# Introduction to (Networked) Game Theory

Networked Life
NETS 112
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# Game Theory for Fun and Profit

- The “Beauty Contest” Game
- Write your name and an integer between 0 and 100
- Let X denote the average of all the numbers
- Whoever’s number is closest to (2/3)X wins $10
- Split in case of ties

# Game Theory

- A mathematical theory designed to model:
  - how *rational individuals* should behave
  - when individual outcomes are determined by *collective* behavior
  - *strategic* behavior
- *Rational* usually means selfish --- but not always
- Rich history, flourished during the Cold War
- Traditionally viewed as a subject of economics
- Subsequently applied by many fields
  - evolutionary biology, social psychology… now computer science
- Perhaps the branch of pure math most widely examined outside of the “hard” sciences

# Games for Two

- Prisoner’s Dilemma
- Chicken
- Matching Pennies

# Prisoner's Dilemma

| | cooperate | defect |
|---|---|---|
| cooperate | -1, -1 | -10, -0.25 |
| defect | -0.25, -10 | -8, -8 |

- Cooperate = deny the crime; defect = confess guilt of both
- Claim that (defect, defect) is an *equilibrium:*
  - if I am definitely going to defect, you choose between -10 and -8
  - so you will also defect
  - same logic applies to me
- Note *unilateral* nature of equilibrium:
  - I fix a behavior or strategy for you, then choose my best response
- Claim: no other pair of strategies is an equilibrium
- But we would have been so much better off *cooperating…*

# Penny Matching

| | heads | tails |
|---|---|---|
| heads | 1, 0 | 0, 1 |
| tails | 0, 1 | 1, 0 |

- What are the equilibrium strategies now?
- There are none!
  - if I play heads then you will of course play tails
  - but that makes me want to play tails too
  - which in turn makes you want to play heads
  - etc. etc. etc.
- But what if we can each (privately) *flip coins?*
  - the strategy pair (1/2, 1/2) is an equilibrium
- Such randomized strategies are called *mixed strategies*

# The World According to Nash

- A mixed strategy for a player is a *distribution* on their available actions
  - e.g. 1/3 rock, 1/3 paper, 1/3 scissors
- *Joint* mixed strategy for N players:
  - a probability distribution for each player (possibly different)
  - assume *everyone* knows all the *distributions*
  - but the “coin flips” used to *select* from player P’s distribution known *only* to P
    - “private randomness”
    - so only player P knows their actual choice of action
    - can people randomize? (more later)
- Joint mixed strategy is an *equilibrium* if:
  - for every player P, their distribution is a *best response* to all the others
    - i.e. cannot get higher (average or expected) payoff by changing distribution
    - only consider *unilateral* deviations by each player!
  - Nash 1950: *every game* has a mixed strategy equilibrium!
  - no matter how many rows and columns there are
  - in fact, no matter how many players there are
- Thus known as a *Nash equilibrium*
- A major reason for Nash’s Nobel Prize in economics

# Facts about Nash Equilibria

- While there is always at least *one*, there might be *many*
  - zero-sum games: all equilibria give the same payoffs to each player
  - non zero-sum: different equilibria may give different payoffs!
- Equilibrium is a *static* notion
  - does not suggest how players might *learn* to play equilibrium
  - does not suggest how we might *choose* among multiple equilibria
- Nash equilibrium is a *strictly competitive* notion
  - players cannot have "pre-play communication"
  - bargains, side payments, threats, collusions, etc. not allowed
- *Computing* Nash equilibria for large games is difficult

# Behavioral Game Theory:
# What do People *Really* Do?

**(Slides adapted from Colin Camerer, CalTech)**

# Behavioral Game Theory and Game Practice

- Game theory: how rational individuals should behave
- Who are these rational individuals?
- BGT: looks at how people actually behave
  - experiment by setting up real economic situations
  - account for people’s economic decisions
  - don’t break game theory when it works
- Fit a model to observations, not “rationality”

# Beauty Contest Analysis

Some number of players try to guess a number that is 2/3 of the average guess.

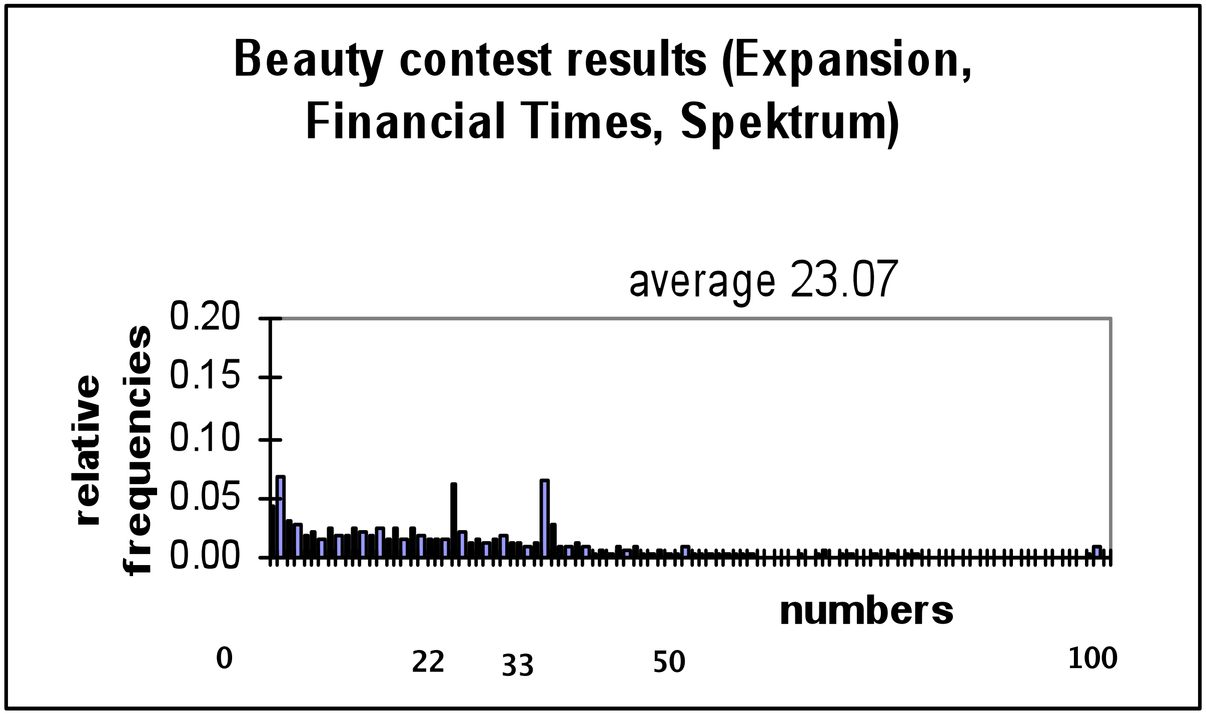
The answer can't be between 68 and 100 - no use guessing in that interval. It is *dominated.*

But if no one guesses in that interval, the answer won't be greater than 44.

But if no one guesses more than 44, the answer won't be greater than 29…

Everyone should guess 0! And good game theorists might…

But they'd lose…


Beauty contest results (Expansion, Financial Times, Spektrum)
average 23.07
0.20
0.15
0.10
0.05
0.00
relative frequencies
numbers
0
22
33
50
100

# Ultimatum Game

- Proposer has \$10
- Offers x to Responder (keeps \$10-x)
- What should the Responder do?
  - Self-interest: Take any $x > 0$
  - Empirical: Reject $x = \$2$ half the time

# How People Ultimatum-Bargain

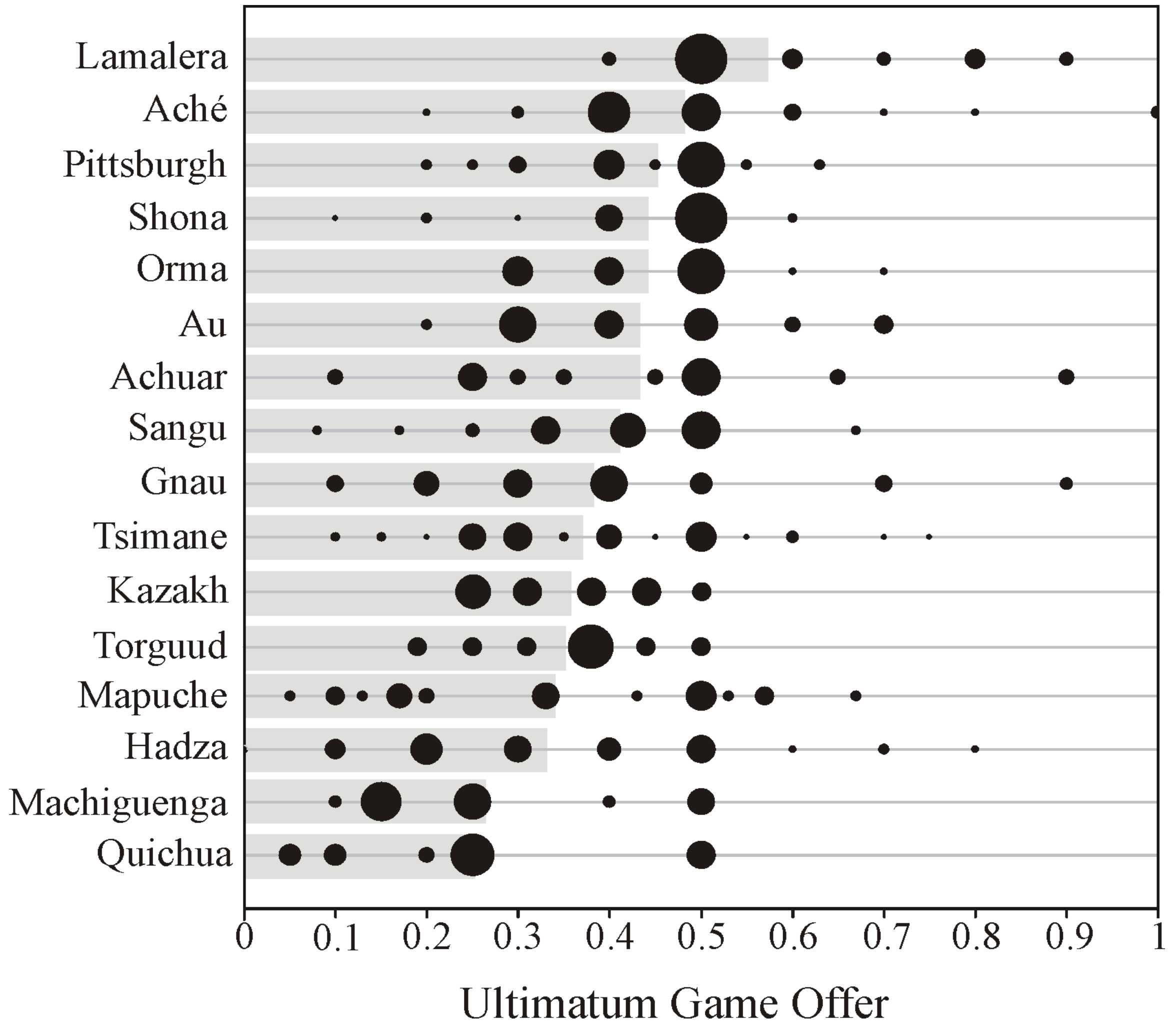
*Thousands* of games have been played in experiments…

- In different cultures around the world
- With different stakes
- With different mixes of men and women
- By students of different majors
- Etc. etc. etc.

Pretty much always, two things prove true:

1. Player 1 offers close to, but less than, half (40% or so)
2. Player 2 rejects low offers (20% or less)

# Ultimatum offers across societies (mean shaded, mode is largest circle…)

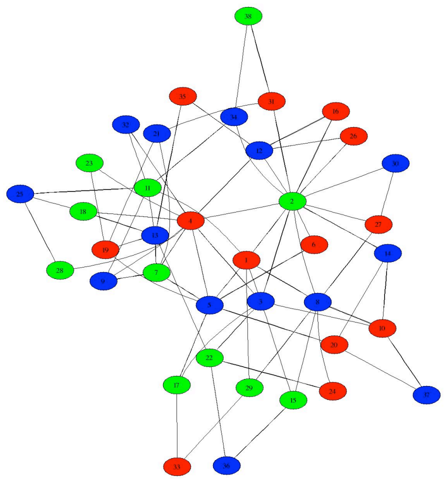


# Behavioral Game Theory: Some Key Themes

- Bounded Rationality: Humans don’t have unlimited computational/reasoning capacity (Beauty Contest)
- Inequality Aversion: Humans often deviate from equilibrium towards “fairness” (Ultimatum)
- Mixed Strategies: Humans can generate “random” values within limits; better if paid.

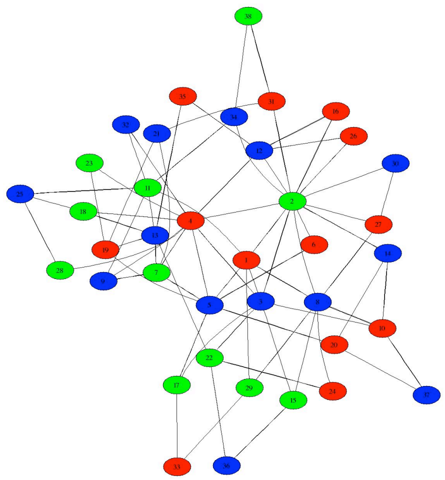
# Game Theory Review

- Specify a game by payoffs to each player under all possible joint actions
  - matrix or “normal form” games
- Nash equilibrium: choice of actions (a1,a2) for the players such that
  - a1 is a best response to a2, a2 is a best response to a1 (e.g. (confess, confess) in PD)
  - neither player can unilaterally improve their payoff
  - More generally, every player is best-responding to the other N-1 players
- Nash equilibria always exist; players may need to randomize
- A static, instantaneous concept
  - no notion of dynamics, repeated or gradual play, learning, etc.
- Examples so far:
  - small number of players (2)
  - small number of actions per player (e.g. deny or confess)
  - no notion of network



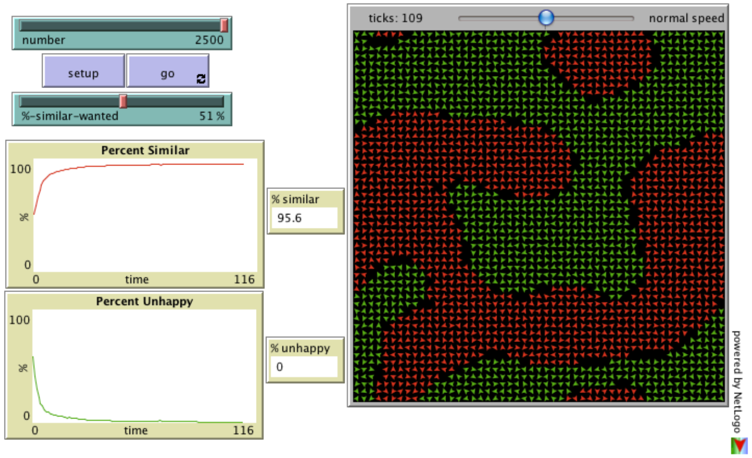
# Games on Networks

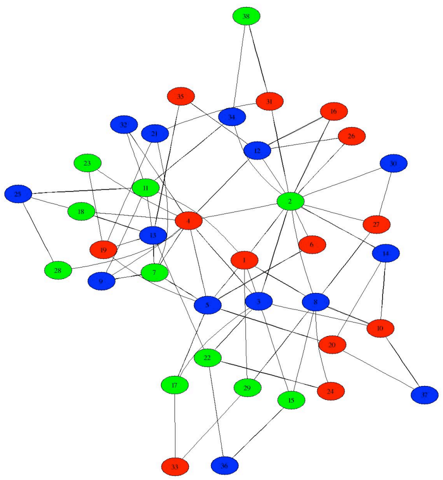
- Large number of players
- Large number of actions
- Network mediates the interactions between players and payoffs
  - player’s payoff depends only on local interactions
- Don’t need exhaustive table to specify payoffs
  - instead specify payoffs for each configuration of the local neighborhood
- Often consider dynamic, gradual interactions
  - but (Nash) equilibrium still a valuable guide



# Example: Schelling's Segregation Model

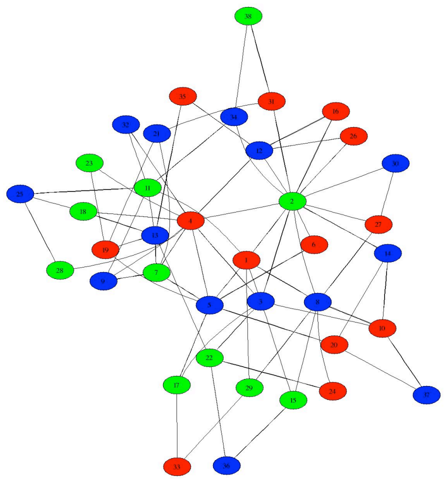
- Large number of players: 2500 in demo
- Large number of actions: all currently empty cells
- Network mediates the interactions: grid network
  - any player's payoff depends on only their neighboring cells
- Don't need exhaustive table to specify payoffs
  - payoff = 1 if at least X% like neighbors; else payoff = 0
- Often consider dynamic, gradual interactions
  - unhappy (payoff=0) players move to empty cell, may improve payoff
  - simulation converges to a Nash equilibrium (all players payoff=1)



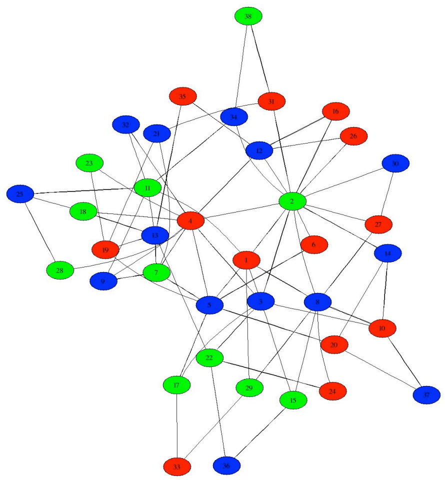
# Example: Driving to Work

- "Players" are commuters driving to work (large number)
  - each has their own origin and destination
  - wants to minimize their driving time
- Actions are routes they could take (large number)
  - multiple freeway choices, surface roads, etc.
- Network of roads intermediates payoffs
  - player's driving time depends only on how many other players are driving same roads
  - cost (= -payoff): sum of latencies on series of roads chosen
- Very complex game; still has a Nash equilibrium
- Equivalent to Internet routing
- How inefficient can the equilibrium outcome be?



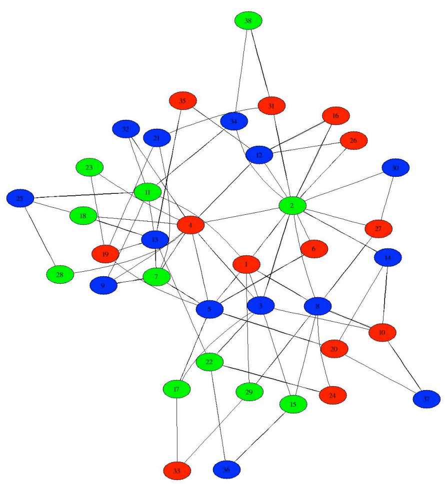
# Consensus and Coordination in Networks

- Players are individuals in a social network
- Actions are simple choices of colors to adopt
- Social network intermediates payoffs and information
  - only see color choices of your neighbors
  - payoff determined by your color choices and neighbors'
- Consensus: want to agree on common color
- Differentiation: want to be a different color than neighbors
- Biased voting: want to agree on a common color, but "care" which color
- How does network structure influence individual and collective behavior?



# Trading and Bargaining in Networks

- Players are individuals in a social network
- Actions are financial
  - trading: barter offers (e.g. trade 1 unit of Milk for 2 units of Wheat)
  - bargaining: proposals for splitting $1 (as in Ultimatum Game)
- Social network intermediates payoffs and information
  - Can only trade/bargain with your neighbors
  - payoff determined by what deals you strike with neighbors
- How does network position influence player wealth?
- What does equilibrium predict, and what do players actually do?



# Summary

- Coming lectures examine games and economic interactions on networks
- Will move back and forth between theory and experimental results
- Experiments conducted in offline class at University of Pennsylvania
- Common themes:
  - equilibrium predictions vs. behavior
  - effects of network structure on individual and collective outcome