NETS 112 Final Review

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Shahin Jabbari and Ryan Rogers
Networks

Structural Properties:
- Degree, Degree Distribution, Distance, Diameter, Clustering Coefficient.
Examples from Class

• Erdos Number Project
• Last Name Experiment
• Toss a ball from one person in the class to another to reach a specific person (local vs global info)
Dynamics in Networks

- Network Structure (static) definitely impacts a process (dynamics) in a network
- 2 Demos showing Contagion in Networks:
  - Forest Fire
  - Average Degree
Forest Fire Demo

• What Network did this use?
  • Lattice and then deleted some nodes

• What parameter can we adjust?
  • Probability of keeping a node

• Describe what is meant by a tipping point at $p = 0.6$
  • There is a sharp increase in the number of infected nodes from when $p$ is slightly below 0.6 and then when $p = 0.6$. 
Average Degree (Viral Spread) Demo

• What Network did this use?
  • N nodes with no edges and then add edges to random pairs until there are \( E = d \times N/2 \) edges

• What parameter can we adjust?
  • Average node degree \( d \)

• Describe what is meant by a tipping point at \( d=3 \)
  • There is a sharp increase in the number of infected nodes when we select a random node to infect
Contagion in Economic Exchange

• Given a Graph, we let each node start with $1.
• Each vertex divides their current cash equally among their neighbors.
• Repeat.
• Equilibrium when players give away as much as they receive.
• Equilibrium Wealth for each node $u$: $w(u)$
• $w(u) = (\text{Total Wealth}) \times \frac{\text{deg}(u)}{\text{Sum of all Degrees}}$
Equilibrium Wealth for each node $u$: $w(u)$

$w(u) = \frac{(\text{Total Wealth}) \times \text{deg}(u)}{(\text{Sum of all Degrees})}$
Navigation in Networks

• Travers and Milgram, Dodds et al.

• To find short paths in a graph:
  – Graph must have a small diameter
  – Players must be able to find the short paths with only local info.
Kleinberg’s Model

• Begin with a grid network
• Add an edge from a node to another node that is a distance d away with probability

\[ p(d) = \text{constant} \times \left(\frac{1}{d}\right)^r \]

What happens for large r and for small r?
Kleinberg’s Model

• How do player’s move in the network?
  – Players know the address of their destination and their neighbors
  – Players only forward to the neighbor who is closest to the destination

• What happens when $r = 2$?
  – Efficient navigation $\sim \log(N)$ steps to any node using the procedure.
The “Look” of Real Networks

- Heavy Tailed Distribution: $\text{Prob}(x) \sim (1/x)^B$
  - Power Law $(1/x)^B \Rightarrow \log((1/x)^B) = -B \log(x)$

- Normal Distribution
  - $\log(\text{Prob}) = \text{Curved!}$

Look at Log – Log Plots!
Example

Degree Distribution for the Erdos Network

Log-Log Plot

- Log(Degree)
- Log(Number of Authors)
Models of Network Formation

- **Erdos-Renyi**
  - Nodes are fixed, add edges

- **Small World**
  - e.g., rewiring model

- **Preferential Attachment**
  - Start with two connected nodes, add a node, add an edge, continue;
## Models of Network Formation

<table>
<thead>
<tr>
<th>Model</th>
<th>Small Diameter</th>
<th>Giant Component</th>
<th>High Clustering Coefficient</th>
<th>Heavy Tail</th>
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</thead>
<tbody>
<tr>
<td>Erdos-Renyi</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Small World</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Preferential Attachment</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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</table>
Attendance Dynamic

stable equilibrium: 50%
Attendance Dynamic

- S: Start of the process
- U: Unmet demand
- S: Steady state

Graph showing the relationship between actual and expected number of people.
Attendance Dynamic

- What will happen if we start at 1?
- Unstable vs. Stable
Networked Games

“first neighborhood” view
Coloring

Small Worlds Family

Simple Cycle

Preferential Attachment, \( \nu = 3 \)
Consensus
Game Theory

• What is different from a Pure Strategy Nash Equilibrium and a Mixed Strategy Nash Equilibrium?
  – Players can randomize over actions. The probability distribution is public, but the realization, i.e. what action is actually chosen, is private. Think Rock-Paper-Scissors!

• (Mixed) Nash Equilibrium always exist in games with finite number of players and actions.
### Mixed Strategy Equilibrium

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<tr>
<th>Actions</th>
<th>A</th>
<th>B</th>
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<tr>
<td>A</td>
<td>-3, -3</td>
<td>-1, 2</td>
</tr>
<tr>
<td>B</td>
<td>2, -1</td>
<td>-10, -10</td>
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<th>Actions</th>
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<tr>
<td>A</td>
<td>2, 4</td>
<td>1, 3</td>
</tr>
<tr>
<td>B</td>
<td>2, 1</td>
<td>1, -2</td>
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## Game Theory

### Row Player Actions

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<tr>
<td></td>
<td>A</td>
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<tr>
<td>A</td>
<td>6,6</td>
</tr>
<tr>
<td>B</td>
<td>10,0</td>
</tr>
<tr>
<td>C</td>
<td>8,0</td>
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- Nash Equilibrium at (C,C) for a payoff of 8.
- Optimal Strategy (A,B) gives payoff of 28.
- POA = 28/8 > 1
Network Trading

A — B — C

D — E — F
Network Trading

Maximize \( v(S) = \frac{|S|}{|N(S)|} = 2 \)

\( \Rightarrow \) B, C can get at most \( \frac{1}{2} \) and F gets at least 2.
Maximize $v(S) = \frac{|S|}{|N(S)|} = 2$ => D, E can get at most $\frac{1}{2}$ and A gets at least 2.
Trades: D and E give all their good to A, and A gives $\frac{1}{2}$ of her good each to D and E. Similarly, B and C give all their good to F, and F gives $\frac{1}{2}$ of his good to B and C.
Network Trading (in-class experiment)

Figure 1: Network Trading - In Class Example
Selfish Routing

Diagram of a network with nodes S, A, B, C, D, and T, and edges labeled with values 1, 2X, X, 0, and 1.
Selfish Routing

\[Cost = 4\]

Equilibrium?  NO!
Selfish Routing

Cost = \( \frac{8}{3} = 2.6667 \)

Equilibrium? Yes!
Selfish Routing

Cost = 2.5 = OPT
Equilibrium? NO!
Sponsored Search
Sponsored Search

- Sponsored web search: a market for ad placement
- Advertiser submit their bids
- The winners get their ads displayed
- Advertisers pay only if their ads get clicked
- Second price auction