

FINAL EXAMINATION
Networked Life (NETS 112)
December 13, 2013
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: _____

Penn ID: _____

Problem 1: _____/10

Problem 2: _____/15

Problem 3: _____/15

Problem 4: _____/15

Problem 5: _____/20

Problem 6: _____/10

Problem 7: _____/15

TOTAL: _____/100

Problem 1 (10 points) For each lettered item on the left below, write the number of the item on the right that matches it best.

- | | |
|---------------------------------------|--------------------------------------------|
| a) Penny matching game _____ | 1. Equilibrium welfare vs. maximum welfare |
| b) Magic number 150 _____ | 2. Inequality aversion |
| c) Beauty contest game _____ | 3. Efficient navigation |
| d) Volleyball _____ | 4. Quality of service |
| e) Braess's Paradox _____ | 5. Multiple pure strategy equilibria |
| f) Paris Metro Pricing _____ | 6. Social channel capacity |
| g) $r = 2$ in Kleinberg's model _____ | 7. Attendance dynamics |
| h) Ultimatum game _____ | 8. Bounded rationality |
| i) "Chicken" game _____ | 9. No pure strategy equilibrium |
| j) Price of Anarchy _____ | 10. Selfish routing |

Problem 3 (15 points)

We have discussed several network formation models in which the choice of a parameter controls the structure and dynamics of the resulting network. For each of the following, describe the key properties of the network formed by the given model and parameter choice.

a) Kleinberg's model, $r = 0$. Recall that the probability of adding an edge to a vertex at grid distance d is proportional to $\left(\frac{1}{d}\right)^r$.

b) Erdos-Renyi model, $p > \frac{1}{N}$, where p is the edge density and N is the network size.

c) Alpha model, $\alpha \rightarrow 0$. Recall that the probability of connecting two vertices u and v is proportional to $p + \left(\frac{x}{N}\right)^\alpha$, where p is the background edge density, x is the number of current common neighbors of u and v , and N is the network size.

d) Ring-rewiring model, $q = 1$, where q is the probability of rewiring an edge.

Problem 4 (15 points)

In each part of this problem, you should draw a bipartite network between 4 "Wheat" traders and 4 "Milk" traders who each start with an endowment of 1.0 units of their initial good. The network you draw should have the properties described.

(a) Each vertex has degree 2, and at equilibrium all traders' wealths are equal to 1.0

(b) The deletion of any single edge leaves equilibrium wealths unchanged.

(c) All equilibrium wealths fall in the set $\{1/2, 2/3, 3/2, 2\}$.

(d) The network is connected, and all equilibrium wealths fall in the set $\{1/2, 1, 2\}$.

Problem 5 (20 points)

Imagine that you are the marketing director at a large advertising agency, and are responsible for developing a viral marketing campaign for a new product on Facebook. Your plan is to give the product away to a limited number of carefully chosen "seed" users, in the hopes of gathering "likes" for the product. While Facebook declined your request to see a complete description of their social network, fortunately you were able to download it from the National Security Agency website. Suppose that a typical Facebook user will "like" a product as long as $f(x) > y$, where x is the fraction of the user's neighbors who currently "like" the product, and y is a random number between 0 and 1 that measures the "immunity" of the user.

(a) Suppose the Facebook network structure is highly clustered, and that $f(x) = \sqrt{x}$. Describe how you would choose your seed users, and justify your strategy.

(b) Suppose the Facebook network structure is highly clustered, and that $f(x) = x^2$. Describe how you would choose your seed users, and justify your strategy.

(c) Suppose the Facebook network structure consists of two separate dense and highly clustered groups, with little connectivity between them, and that $f(x) = \sqrt{x}$. Describe how you would choose your seed users, and justify your strategy.

(d) Suppose the Facebook network structure consists of two separate dense and highly clustered groups, with little connectivity between them, and that $f(x) = x^2$. Describe how you would choose your seed users, and justify your strategy.

(e) Suppose the Facebook network structure was generated by preferential attachment, and that $f(x) = x$. Describe how you would choose your seed users, and justify your strategy.

Problem 6 (10 points)

Suppose you make a Skype call from your laptop to a friend in Europe. In as much detail as possible, describe both the technical process by which the call is conducted over the Internet, and the potential strategic, economic or game-theoretic issues that such a process entails. Then briefly describe any aspects of your answer that would change if instead of a Skype call, you send a short email to your friend.

Some topics you might want to touch upon in your answers: packets, routing, the IP protocol, IP addresses, TCP/IP, autonomous systems, the border gateway protocol, Quality of Service guarantees, bandwidth, latency, congestion, the Price of Anarchy, customer-provider, peer networks.

Problem 7 (15 points)

Throughout the course, we considered a variety of models for network formation in which the decisions to connect vertices are made probabilistically or randomly (as in Erdos-Renyi and Preferential Attachment). In this problem, we consider network formation from a game-theoretic perspective.

Consider a one-shot, simultaneous-move game in which each of N players must choose which other players they will purchase an edge to. The collection of all edges purchased by the players results in some overall network G . The payoff to a particular player i is then defined to be $B(G, i) - K \times C$, where K is the number of edges purchased by player i , C is some fixed cost per edge, and $B(G, i)$ measures the "network benefit" enjoyed by player i in the overall network G . For example, $B(G, i)$ might be the PageRank of i in G (so each player would like to balance being "important" with their edge expenditures), or $B(G, i)$ might be the clustering coefficient of i in G (so each player wants to be in a tight-knit circle of friends without spending too much on edges). Note that in such games, a player may benefit from the edges purchased by other players, not just their own edge purchases.

(a) Must it be that for any choice of the function $B(G, i)$ and any value for C , the resulting game has a Nash equilibrium? Why or why not?

(b) Suppose $B(G, i)$ is the size of the connected component of player i in G , and that the edge cost C is very small (say, just slightly larger than 0). What can you say about the networks that can be formed at equilibrium? For instance, will they be connected? Why or why not? Can they contain cycles? Why or why not? Can the equilibrium networks have large diameter? Why or why not?

(c) Briefly describe any strengths or weaknesses you perceive regarding such game-theoretic network formation models, relative to the probabilistic ones we studied in the course. You may want to take into consideration the various properties of the probabilistic models we considered, and the motivation behind them.