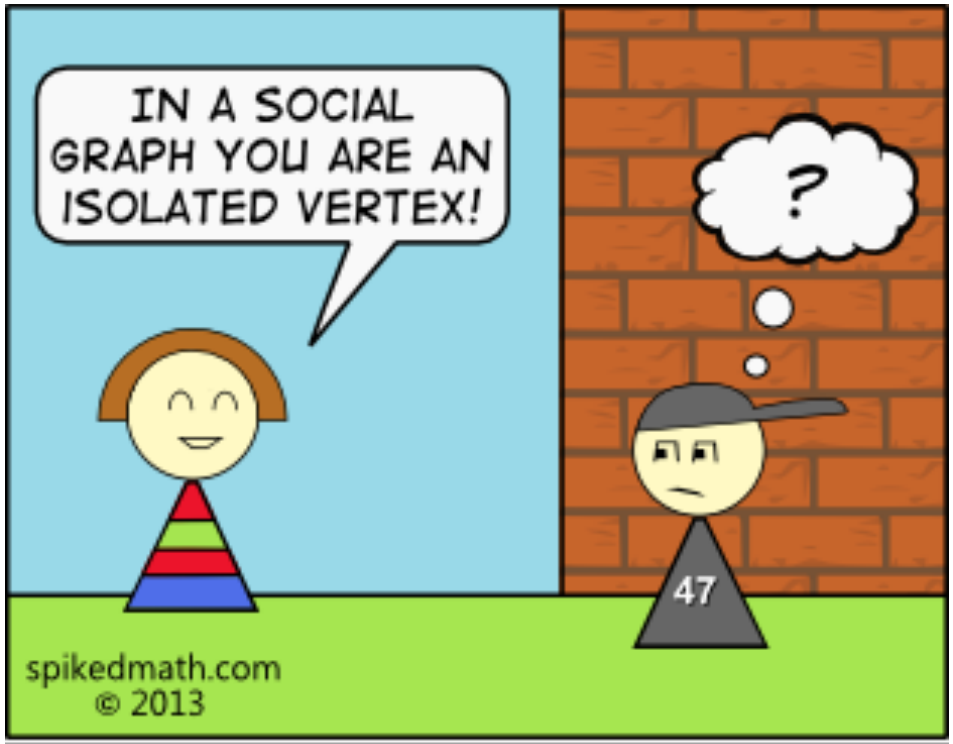


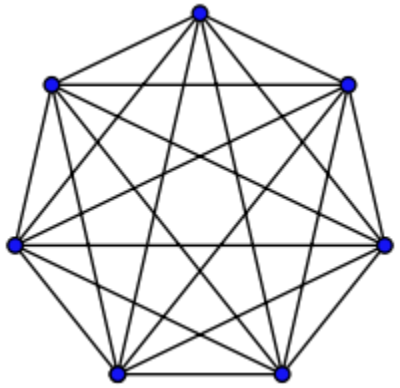
Undefined



Discrete Bullying

Part 1: Network Structure

We say a network is *complete* if there is an edge between every pair of vertices.



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1

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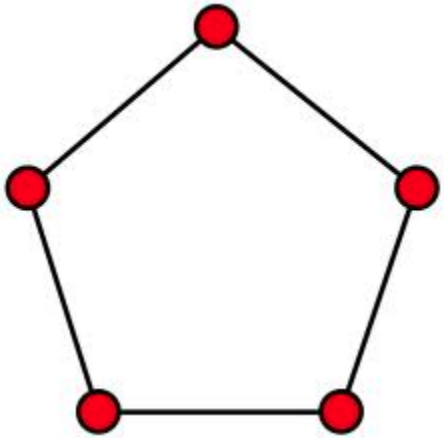

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4/3

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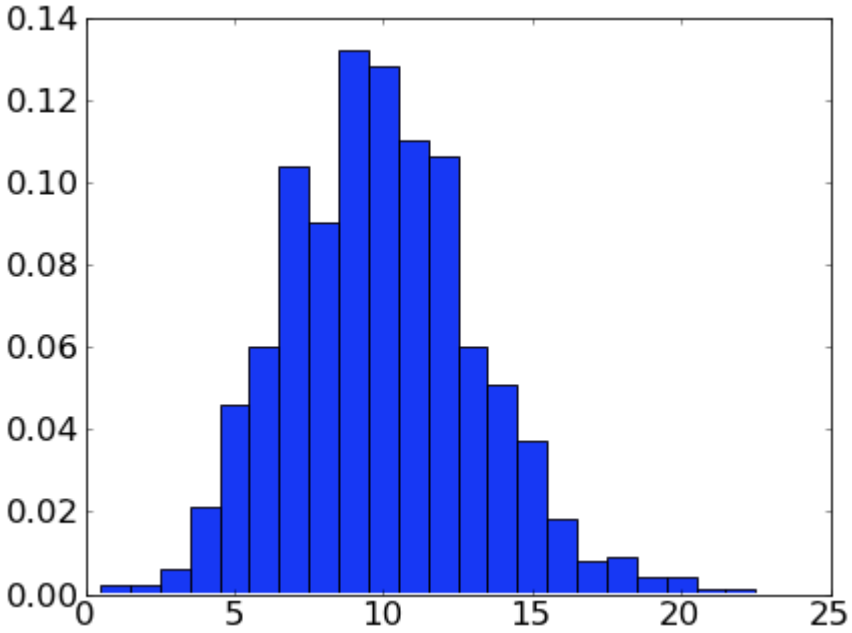

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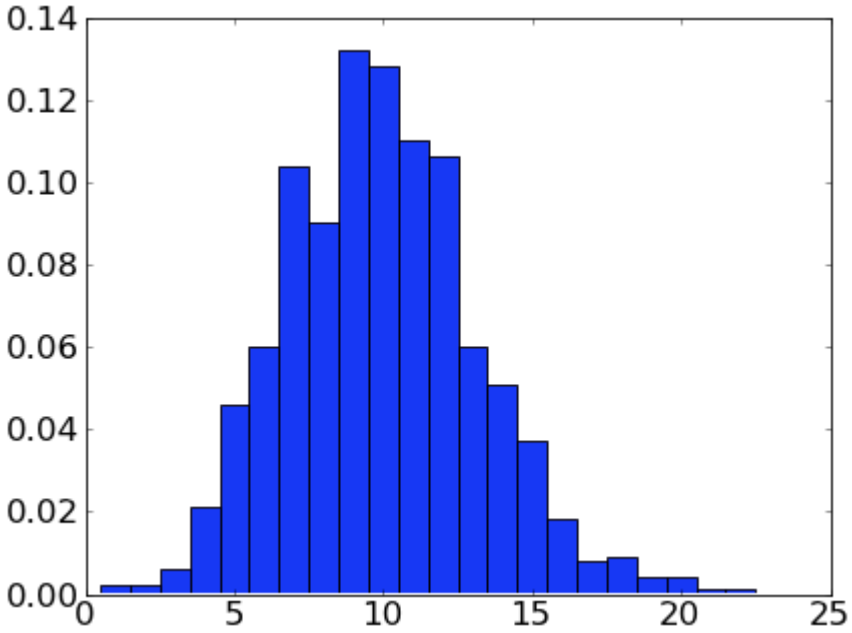

What are the x-axis and y-axis labels of a degree distribution?

x-axis: degree

y-axis: number of nodes with that degree

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What does the degree distribution of a complete network look like?

One bar of at degree N - 1 with height N

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True/False: The sum of the degrees in a network must always equal the number of edges.

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True

Part 2: Erdos Numbers

In the Erdos collaboration network, each vertex is a mathematician or computer scientist, and there is an edge between two vertices if they have coauthored a paper together.

An individual's *Erdos number* is the distance from their vertex to Paul Erdos' vertex.

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How many individuals have an Erdos number of 0?

1 (Paul Erdos)

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Suppose Alex has an Erdos number of 5, and now writes a paper with Sarah and Mike. Sarah has an Erdos number of 3 and Mike has an Erdos number of 7. What is Alex's new Erdos number?

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Might decrease or stay the same, but cannot increase.

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Is your degree related to your Erdos number?

Not necessarily, but there is probably a correlation

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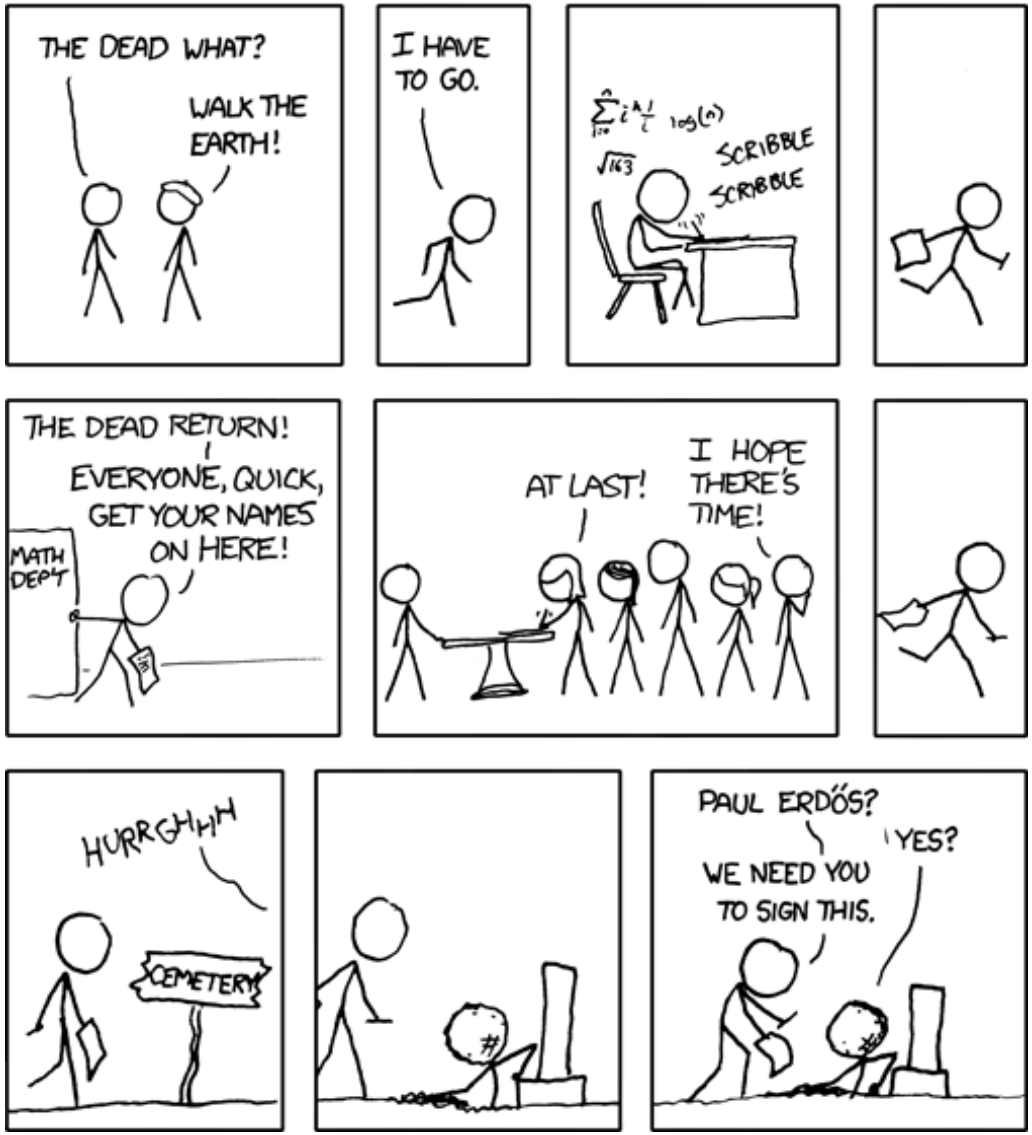
Probably increase


THE APOCALYPSE! THE SKIES BURN, THE SEAS TURN TO BLOOD, AND THE DEAD WALK THE EARTH!


THE DEAD WHAT?
WALK THE EARTH!
I HAVE TO GO.
SCRIBBLE SCRIBBLE
THE DEAD RETURN!
EVERYONE, QUICK, GET YOUR NAMES ON HERE!
MATH DEPT
AT LAST!
I HOPE THERE'S TIME!
HURRGHHH
CEMETERY
PAUL ERDŐS?
YES?
WE NEED YOU TO SIGN THIS.

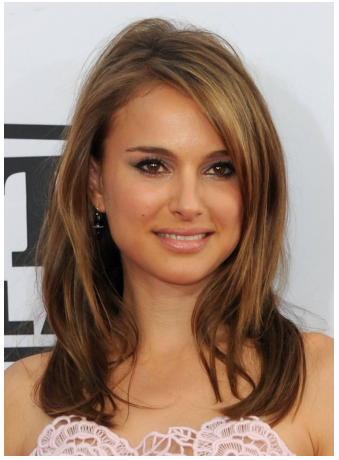
Part 2: Erdos Numbers

Bacon number: Degrees of separation from Kevin Bacon in actor co-star network

Erdos-Bacon number: Sum of Erdos and Bacon numbers

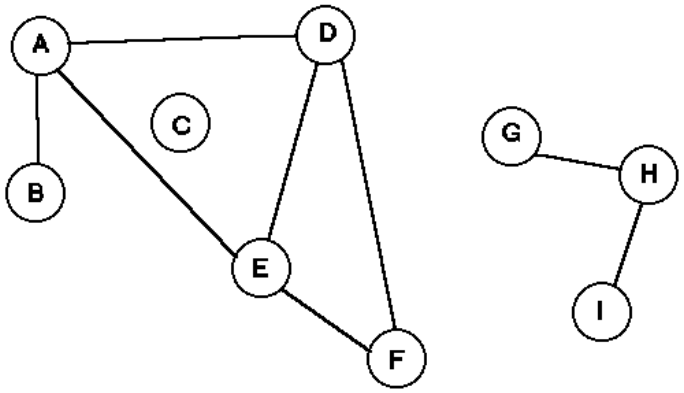
Who has a finite Erdos-Bacon number?

- Natalie Portman (7)
- Colin Firth (7)
- Carl Sagan (6)
- Noam Chomsky (7)



Part 3: Contagion

Consider the network below:



In the forest fire model, if we start a fire at vertex D, how many vertices will “burn”?

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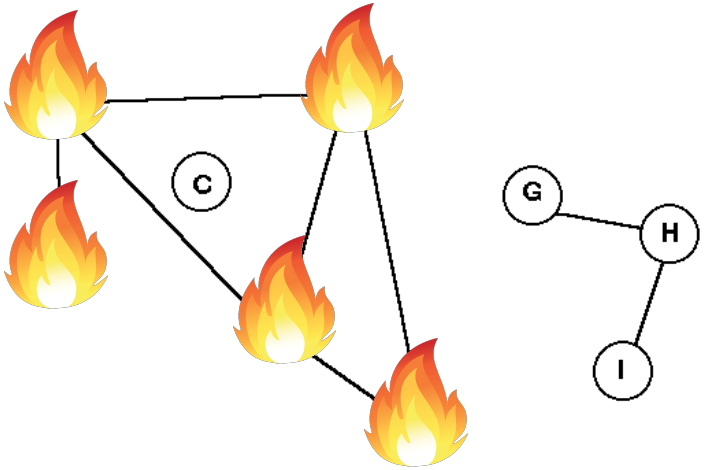

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Fire spread to all adjacent cells

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How did the viral spread demo work?

Rewired local edges to random destination with certain probability

Virus spread to adjacent cells with certain probability

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How can we design a network so as to maximize contagion?

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What was special about the equilibrium outcome of the "altruistic contagion" model?

Entirely determined by network structure (degrees)

Part 4: Navigation

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a) How did we find paths between a source and a target?

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c) Would our approach in step b work in the “real world”?

No -- quickly “blows up”, not scalable

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What is the relationship between a heavy-tailed degree distribution and efficient navigation?

Heavy-tailed degree distribution -> small diameter -> short paths exist

Why can’t a network have a large number of vertices, a small maximum degree, and a small diameter at the same time?

Small max degree -> no connectors -> can’t make big jumps across network

Part 4: Navigation

Why is small diameter necessary but not sufficient for efficient navigation?

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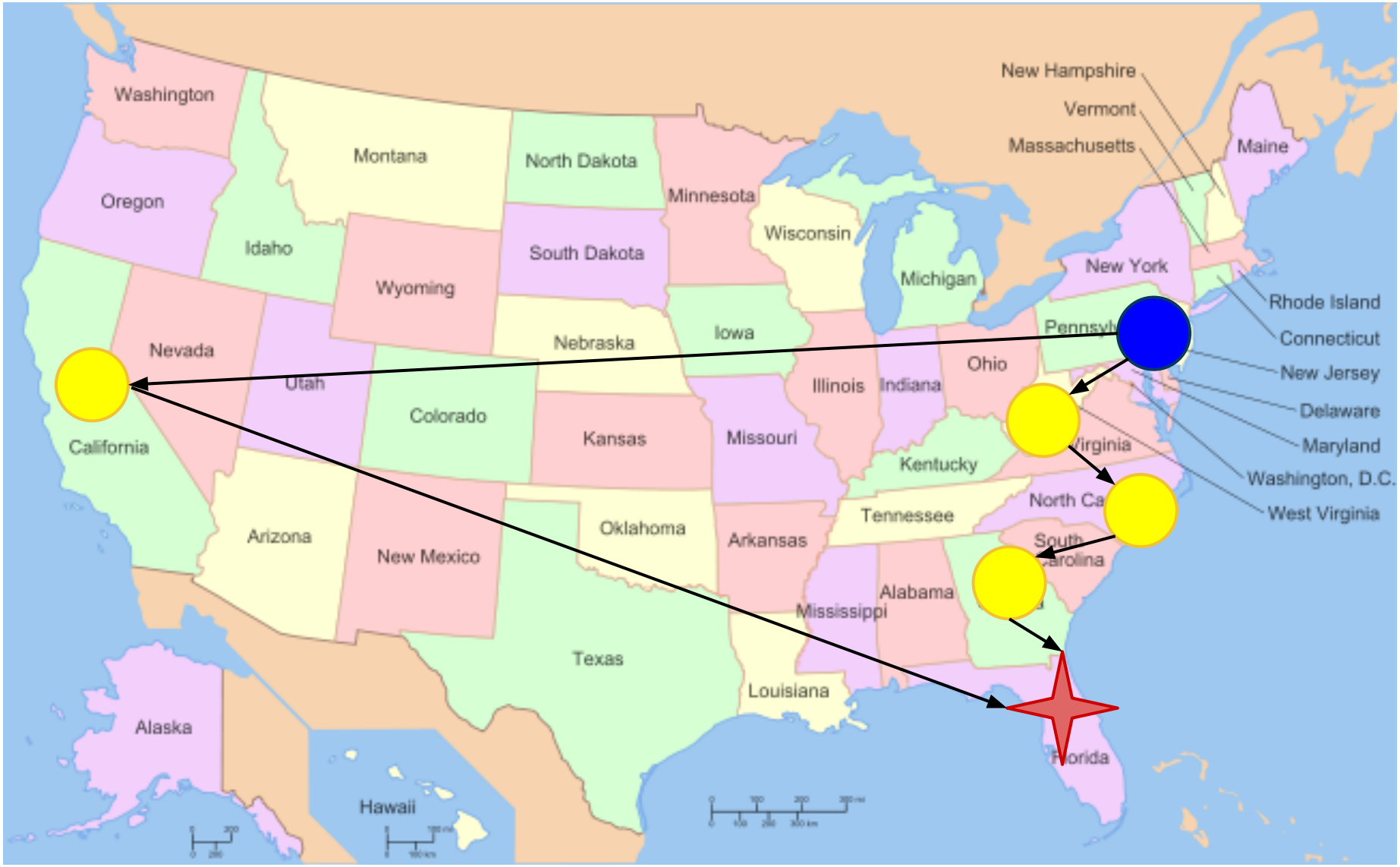
Local / distributed algorithm may not be able to find the short paths


Washington
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Nevada
Idaho
Montana
Wyoming
Utah
Arizona
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Alabama
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Florida
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North Carolina
Virginia
West Virginia
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Delaware
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Rhode Island
Massachusetts
Vermont
New Hampshire
Maine
Washington, D.C.
Alaska
Hawaii

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1/4

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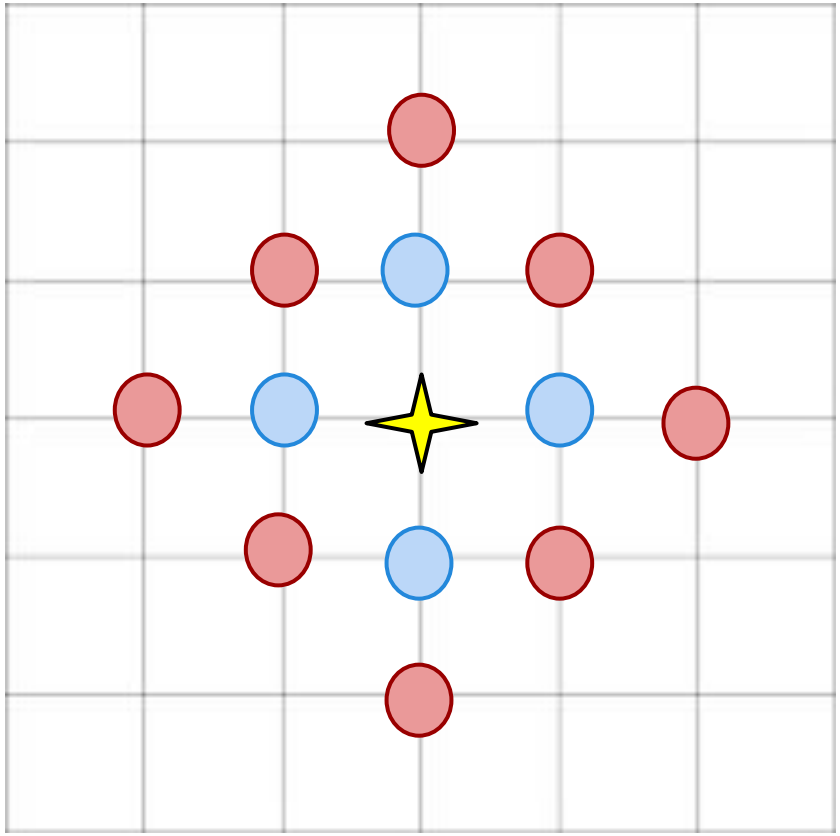
In a grid network, there are 4 vertices at distance 1 away from A, and 8 vertices at distance 2 away from A.



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The probability of adding an edge to of distance 2 to A is:

2/6 = 1/3

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Only r = 2 permits navigation in log(n) steps

Fourth Column

http://thewikigame.com/