Networked Life
CSE 112
Prof. Michael Kearns
Final Examination
May 1, 2007

NAME: ___________________________________

PENN ID: __________________________________

Exam Score:

Problem 1:__________/10
Problem 2:__________/10
Problem 3:__________/10
Problem 4:__________/10
Problem 5:__________/10
Problem 6:__________/10
Problem 7:__________/10
Problem 8:__________/10
Problem 9:__________/10
Problem 10:_________/10

TOTAL:____________/100

This is a closed-book exam; you should have no materials other than this exam and a pencil or pen.

If you need more space to answer a problem, use the reverse side of the page, but clearly indicate where your answers are.

1. (10 points) For each item on the left, write the index of the item on the right which is the best match.

a. connectors __7__	1. fewest colors required
b. market for lemons __3__	2. neural network
c. dollar bill migration _8___	3. cascading
d. C. Elegans _2___	4. Nash equilibrium
e. baggage screening _10___	5. no wealth variation
f. perfect matching _5___	6. independent set
g. forest fire __9__	7. the heavy tail
h. complement of clique __6__	8. scaling laws of human travel
i. chromatic number _1___	9. viral spread
j. no unilateral gain _4___	10. interdependent security

2. (10 points)

(a) (5 points) Draw a network with 10 vertices in which the clustering coefficient and the overall rate of connectivity (i.e. the fraction of all possible edges in the network that are present) are both low. What network formation model studied in class would give rise to networks with this property?

One (not the only) possibility is something like a cycle --- very regular structure, very sparse.

Formation model: Erdos-Renyi with small value of p (points off if they do not specify small p). Note that the phrasing of the problem does NOT require the specific graph they draw to match the formation model they name.

(b) (5 points) Draw a network with 10 vertices in which the clustering coefficient is relatively high, but the overall rate of connectivity is relatively low. What network formation model studied in class would give rise to networks with this property?

One possibility is something like a chain of cliques.

Possible formation models: alpha model, rewired chain of cliques

3. (10 points) Consider the behavioral network science experiments in which the games being played were Coloring, Consensus, and Kings and Pawns (without tips). In each of these games, there was a precise specification of how individuals would be paid in response to their own actions and those of their neighbors in the network, so we can discuss both the maximum social welfare states (the global configurations in which the total payoff to the population is highest), and the Nash equilibria. For each of the three games, answer the following questions:

   (i) Is a maximum social welfare state always a Nash equilibrium? Explain.

   (ii) Is a Nash equilibrium always a maximum social welfare state? Explain.

Coloring:
(i)
Yes. Max social welfare corresponds to a proper coloring, in which everyone is paid the maximum possible amount, so this is also Nash.
(ii)
No. It is possible for a player to have the colors used in its neighborhood in a Nash equilibrium (think of a red-blue-blue-red chain) and thus not be able to improve its payoff from 0.

Consensus:
(i)
Yes. Max welfare corresponds to consensus, in which everyone receives the max possible amount, so this is also Nash.
(ii)
No --- think of a cycle with three colorings alternating around it --- nobody is getting paid anything, but also cannot do anything about it since both their neighbors have different colors, so it is Nash

Kings and Pawns without tips:
(i)
Yes. Max welfare is a max independent set, and the pawns cannot improve their payoff.

(ii)
No --- any maximal independent set is Nash but may not be maximum.

4. (10 points) Consider the assigned reading “The Scaling Laws of Human Travel”.

(a) (3 points) Briefly summarize the source and nature of the data analyzed in the paper.

Data source is from wheresgeorge.com, a web-based dollar bill tracking service. The data consists of the sequence of distances traveled by individual dollar bills in consecutive “sightings”. The main interest of the paper is in studying the empirical distribution or histogram of distances traveled.

(b) (4 points) Briefly summarize the main empirical findings of the paper.

The main empirical claim of the paper is that the empirical distribution of distances traveled has a heavy tail, specifically with a decay ~ 1/r^(1.59) where r is the distance traveled. (Full credit if they at least state this finding clearly.) They also point out that the initial hop after the first sighting is often a very short one, leading the distribution to be slightly more peaked near small r and thus causing some departure from a line on a log-log plot.

(c) (3 points) Discuss the implications of the empirical findings for the network formation model and theoretical results of Kleinberg’s paper “Navigation in a Small World”.

They should first point out that the exponent found in the Scaling Laws paper is different than the specific exponent of 2 required in Kleinberg’s work for efficient navigation. Then they should either point out that a) these two exponents (1.59 and 2) are different enough that there is inconsistency between the two papers, and that if we believe that humans can in fact solve efficient navigation then Kleinberg’s theory needs amending; or b) that these exponents are close enough to be broadly consistent and thus render Kleinberg’s explanation plausible. Either argument is fine as long as they make it clearly enough.

5. (10 points)

(a) (6 points) Briefly discuss what is being illustrated in the figure below, and what points are demonstrated by the diagram.

The figure is from the IDS baggage security case study. There are 36 tiny plots corresponding to the 36 largest carriers by volume. x-axis of each plot is simulation time, y-axis is fraction made of possible security investment. Dynamics of simulation is that at each step, each carrier incrementally adjusts their investment upwards if doing so improves their payoff (reduces their overall cost). The diagram shows that the Price of Anarchy is large --- while smaller carriers invest fully at equilibrium the largest ones do not.

(b) (2 points) Briefly discuss how the figure below differs from that in (a), and what point is being demonstrated.

In this figure the two largest carriers have been clamped at full investment, representing (e.g.) government subsidy for their investment. Now all of the carriers eventually converge to full investment, showing that the two largest carriers form a “tipping set” for the entire population. The figure also demonstrates cascading behavior, as there is a distinct order in which the larger carriers begin investing, in response to the investments of their neighbors.

(c) (2 points) Briefly discuss how the figure below differs from that in (a) and (b), and what point is being demonstrated.

In this figure only the largest carrier is subsidized, and there is again a high Price of Anarchy --- many of the large carriers still do not invest. So subsidizing both of the largest really was required to get the result in part (b).

6. (10 points) Consider the network Milk-Wheat economic exchange model considered extensively in class. Draw the smallest bipartite network you can in which the number of Milk players and Wheat players is the same, each player begins with an endowment of 1.0 of their respective good, the network is connected (i.e. there is a path between any pair of players), and the equilibrium wealths are *not* all equal. Annotate your diagram with the wealth of each player at equilibrium.

Any example that is simple and meets all the criteria of the statement is acceptable. The connectivity requirement excludes trivial solution like A connected to 1 and 2 on the other side, and 3 connected to B and C, giving two disjoint components with wealth variation.

7. (10 points). The following assertions all refer to the behavioral network science experiments from this semester and/or the ones from last year described in the paper “An Experimental Study of the Coloring Problem on Human Subject Networks”. For each assertion, circle True or False.

(a) All networks used in the Kings and Pawns experiments were bipartite.
TRUE FALSE

(b) In the network formation model used for this semester’s coloring experiments, larger values of p tended to increase the time to solution
TRUE FALSE

(c) In the network formation model used for this semester’s consensus experiments, larger values of p tended to increase the time to solution
TRUE FALSE

(d) Allowing the exchange of tips tended to reduce social welfare in Kings and Pawns
TRUE FALSE

(e) In last year’s coloring experiments, showing participants the entire network increased the time to solution, regardless of the network structure
TRUE FALSE

(f) In Kings and Pawns with tips allowed, the instantaneous social welfare approached its maximum possible value at some point during most experiments
TRUE FALSE

(g) In last year’s coloring experiments, the Leader Cycle networks yielded the smallest average time to solution
TRUE FALSE

(h) Across all of the experiments from both last year and this year, smaller network diameter tended to lead to faster solution, regardless of the game type
TRUE FALSE

(i) Across all of the experiments from both last year and this year, higher network clustering coefficient tended to lead to faster solution, regardless of the game type
TRUE FALSE

(j) Allowing the exchange of tips dramatically improved the social welfare in Kings and Pawns when play occurred in isolated pairs
TRUE FALSE

8. (10 points) Consider a bipartite network in which one set of vertices represents movies, the other set of vertices represents actors, and there is an edge between a movie and an actor if and only if that actor appeared in that movie.

(a) (3 points) Suppose there is an actor whose degree is very large. Does this imply that there is a movie with very large degree? Explain your answer.

No --- as long as the number of movies is large compared to the max actor degree (e.g. if the number of movies is at least as large as the number of actors, a reasonable assumption), the high degree of this actor could be "spread out" over many movies, each of small degree.

(b) (3 points) Suppose the degree distribution of the actors is heavy-tailed. Does this imply that the degree distribution of the movies is heavy-tailed? Explain your answer.

No --- imagine first specifying the degrees of the actors in a way that gives them a heavy-tailed distribution. But now choose the destination movies of each of the edges uniformly at random. Then the degree distribution of the movies will be sharply peaked, not heavy-tailed.

(c) (4 points) Suppose that the number of movies and actors is the same, and that every actor has appeared in at least d movies. What can you say about the degrees of the movies? Explain your answer.

The average degree of the movies must be at least d by a simple counting argument --- if there are N actors and N movies, and each actor has degree at least d, then there are at least N*d edges total, and thus the average movie must have at least d edges or actors.

9. (10 points) Consider a population of N people. Suppose there is a group activity for this population that obeys the following dynamics:
    - If k people participated in the activity last time, then N-k will participate this time.

To be fair to all students, I want to make this problem worth only 5 extra credit points total. In order to get the 5 points, they need to simply observe that if you start with 0 you will oscillate forever between 0 and N, and that more generally if you start with k you will oscillate forever between k and N-k. There is no need for them to draw a diagram.

(a) (3 points) Draw a diagram below of the type found in Schelling's book to represent these dynamics. Be sure to label your axes precisely.

(b) (4 points) Re-draw your diagram from part (a) below and use it to compute the eventual number of people participating if the initial number of participants is 0.

(c) (3 points) Is there any number of initial participants that will cause the eventual number of people participating to be different from your answer to part (b)?

10. (10 points)

(a) (5 points) Give an example of a model of economic exchange taking place over networks in which the wealth of individuals is entirely determined by their degrees. Be precise in your description.

The example I had in mind was the "brain-dead" model from early in the course --- each vertex starts with $1, and at each time step gives away all of its money to its neighbors, equally divided. Thus at each step every vertex gives all its money away, but also receives new money from its neighbors. This process converges to a player's average wealth being exactly proportional to its degree.

Other examples are acceptable (including original ones) as long as they are precise and correct.

(b) (5 points) Give an example of a model of economic exchange taking place over networks in which the wealth of individuals is *not* entirely determined by their degrees, and explain why it is not. Be precise in your description.

The most obvious example would be the bipartite Milk-Wheat exchange model from class, where it was pointed out at some length that the degree distribution does NOT determine the equilibrium wealth in all cases --- e.g. if you have two neighbors that have no neighbors besides you, you will make $2 at equilibrium, but if you have degree two and the network has a perfect matching you will make only $1.