Contagion in Networks

Networked Life
NETS 112
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Two Models of Network Formation

• Start with a grid, remove random fraction of vertices
  - “local” or “geographic” connectivity
• Start with N isolated vertices, add random edges
  - “long distance” connectivity
• Examine a deterministic contagion model
• Widespread infection occurs at “tipping point” of connectivity
“Mathematizing” the Forest Fire
(see Coursera “Contagion” video)

• Start with a regular 2-dimensional grid network
  - this represents a complete forest
• Delete each vertex (and all 4 of its edges) with probability 1-p
  - p is fraction of forest, 1-p is fraction of parking lots or clear-cut
• Choose a random remaining vertex v
  - this is my campsite
• Q: What is the expected size of v's connected component?
  - i.e. the number of vertices reachable from v
  - this is how much of the forest is going to burn
• Observe a “tipping point” around $p = 0.6$
“Mathematizing” the Average Degree Demo
(see Coursera “Contagion” video)

• Let $d$ be the desired average degree in a network of $N$ vertices
• Then the total number of edges should be

$$e = d \times N / 2$$

• Just start connecting random pairs of vertices until you have $e$ edges
• Pick a random vertex $v$ to infect
• What is the size of $v$’s connected component?
• Observe a “tipping point” around $d=3$
Some Remarks on the Demos

• Connectivity patterns were either \textit{local} or \textit{random}
  - will eventually formalize such models
  - what about other/more realistic structure?

• Tipping was inherently a \textit{statistical} phenomenon
  - probabilistic nature of connectivity patterns
  - probabilistic nature of disease spread
  - model \textit{likely} properties of a large \textit{set} of possible outcomes
  - can model either inherent randomness or variability

• Formalizing tipping in the forest fire demo:
  - might let grid size $N \to \infty$, look at fixed values of $p$
  - is there a threshold value $q$:
    • $p < q \Rightarrow$ expected fraction burned $< 1/10$
    • $p > q \Rightarrow$ expected fraction burned $> 9/10$
Structure and Dynamics Case Study: A “Contagion” Model of Economic Exchange

- Imagine an undirected, connected network of individuals
  - no model of network formation
- Start each individual off with some amount of currency
- At each time step:
  - each vertex divides their current cash equally among their neighbors
  - (or chooses a random neighbor to give it all to)
  - each vertex thus also receives some cash from its neighbors
  - repeat
- A transmission model of economic exchange --- no “rationality”
- Q: How does network structure influence outcome?
- A: As time goes to infinity:
  - vertex i will have fraction $\frac{\text{deg}(i)}{D}$ of the wealth; $D =$ sum of $\text{deg}(i)$
  - degree distribution entirely determines outcome!
  - “connectors” are the wealthiest
  - not obvious: consider two degree $= 2$ vertices...
- How does this outcome change when we consider more “realistic” dynamics?
  - e.g. we each have goods available for trade/sale, preferred goods, etc.
- What other processes have similar dynamics?
  - looking ahead: models for web surfing behavior