Trading within each group permitted
Anyone can trade with Group S(uper)
Group A: 
10W, 3M 
W surplus: 536 
M surplus: 538

Group B: 
3W, 10M 
W surplus: 573

Group C: 
10W, 3M 
M surplus: 567

Group D: 
3W, 10M 

Group E: 
10W, 10M 
W surplus: 29

Group F: 
5W, 1M 
W surplus: 215

Group G: 
1W, 5M 
W surplus: 375

Group S: 
5W, 5M 

W surplus: 0 

Group S received: 
210 
191 
156 
152 
130 
100 (5)
What “should” have happened?
• Group E trades internally 1-for-1
• Group A,C,F Ws trade with A,C,F,S Ms
  Exchange rate: 25/12 W for 1 M
• Group B,D,G Ms trade with B,D,G,S Ws
  Exchange rate: 25/12 M for 1 W
Roadmap

- Networked trading motivation
- A simple model and its equilibrium
- A detailed example
Trading in Networks:
I. Model

Prof. Michael Kearns
Networked Life
NETS 112
Fall 2014
Roadmap

- Networked trading motivation
- A simple model and its equilibrium
- A detailed example
Networked Games vs. Trading

- **Models and experiments so far (coloring, consensus, biased voting):**
  - simple coordination games
  - extremely simple actions (pick a color)
  - “trivial” equilibrium theories (“good” equilibrium or “trapped” players)
  - no equilibrium predictions about network structure and individual wealth

- **Networked trading:**
  - a “financial” game
  - complex action space (set of trades with neighbors)
  - nontrivial equilibrium theory
  - detailed predictions about network structure and individual wealth
Networked Trading: Motivation

- Settings where there are restrictions on who can trade with whom
- International trade: restrictions, embargos and boycotts
- Financial markets: some transactions are forbidden
  - e.g. trades between brokerage and proprietary trading in investment banks
- Geographic constraints: must find a local housecleaning service
- Natural to model by a network:
  - vertices representing trading parties
  - presence of edge between u and v: trading permitted between parties
  - absence of edge: trading forbidden
A Simple Model of Networked Trading

• Imagine a world with only two goods or commodities for trading
  - let’s call them Milk and Wheat

• Two types of traders:
  - Milk traders: start game with 1 unit (fully divisible) of Milk, but only value Wheat
  - Wheat traders: start game with 1 unit of Wheat, but only value Milk
  - trader’s payoff = amount of the “other” good they obtain through trades
  - “mutual interest in trade”
  - equal number of each type → same total amount of Milk and Wheat

• Only consider bipartite networks:
  - all edges connect a Milk trader to a Wheat trader
  - can only trade with your network neighbors!
  - all trades are irrevocable
  - no resale or arbitrage allowed
Equilibrium Concept

- Imagine we assigned a price or exchange rate to each vertex/trader
  - e.g. “I offer my 1 unit of Milk for 1.7 units of Wheat”
  - e.g. “I offer my 1 unit of Wheat for 0.8 units of Milk”
  - note: “market” sets the prices, not traders (“invisible hand”)
  - unlike a traditional game --- traders just react to prices

- Equilibrium = set of prices + trades such that:
  - 1. market clears: everyone trades away their initial allocation
  - 2. rationality (best responses): a trader only trades with best prices in neighborhood
  - e.g. if a Milk trader’s 4 neighbors offer 0.5, 1.0, 1.5, 1.5 units Wheat, they can trade only with those offering 1.5
  - note: set of trades must ensure supply = demand at every vertex

- Simplest example: complete bipartite network
  - every pair of Milk and Wheat traders connected by an edge
  - equilibrium prices: everyone offers their initial 1 unit for 1 unit of the other good
  - equilibrium trades: pair each trader with a unique partner of other type
  - market clears: everyone engages in 1-for-1 trade with their partner
  - rationality: all prices are equal, so everyone trading with best neighborhood prices
A More Complex Example

- equilibrium prices as shown (amount of the other good demanded)
- equilibrium trades:
  - a: sends $\frac{1}{2}$ unit each to w and y, gets 1 from each
  - b: sends 1 unit to x, gets $\frac{2}{3}$ from x
  - c: sends $\frac{1}{2}$ unit each to x and z, gets $\frac{1}{3}$ from each
  - d: sends 1 unit to z, gets $\frac{2}{3}$ from z
- equilibrium check, blue side:
  - w: traded with a, sent 1 unit
  - x: traded with b and c, sent 1 unit
  - y: traded with a, sent 1 unit
  - z: traded with c and d, sent 1 unit
Remarks

• How did I figure this out? Not easy in general  
• Some edges unused by equilibrium  
• Trader wealth = equilibrium price at their vertex  
• If two traders trade, their wealths are reciprocal (w and 1/w)  
• Equilibrium *prices (wealths)* are always unique  
• Network structure led to *variation* in wealth
• Suppose we add the single green edge
• Now equilibrium has no wealth variation!
Summary

• (Relatively) simple networked trading model
• Equilibrium = prices + trades such that market clears, traders rational
• Some networks don’t have wealth variation at equilibrium, some do
• Next: What is the general relationship between structure and prices?
Trading in Networks:  
II. Network Structure and Equilibrium

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Roadmap

- Perfect matchings and equilibrium equality
- Characterizing wealth inequality at equilibrium
- Economic fairness of Erdös-Renyi and Preferential Attachment
Trading Model Review

- Bipartite network, equal number of Milk and Wheat traders
- Each type values only the other good
- Equilibrium = prices + trades such that market clears, traders rational
Perfect Matchings

- A pairing of reds and blues so everyone has exactly one partner.
- So really a subset of the edges with each vertex in exactly one edge.
- Some networks may have many different perfect matchings.
- Some networks may have no perfect matchings.
Perfect Matchings

- A pairing of reds and blues so everyone has *exactly one partner*
- So really a subset of the edges with each vertex in exactly one edge
- Some networks may have many different perfect matchings
- Some networks may have no perfect matchings
Examples

- Has no perfect matching

- Has a perfect matching
Perfect Matchings and Equality

- Theorem: There will be no wealth variation at equilibrium (all exchange rates = 1) if and only if the bipartite trading network contains a perfect matching.
- Characterizes sufficient "trading opportunities" for fairness
- What if there is no perfect matching?
Neighbor Sets

- Let $S$ be any set of traders on one side
- Let $N(S)$ be the set of traders on the other side connected to any trader in $S$; these are the only trading partners for $S$ collectively
- Intuition: if $N(S)$ is much smaller than $S$, $S$ may be in trouble
- $S$ are “captives” of $N(S)$
- Note: If there is a perfect matching, $N(S)$ always at least as large as $S$
Characterizing Inequality

- For any set $S$, let $v(S)$ denote the ratio $(\text{size of } S)/(\text{size of } N(S))$.
- Theorem: If there is a set $S$ such that $v(S) > 1$, then at equilibrium the traders in $S$ will have wealth at most $1/v(S)$, and the traders in $N(S)$ will have wealth at least $v(S)$.
- Example: $v(S) = 10/3 \rightarrow S$ gets at most $3/10$, $N(S)$ at least $10/3$
- Greatest inequality: find $S$ maximizing $v(S)$
- Can iterate to find all equilibrium wealths
- Corollary: adding edges can only reduce inequality
- Network structure completely determines equilibrium wealths
- Note: trader/vertex degree not directly related to equilibrium wealth
Examples Revisited

Has no perfect matching

Has a perfect matching
Group D: 3W, 10M
Group A: 10W, 3M
Group C: 10W, 3M
Group B: 3W, 10M
Group F: 5W, 1M
Group G: 1W, 5M
Group E: 10W, 10M
Group S: 5W, 5M

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Anyone can trade with Group S(uper)
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What “should” have happened?
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Inequality in Formation Models

• Bipartite version of Erdös-Renyi: even at low edge density, very likely to have a perfect matching \(\rightarrow\) **no wealth variation** at equilibrium
• Bipartite version of Preferential Attachment: wealth variation will **grow rapidly** with population size
• Erdös-Renyi generates economically “fairer” networks
Summary

- Ratios $v(S)$ completely characterize equilibrium
- Determined entirely by network structure
- More subtle and global than trader degrees
- Next: comparing equilibrium predictions with human behavior
Trading in Networks:
III. Behavioral Experiments

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Roadmap

- Experimental framework and trading mechanism/interface
- Networks used in the experiments
- Visualization of actual experiments
- Results and comparison to equilibrium theory predictions
Equilibrium Theory Review

• Equilibrium prices/wealths entirely determined by network structure
• Largest/smallest wealths determined by largest ratios:

\[ v(S) = \frac{\text{size of } S}{\text{size of } N(S)} \quad \text{N(S) “winners”, S “losers”} \]

• Network has a perfect matching: all wealths = 1
Experimental Framework

• Same framework as coloring, consensus and biased voting experiments
• 36 simultaneous human subjects in lab of networked workstations
• In each experiment, subjects play our trading model on varying networks
• In equilibrium theory, prices are magically *given* (“invisible hand”)
• In experiments, need to provide a mechanism for price *discovery*
• Experiments used simple *limit order* trading with neighbors
  - networked version of standard financial/equity market mechanism
• Each player starts with 10 fully divisible units of Milk or Wheat
  - payments proportional to the amount of the other good obtained
Pairs

2-Cycle

4-Cycle

Clan

Clan + 5%

Clan + 10%

Erdos-Renyi, p=0.2

E-R, p=0.4

Pref. Att. Tree

Pref. Att. Dense

[movies]
Collective Performance and Structure

- overall behavioral performance is strong
- structure matters: many (but not all) pairs distinguished

overall mean ~ 0.88

fraction of possible wealth realized
Equilibrium vs. Behavior

- correlation $\sim -0.8$ (p $< 0.001$)
- correlation $\sim 0.96$ (p $< 0.001$)

- greater equilibrium variation $\rightarrow$ behavioral performance degrades
- greater equilibrium variation $\rightarrow$ greater behavioral variation
Best Model for Behavioral Wealths?

- The equilibrium wealth predictions are better than:
  - degree distribution and other centrality/importance measures
  - uniform distribution
- Best behavioral prediction: $0.75(\text{equilibrium prediction}) + 0.25(\text{uniform})$
- “Networked inequality aversion” (recall Ultimatum Game)
Summary

• Trading model most sophisticated “rational dynamics” we’ve studied
• Has a detailed equilibrium theory based entirely on network structure
• Equilibrium theory matches human behavior pretty well