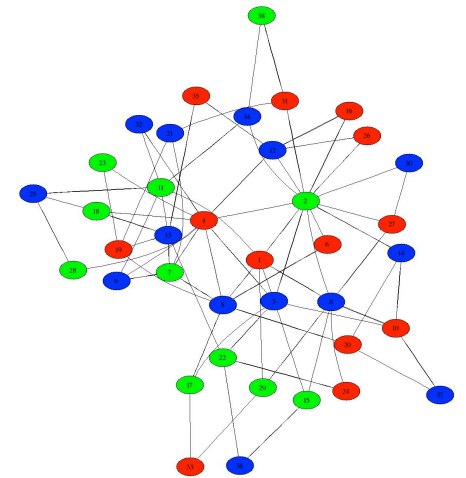


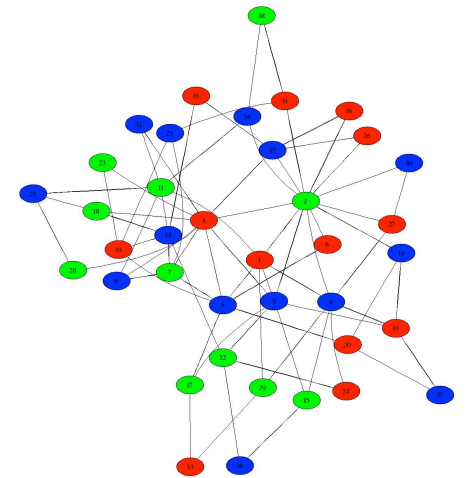
Trading in Networks: I. Model

Prof. Michael Kearns
Networked Life
NETS 112
Fall 2013



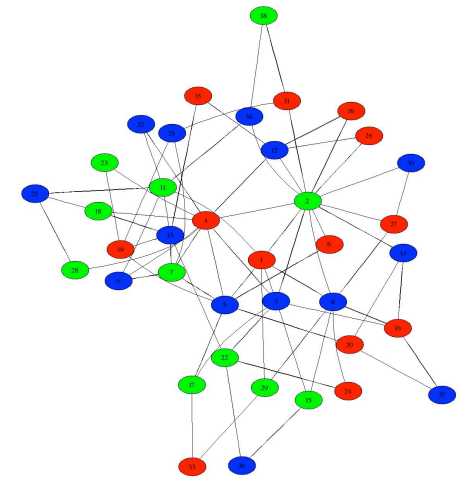
Roadmap

- Networked trading motivation
- A simple model and its equilibrium
- A detailed example



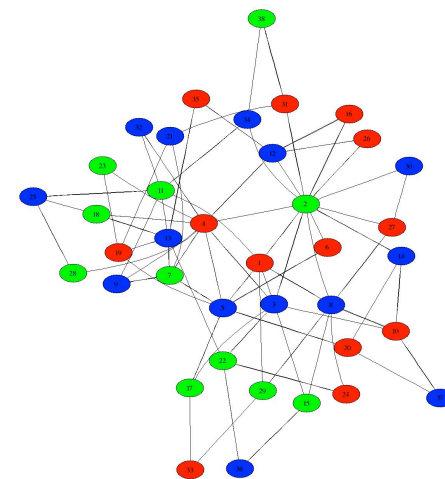
Networked Games vs. Trading

- Models and experiments so far (coloring, consensus, biased voting):
 - simple coordination games
 - extremely simple actions (pick a color)
 - “trivial” equilibrium theories (“good” equilibrium or “trapped” players)
 - no equilibrium predictions about network structure and individual wealth
- Networked trading:
 - a “financial” game
 - complex action space (set of trades with neighbors)
 - nontrivial equilibrium theory
 - detailed predictions about network structure and individual wealth



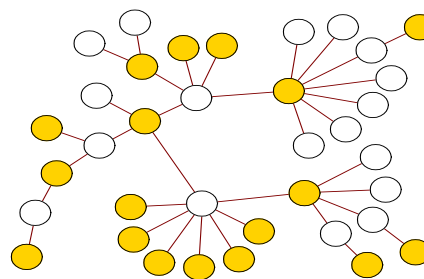
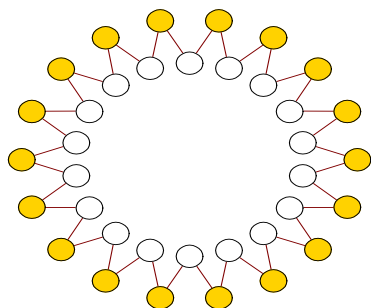
Networked Trading: Motivation

- Settings where there are restrictions on who can trade with whom
- International trade: restrictions, embargos and boycotts
- Financial markets: some transactions are forbidden
 - e.g. trades between brokerage and proprietary trading in investment banks
- Geographic constraints: must find a local housecleaning service
- Natural to model by a network:
 - vertices representing trading parties
 - presence of edge between u and v : trading permitted between parties
 - absence of edge: trading forbidden



A Simple Model of Networked Trading

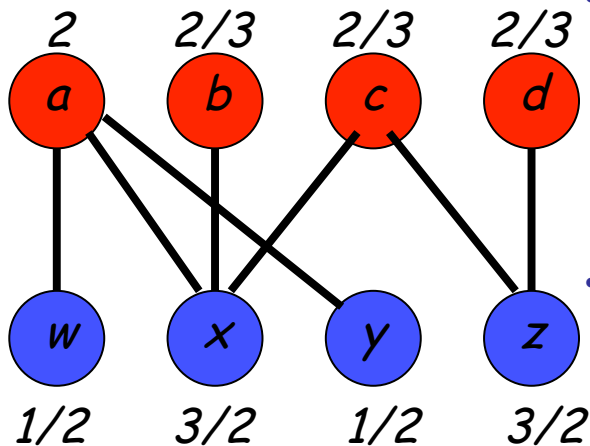
- Imagine a world with only two goods or commodities for trading
 - let's call them Milk and Wheat
- Two types of traders:
 - Milk traders: start game with 1 unit (fully divisible) of Milk, but only value Wheat
 - Wheat traders: start game with 1 unit of Wheat, but only value Milk
 - trader's payoff = amount of the "other" good they obtain through trades
 - "mutual interest in trade"
 - equal number of each type \rightarrow same total amount of Milk and Wheat
- Only consider *bipartite* networks:
 - all edges connect a Milk trader to a Wheat trader
 - can only trade with your network neighbors!
 - all trades are irrevocable
 - no resale or arbitrage allowed



Equilibrium Concept

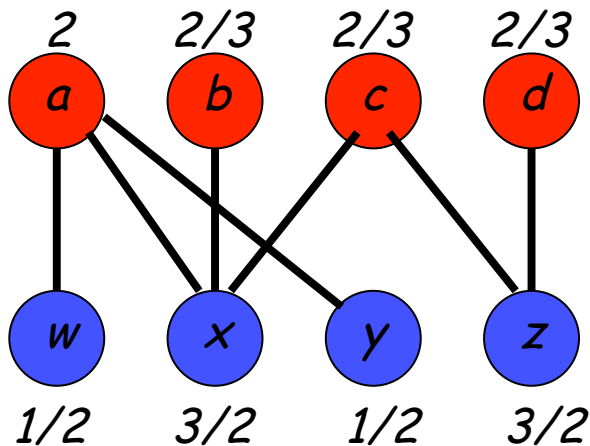
- Imagine we assigned a price or exchange rate to each vertex/trader
 - e.g. "I offer my 1 unit of Milk for 1.7 units of Wheat"
 - e.g. "I offer my 1 unit of Wheat for 0.8 units of Milk"
 - note: "market" sets the prices, not traders ("invisible hand")
 - unlike a traditional game --- traders just react to prices
- Equilibrium = set of prices + trades such that:
 - 1. market *clears*: everyone trades away their initial allocation
 - 2. rationality (best responses): a trader only trades with best prices in neighborhood
 - e.g. if a Milk trader's 4 neighbors offer 0.5, 1.0, 1.5, 1.5 units Wheat, they can trade only with those offering 1.5
 - note: set of trades must ensure supply = demand at every vertex
- Simplest example: complete bipartite network
 - every pair of Milk and Wheat traders connected by an edge
 - equilibrium prices: everyone offers their initial 1 unit for 1 unit of the other good
 - equilibrium trades: pair each trader with a unique partner of other type
 - market clears: everyone engages in 1-for-1 trade with their partner
 - rationality: all prices are equal, so everyone trading with best neighborhood prices

A More Complex Example

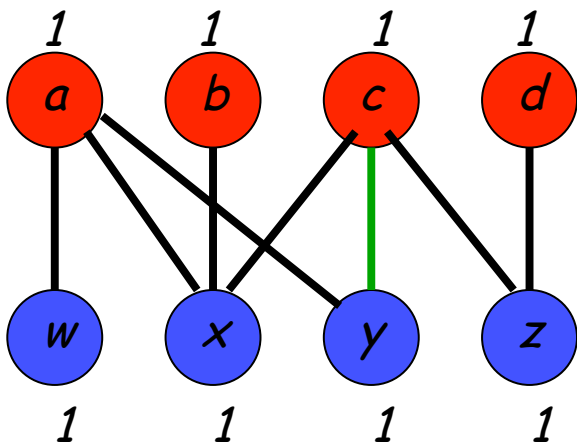


- equilibrium prices as shown (amount of the other good demanded)
- equilibrium trades:
 - a : sends $\frac{1}{2}$ unit each to w and y , gets 1 from each
 - b : sends 1 unit to x , gets $2/3$ from x
 - c : sends $\frac{1}{2}$ unit each to x and z , gets $1/3$ from each
 - d : sends 1 unit to z , gets $2/3$ from z
- equilibrium check, blue side:
 - w : traded with a , sent 1 unit
 - x : traded with b and c , sent 1 unit
 - y : traded with a , sent 1 unit
 - z : traded with c and d , sent 1 unit

Remarks



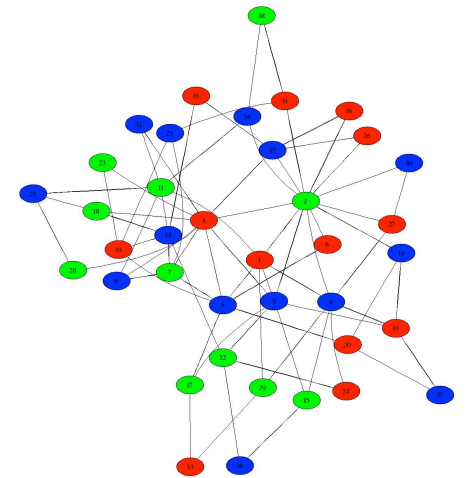
- How did I figure this out? Not easy in general
- Some edges unused by equilibrium
- Trader wealth = equilibrium price at their vertex
- If two traders trade, their wealths are reciprocal (w and $1/w$)
- Equilibrium *prices (wealths)* are always unique
- Network structure led to *variation* in wealth



- Suppose we add the single green edge
- Now equilibrium has **no** wealth variation!

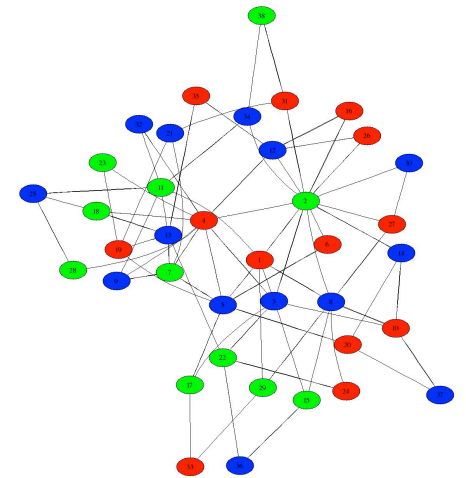
Summary

- (Relatively) simple networked trading model
- Equilibrium = prices + trades such that market clears, traders rational
- Some networks don't have wealth variation at equilibrium, some do
- Next: What is the general relationship between structure and prices?



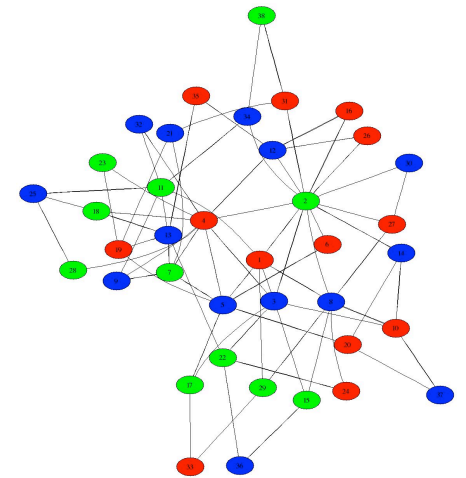
Trading in Networks: II. Network Structure and Equilibrium

Networked Life
Prof. Michael Kearns



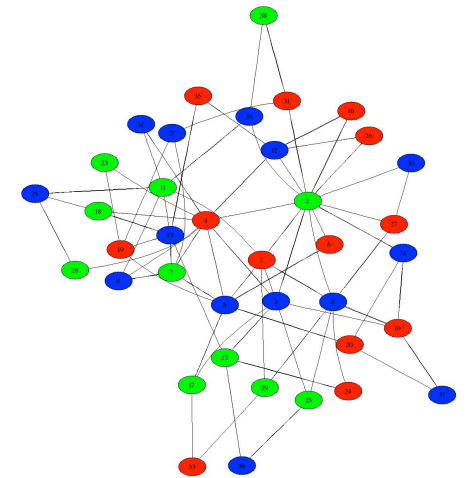
Roadmap

- Perfect matchings and equilibrium equality
- Characterizing wealth inequality at equilibrium
- Economic fairness of Erdős-Renyi and Preferential Attachment

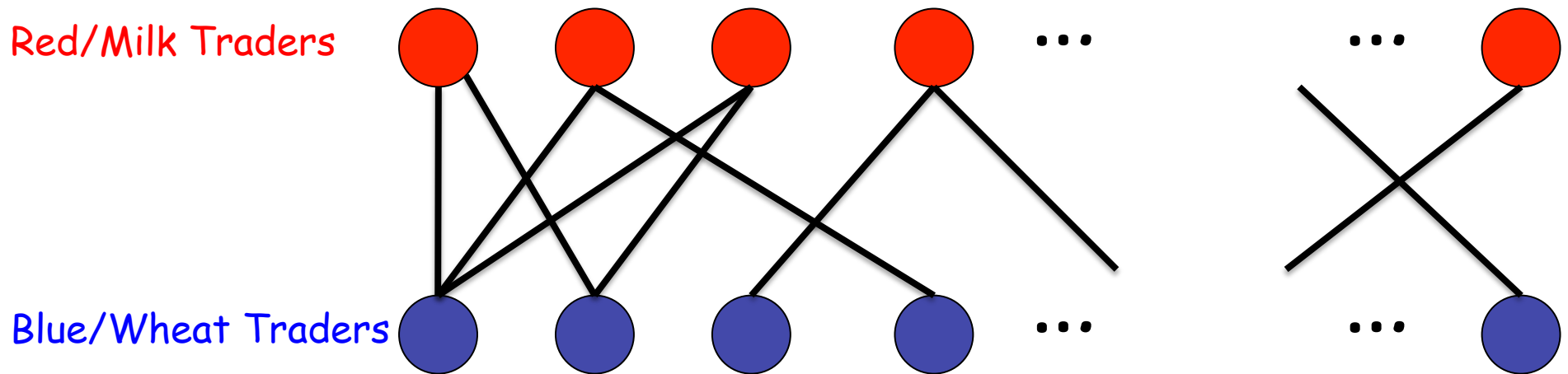


Trading Model Review

- Bipartite network, equal number of Milk and Wheat traders
- Each type values only the other good
- Equilibrium = prices + trades such that market clears, traders rational

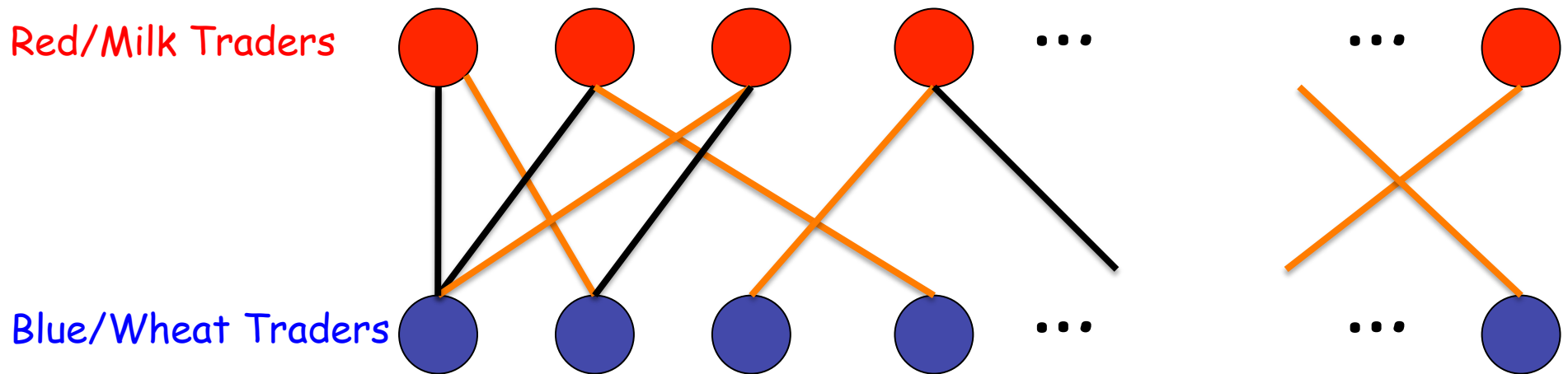


Perfect Matchings



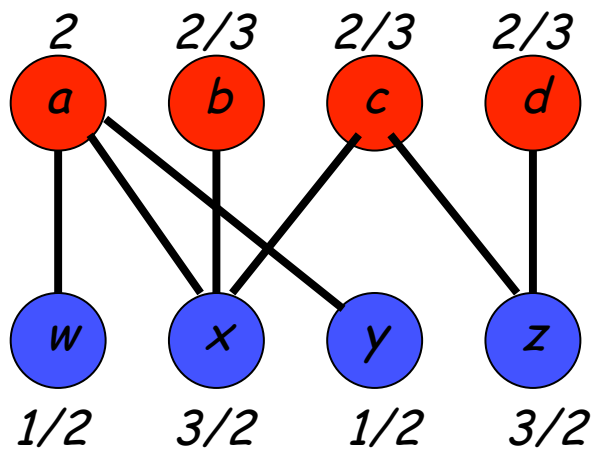
- A pairing of reds and blues so everyone has *exactly one partner*
- So really a subset of the edges with each vertex in exactly one edge
- Some networks may have many different perfect matchings
- Some networks may have no perfect matchings

Perfect Matchings

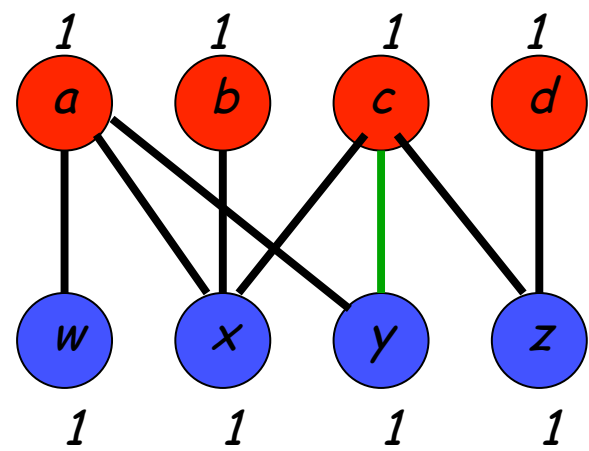


- A pairing of reds and blues so everyone has *exactly one partner*
- So really a subset of the edges with each vertex in exactly one edge
- Some networks may have many different perfect matchings
- Some networks may have no perfect matchings

Examples



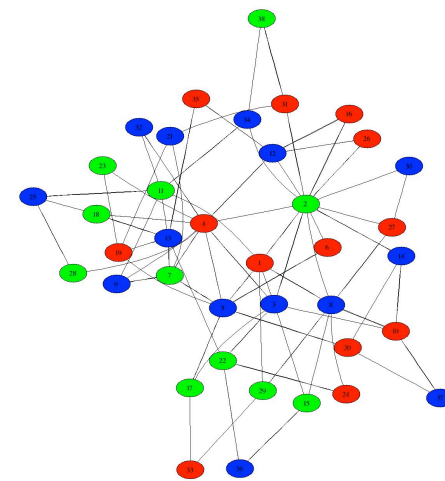
Has no perfect matching



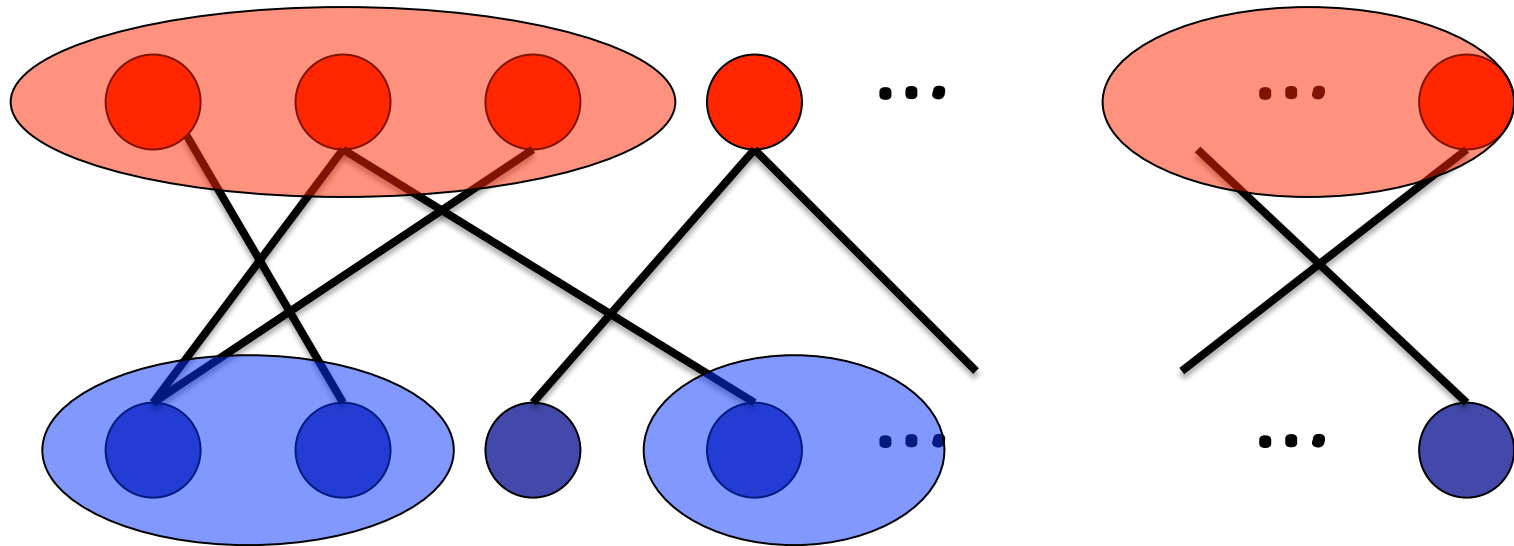
Has a perfect matching

Perfect Matchings and Equality

- Theorem: There will be *no wealth variation* at equilibrium (all exchange rates = 1) if and only if the bipartite trading network contains a perfect matching.
- Characterizes sufficient “trading opportunities” for fairness
- What if there is no perfect matching?



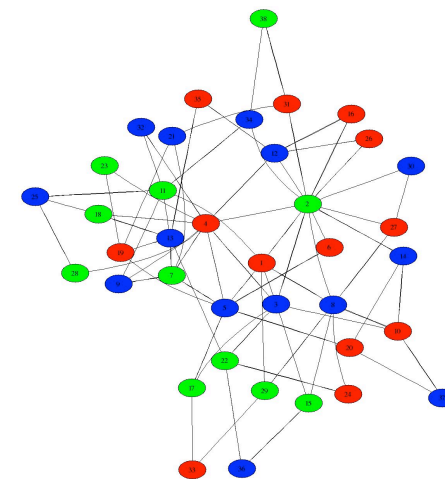
Neighbor Sets



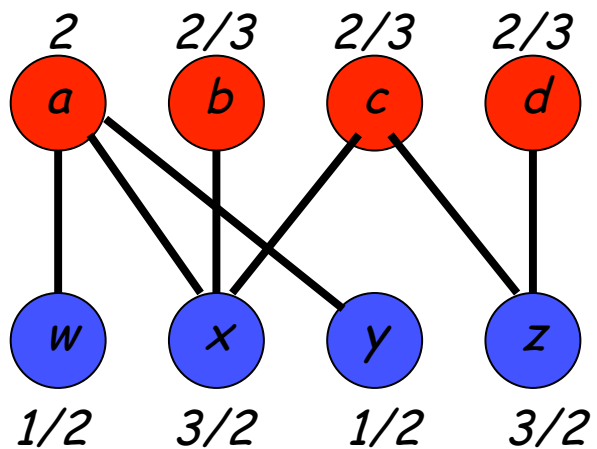
- Let S be any set of traders on one side
- Let $N(S)$ be the set of traders on the other side connected to any trader in S ; these are the only trading partners for S collectively
- Intuition: if $N(S)$ is much smaller than S , S may be in trouble
- S are "captives" of $N(S)$
- Note: If there is a perfect matching, $N(S)$ always *at least as large as* S

Characterizing Inequality

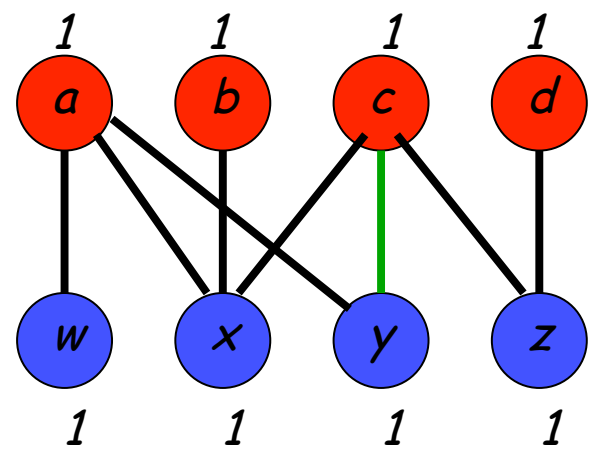
- For any set S , let $v(S)$ denote the ratio (size of S)/(size of $N(S)$)
- Theorem: If there is a set S such that $v(S) > 1$, then at equilibrium the traders in S will have wealth **at most $1/v(S)$** , and the traders in $N(S)$ will have wealth **at least $v(S)$** .
- Example: $v(S) = 10/3 \rightarrow S$ gets at most $3/10$, $N(S)$ at least $10/3$
- Greatest inequality: find S maximizing $v(S)$
- Can iterate to find all equilibrium wealths
- Corollary: adding edges can only **reduce** inequality
- Network structure completely determines equilibrium wealths
- Note: trader/vertex degree not directly related to equilibrium wealth



Examples Revisited



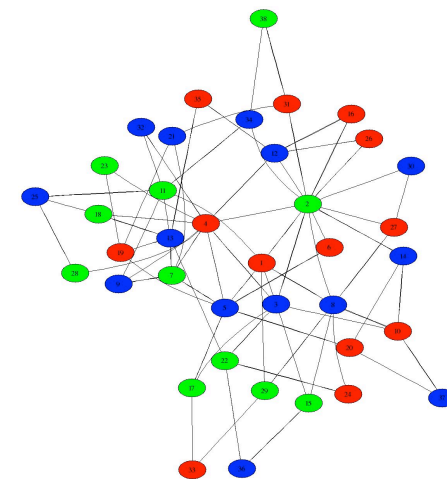
Has no perfect matching



Has a perfect matching

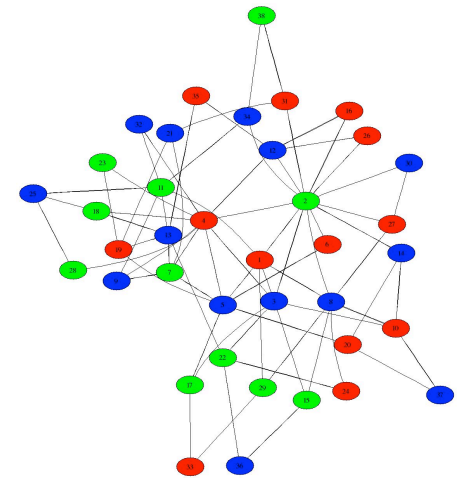
Inequality in Formation Models

- Bipartite version of Erdős-Renyi: even at low edge density, very likely to have a perfect matching → *no wealth variation* at equilibrium
- Bipartite version of Preferential Attachment: wealth variation will *grow rapidly* with population size
- Erdős-Renyi generates economically "fairer" networks



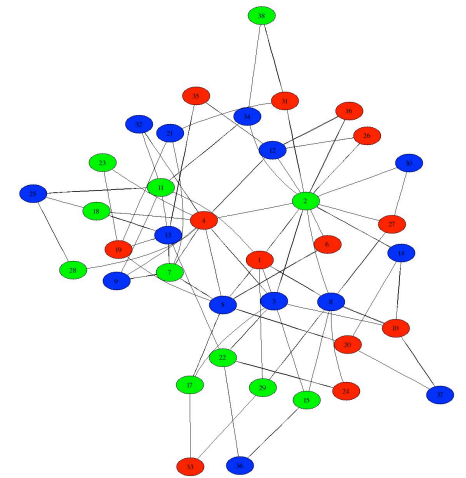
Summary

- Ratios $v(S)$ completely characterize equilibrium
- Determined entirely by network structure
- More subtle and global than trader degrees
- Next: comparing equilibrium predictions with human behavior



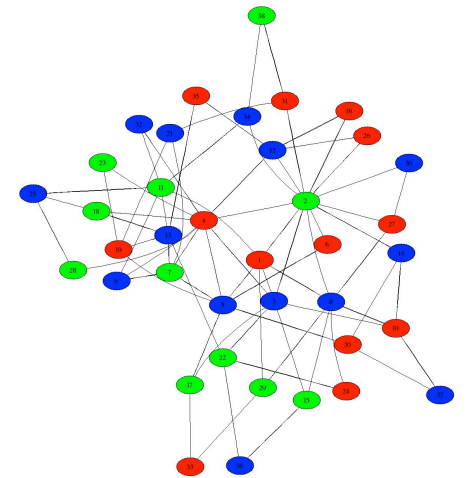
Trading in Networks: III. Behavioral Experiments

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Roadmap

- Experimental framework and trading mechanism/interface
- Networks used in the experiments
- Visualization of actual experiments
- Results and comparison to equilibrium theory predictions

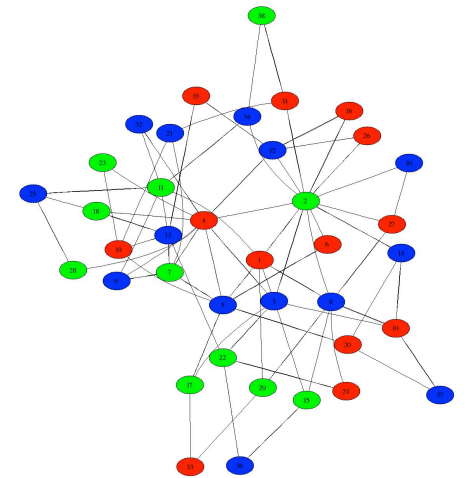


Equilibrium Theory Review

- Equilibrium prices/wealths entirely determined by network structure
- Largest/smallest wealths determined by largest ratios:

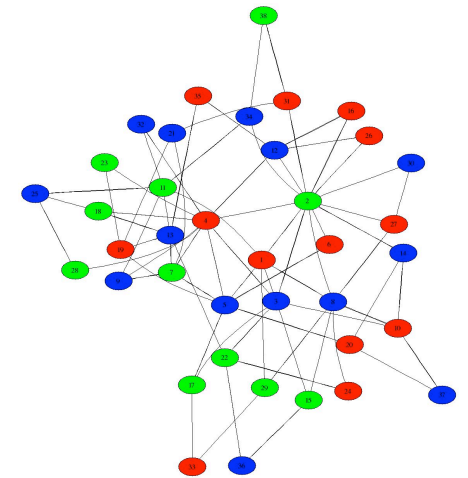
$$v(S) = (\text{size of } S) / (\text{size of } N(S)) \quad N(S) \text{ "winners", } S \text{ "losers"}$$

- Network has a perfect matching: all wealths = 1



Experimental Framework

- Same framework as coloring, consensus and biased voting experiments
- 36 simultaneous human subjects in lab of networked workstations
- In each experiment, subjects play our trading model on varying networks
- In equilibrium theory, prices are magically *given* (“invisible hand”)
- In experiments, need to provide a mechanism for price *discovery*
- Experiments used simple *limit order* trading with neighbors
 - networked version of standard financial/equity market mechanism
- Each player starts with 10 fully divisible units of Milk or Wheat
 - payments proportional to the amount of the other good obtained



BNS Network Games

http://jsjlaptop.local/~sjudd/BNS/games

BNS System kgBNS System BNS Post-Game Survey BNS Survey Results

Behavioural Network Games Donal Dukk

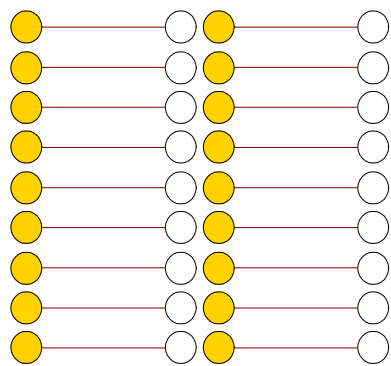
game status: WheatMktGame in progress
 elapsed time: 79%
 accumulated pay: \$2.20

quantity: 1.0 1.0 1.0

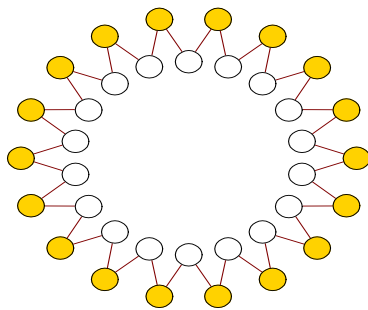
endowment	1.000	giving	1.000	<input type="text" value="2"/>
acquired	11.000	getting	1.450	<input type="text" value="2.9"/>
		buyPrice	0.690	<input type="text" value="0.690"/>

submit

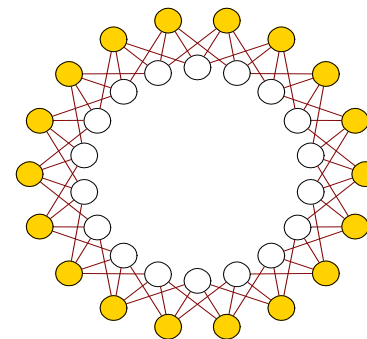
cycle time 198; work time 6, 7, 7



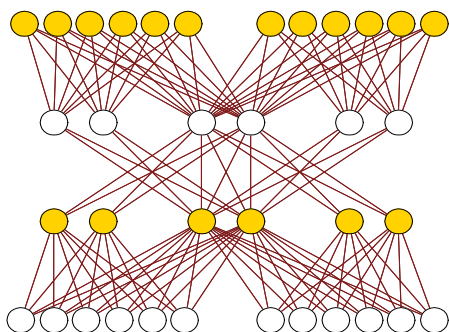
Pairs



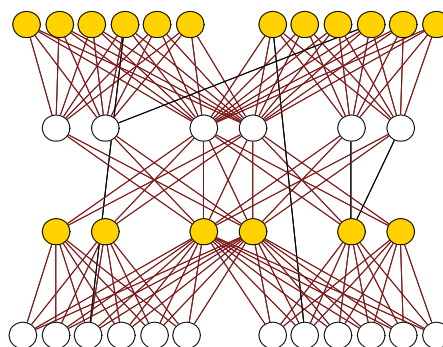
2-Cycle



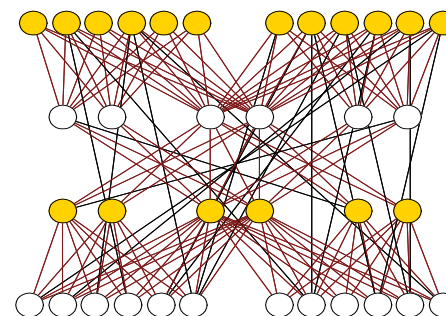
4-Cycle



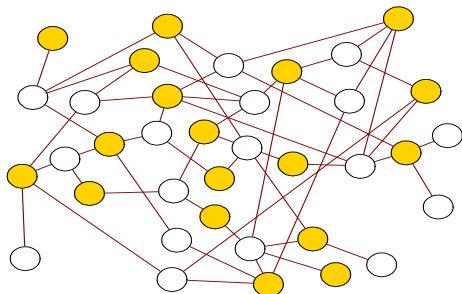
Clan



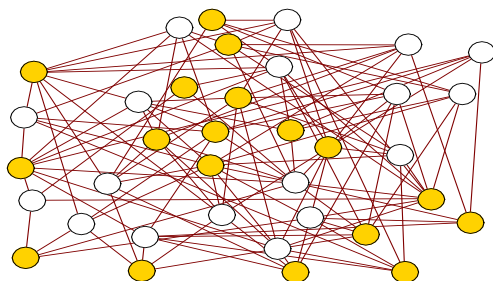
Clan + 5%



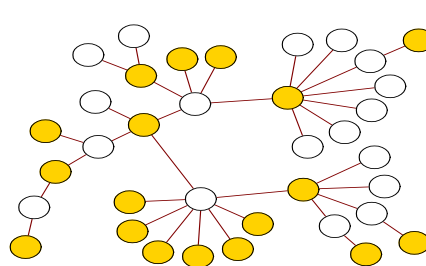
Clan + 10%



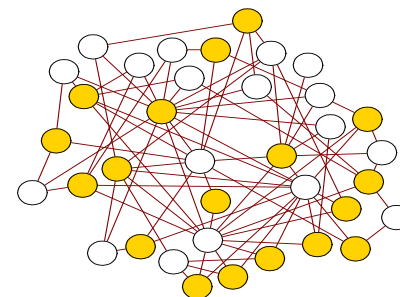
Erdos-Renyi, $p=0.2$



E-R, $p=0.4$



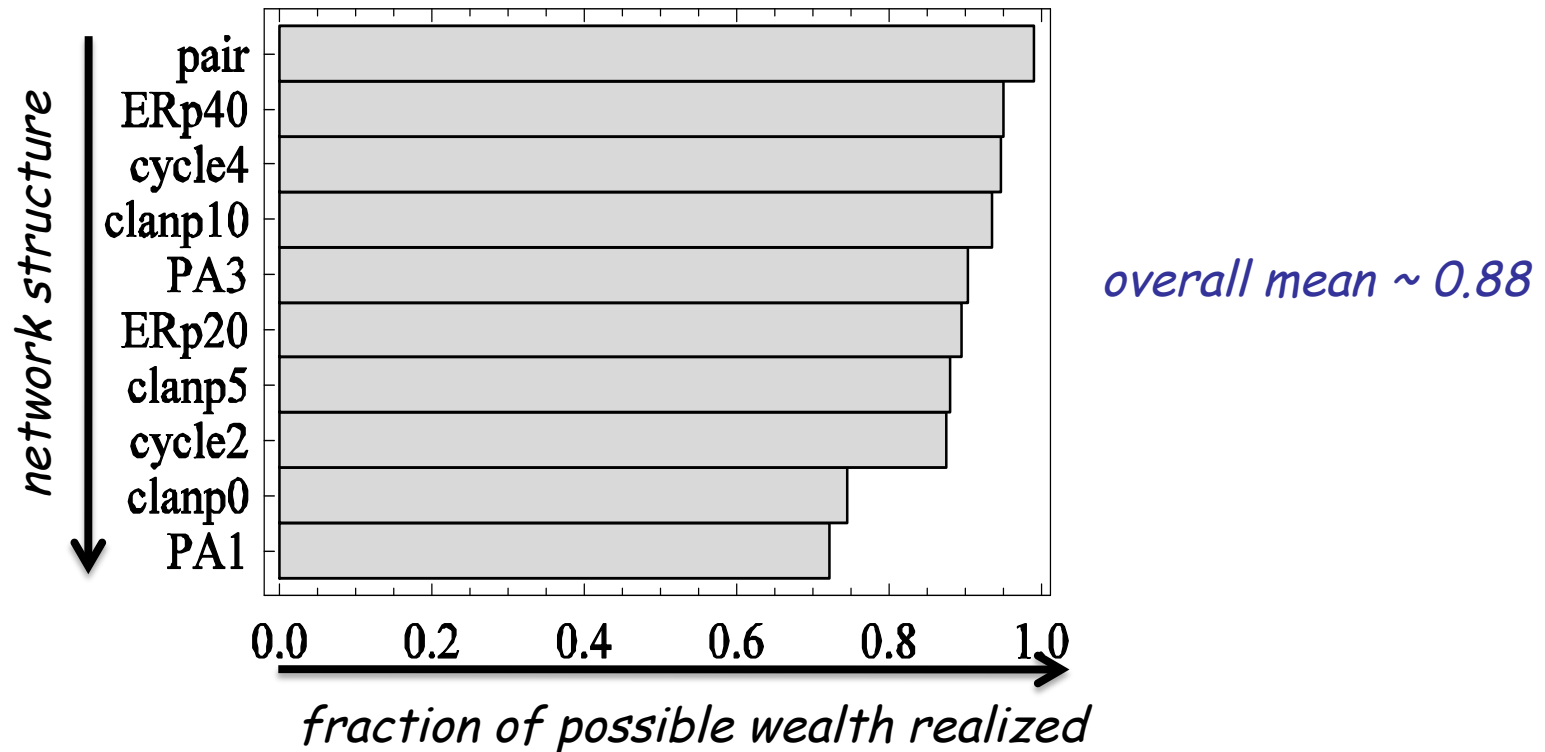
Pref. Att. Tree



Pref. Att. Dense

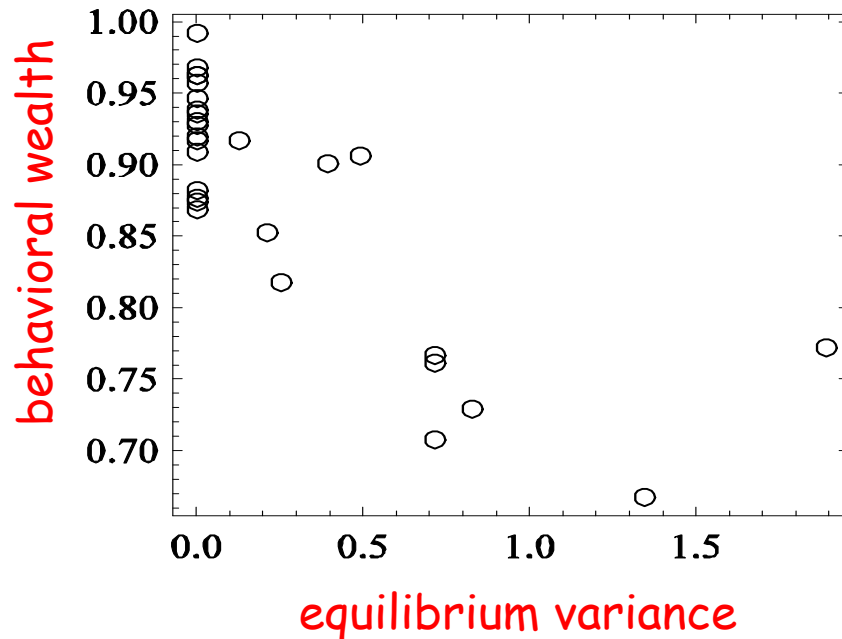
[[movies](#)]

Collective Performance and Structure

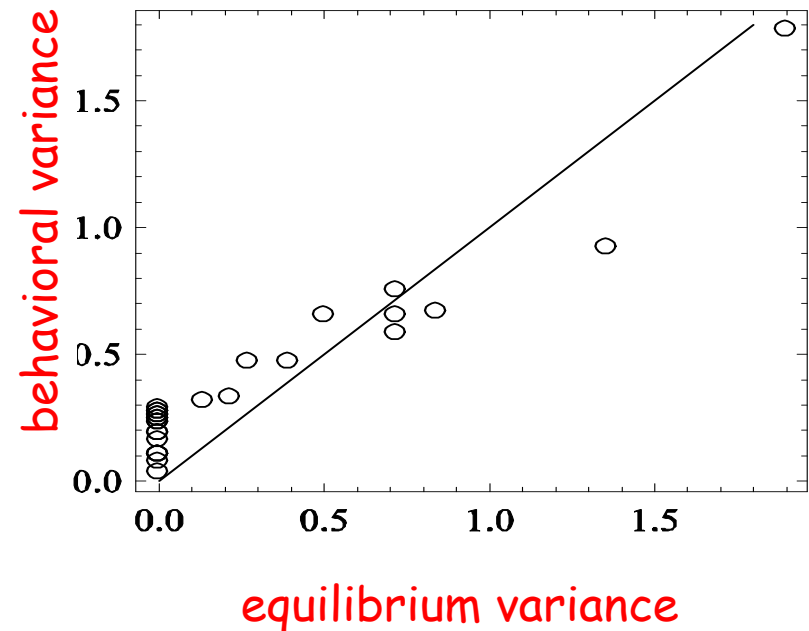


- overall behavioral performance is strong
- structure matters; many (but not all) pairs distinguished

Equilibrium vs. Behavior



correlation ~ -0.8 ($p < 0.001$)

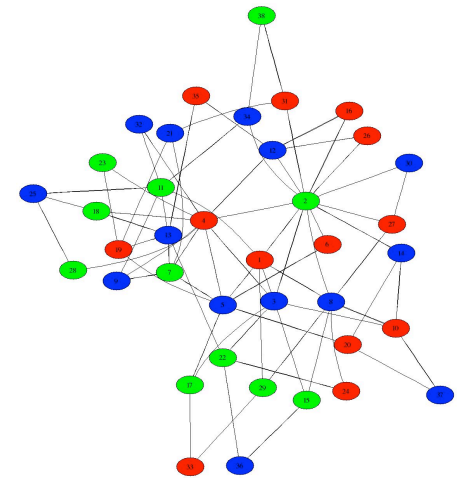


correlation ~ 0.96 ($p < 0.001$)

- greater equilibrium variation \rightarrow behavioral performance degrades
- greater equilibrium variation \rightarrow greater behavioral variation

Best Model for Behavioral Wealths?

- The equilibrium wealth predictions are better than:
 - degree distribution and other centrality/importance measures
 - uniform distribution
- Best behavioral prediction: $0.75(\text{equilibrium prediction}) + 0.25(\text{uniform})$
- "Networked inequality aversion" (recall Ultimatum Game)



Summary

- Trading model most sophisticated “rational dynamics” we’ve studied
- Has a detailed equilibrium theory based entirely on network structure
- Equilibrium theory matches human behavior pretty well

