Homework 2
Networked Life, Fall 2014
Prof Michael Kearns
Due as hardcopy at the start of class, Tuesday December 9

Problem 1 (15 points) Recall the network structure of our in-class trading experiment shown in Figure 1.

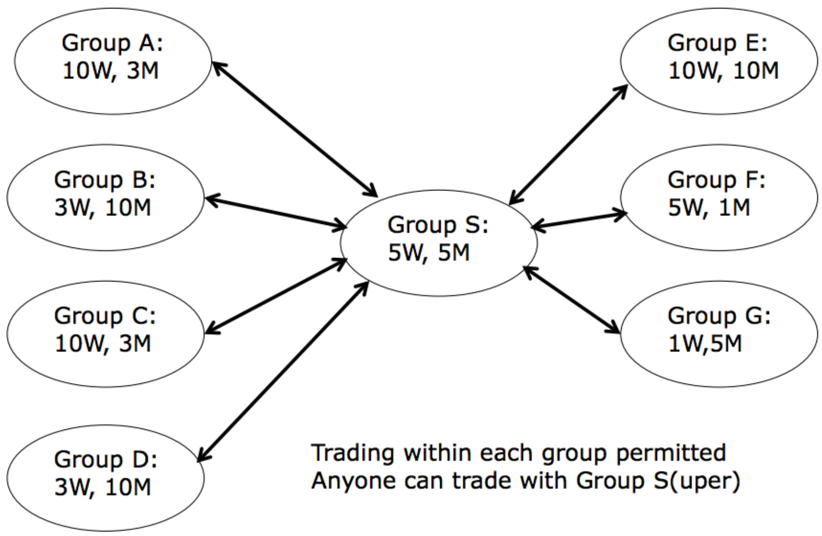

Figure 1: Network Trading - In Class Example

a. (3 points) Carefully apply the theory relating network structure to equilibrium to this network. You must describe exactly how the theory yields the resulting equilibrium wealths/prices, and what parties will trade with each other. No credit will be given for simply describing the equilibrium; you must show how it is derived.

b. (3 points) What is the smallest number of Wheat players that can be added to Group F that will cause the Milk players in Group S to trade only with Group F Wheat players? Justify your answer.

c. (3 points) What is the smallest number of Milk players that can be added to Group B that will cause the Wheat players in Group S to trade only with Group B Milk players? Justify your answer.

d. (3 points) What is the smallest number of edges that can be added to the network so that all equilibrium wealths are equal? Describe in detail which edges should be added.

e. (3 points) Suppose we add another Group H to the network. Let us denote the number of Milk and Wheat players in group H as m_H and w_H, respectively. Assume $w_H > m_H$. Precisely determine the relationship between m_H and w_H such that the Group S Milk players trade only with Group H Wheat players.

Problem 2 (9 points) Consider the competitive contagion experiments in which you participated. Suppose that for some graph G, the distribution of seed pairs chosen by the participants is P. We say that P is an equilibrium if every seed pair (i, j) appearing in P receives the same average payoff (call it x) against P, and there is no seed pair (i, j) not appearing in P that receives a payoff greater than x against P.

a. (3 points) Suppose the graph G is generated by Preferential Attachment. Describe what you think the equilibrium distribution P would look like. Justify your answer.

b. (3 points) Describe, as precisely as possible, a network G for which the equilibrium distribution P will *not* have all players choose the same seed pair. Justify your answer.

c. (3 points) Describe, as precisely as possible, a network G for with the equilibrium distribution P requires that every player play a different seed pair. Justify your answer.

Problem 3 (10 points) Consider a 2-player game where both the row and column players have two actions.

a. (5 points) Suppose the utility of each pair of actions is summarized as in Table 1. Complete the empty cells of the utility matrix such that the game has no pure strategy equilibrium.

Row Player/Column Player	a	b
c	+1, -1	
d		

Table 1: Utility Matrix

b. (5 points) Suppose the utility of each pair of actions is summarized as in Table 2. Complete the empty cells of the utility matrix such that the game has exactly two pure strategy equilibria. Does your game have a mixed strategy Nash-equilibrium?

Row Player/Column Player	a	b
c		-1, 2
d	2, -1	

Table 2: Utility Matrix

Problem 4 (12 points) Remember the attendance dynamic where the horizontal axis denotes the percentage of students who attended the class today and the vertical axis denotes the percentage of students who are going to attend the next class. If possible, plot an attendance dynamic with the properties mentioned in each part. If not, briefly describe why plotting such attendance dynamic is impossible.

a. (3 points) An attendance dynamic with 5 equilibria such that 3 equilibria are stable and 2 are unstable.

b. (3 points) An attendance dynamic with 5 equilibria such that 2 equilibria are stable and 3 are unstable.

c. (3 points) An attendance dynamic with 5 equilibria such that only 1 equilibrium is stable and 4 are unstable.

d. (3 points) An attendance dynamic with 5 equilibria such that all equilibria are unstable.

Problem 5 (5 points) Pick one of the experiments from the assigned papers on behavioral experiments that we have not discussed in class (*e.g.,* Independent Set or Network Bargaining). Briefly but precisely describe the equilibria in the experiment. Then compare these equilibria in terms of the pay-offs that players receive and describe why some equilibria might be more preferable to players than others.

Problem 6 (10 points) Recall the Online Coloring Experiment on 30 graphs and the results that were covered in class. There were three types of graph families (Random, Preferential Attachment, and Small Worlds) that were used in the online experiments and each had the same number of nodes and edges.

a. (5 points) Many students claimed that one strategy they used was to color the maximum degree node first and then color its neighbors in as few colors as possible. Why might this heuristic lead students to faster finish times in Preferential Attachment graphs? Why might this same heuristic cause students to have worse finish times in Small World graphs?

b. (5 points) Another strategy that students claimed to use was to find triangles and then color all the nodes in the triangle differently. What family of graphs (Preferential Attachment or Small World) would you think this would be a better heuristic for? Explain your reason.

Problem 7 (9 points) Recall Network Trading where there are people with one unit of Milk (Red) and people with one unit of Wheat (Blue) that want to trade their good for the other good.

a. (6 points) What is the equilibrium for the Network Trading example in Figure 2? Remember that you must state the set of prices *and* the trades. Does this lead to wealth variation among the players?

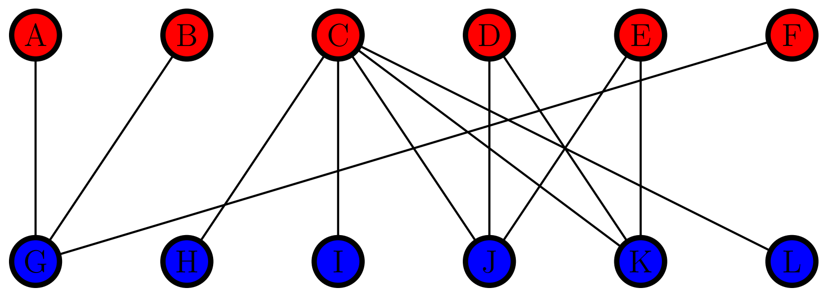

Figure 2: Network Trading

b. (3 points) What is the fewest number of edges you need to add in order for the equilibrium to have no wealth variation? What edges did you add? Why do you know that there is no wealth variation in the new graph?

Problem 8 (12 points) Selfish Routing - Suppose one unit of flow that is fully divisible wants to route from node S to node T in the network given in Figure 3. The latency functions are given on each edge in terms of the fraction of people using that edge. Recall that an equilibrium is a way to route the unit flow such that if any fraction of people deviate, they only get a longer total commute time from S to T.

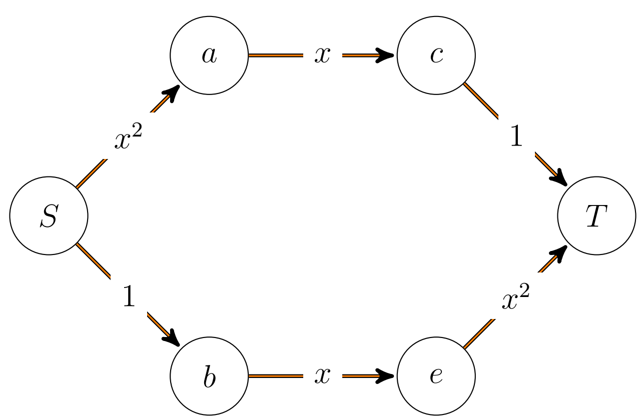

Figure 3: Selfish Routing - Part (a)

a. (4 points) Which of the following is an equilibrium in Figure 3? Give a reason for each one. Also give the total payoff for each strategy. What is the optimal strategy?

- All unit of flow is sent from S to T along the top route ($S \to a \to c \to T$).
- Splitting the unit of flow evenly across the top route and the bottom route.

b. (4 points) Which of the following is an equilibrium in Figure 4? Give a reason for each one. Also give the total payoff for each strategy. What is the optimal strategy?

- All unit of flow is sent from S to T along the zig-zag route ($S \to b \to c \to T$).
- Splitting the unit of flow evenly across the top route and the bottom route.

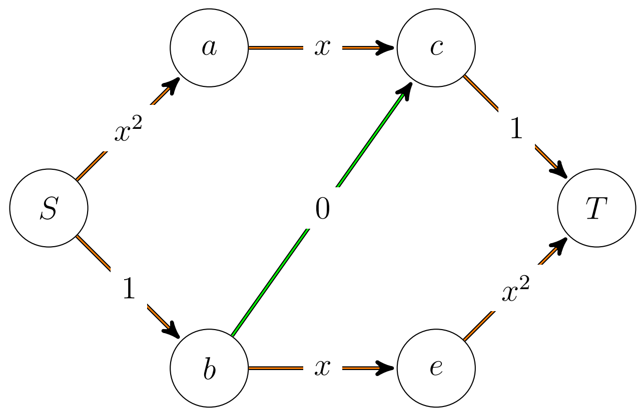

Figure 4: Selfish Routing - Part (b)

c. (4 points) Which of the following is an equilibrium in Figure 5? Give a reason for each one. Also give the total payoff for each strategy. What is the optimal strategy?

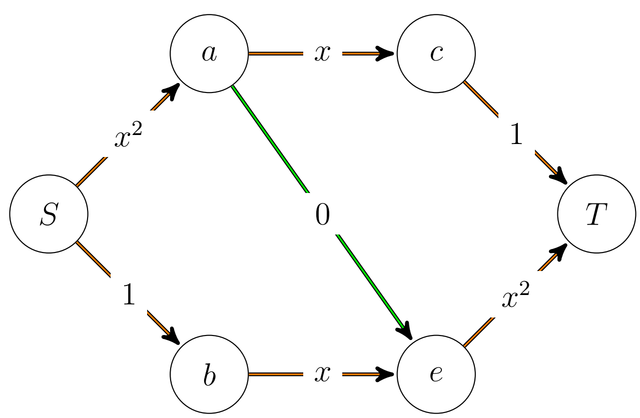

Figure 5: Selfish Routing - Part (c)

- All unit of flow is sent from S to T along the zig-zag route ($S \to a \to e \to T$).
- Splitting the unit of flow evenly across the top route and the bottom route.

Problem 9 (10 points) Price of Anarchy - We refer to the social welfare SW of a pair of strategies for the players as the total payoff to both players under that strategy. As an example, $SW(\text{Confess}, \text{Defect}) = 0 + 10 = 10$ in the Prisoner's Dilemma given in Table 3.

a. (5 points) Consider the following Prisoner's Dilemma given in Table 3 where for each pair of action the entries of the table show the satisfaction of each prisoner after hearing their verdict (so prisoners prefer higher numbers). What is the social welfare SW for the Nash equilibrium for this game? We will call this value $SW(NE)$. Find the strategy OPT that maximizes SW, and calculate its social welfare, *i.e.* $SW(OPT)$. Calculate the price of anarchy POA of the Prisoner's Dilemma by computing $POA = \frac{SW(OPT)}{SW(NE)}$. Explain in non-technical terms what it means to have a POA much larger than 1.

Row Player/Column Player	Cooperate	Defect
Cooperate	9, 9	0, 10
Defect	10,0	1,1

Table 3: Prisoner's Dilemma

b. (5 points) Recall that the Nash equilibrium for the game Rock-Paper-Scissors (given in Table 4 is mixed where each player should choose to play Rock, Paper, or Scissors with equal probability. Calculate the POA for Rock-Paper-Scissors. Be sure to write all of your work. Write in non-technical terms what your value of POA means.

Row Player/Column Player	Rock	Paper	Scissors
Rock	1, 1	0,2	2,0
Paper	2,0	1,1	0,2
Scissors	0,2	2,0	1,1

Table 4: Rock-Paper-Scissors

Problem 10 (8 points) The command *traceroute* determines the route taken by a packet to reach a destination *e.g.* Figure 6 shows the route a packet takes to reach Stanford's Computer Science Department.

a. (4 points) Briefly describe the technological, geographical and economical inferences you can have from Figure 6.

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seas581:~ Shahin$ traceroute cs.stanford.edu
traceroute to cs.stanford.edu (171.64.64.64), 64 hops max, 52 byte packets
 1  seas-apn-gw.router.upenn.edu (158.130.104.1)  1.660 ms  0.935 ms  1.935 ms
 2  huntsman.hnt-brdr.router.upenn.edu (128.91.241.90)  18.406 ms  1.657 ms  1.130 ms
 3  0.hnt-brdr.vag-brdr.router.upenn.edu (128.91.238.205)  1.656 ms  1.068 ms  23.376 ms
 4  vag-brdr.i2-wash.router.upenn.edu (128.91.240.182)  5.100 ms  5.135 ms  5.079 ms
 5  et-5-0-0.104.rtr.atla.net.internet2.edu (198.71.45.6)  18.430 ms  18.769 ms  18.716 ms
 6  et-10-2-0.105.rtr.hous.net.internet2.edu (198.71.45.13)  42.304 ms  42.309 ms  43.198 ms
 7  et-5-0-0.111.rtr.losa.net.internet2.edu (198.71.45.21)  74.867 ms  74.999 ms  74.903 ms
 8  hpr-lax-hpr2--i2-r&e.cenic.net (137.164.26.200)  74.966 ms  75.098 ms  76.112 ms
 9  svl-hpr2--lax-hpr2-10g.cenic.net (137.164.25.38)  85.285 ms
    svl-hpr2--lax-hpr2-10g-2.cenic.net (137.164.25.50)  85.220 ms  90.475 ms
10  hpr-stanford--svl-hpr2-10ge.cenic.net (137.164.27.62)  86.118 ms  85.684 ms  85.863 ms
11  csmx-west-rtr-vl9.sunet (171.66.255.214)  87.900 ms  92.676 ms  87.791 ms
12  cs.stanford.edu (171.64.64.64)  85.996 ms  85.833 ms  85.585 ms
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Figure 6: Output of command *traceroute*

b. (4 points) Consider the network in Figure 7 where user A, who has Comcast as its provider, tries to send a packet to user B, who has Sprint as its provider. Comcast and Sprint operate separately and each uses the shortest path to route the packet through its network. Does this routing scheme result in the packet from user A to user B to travel through the shortest path in the network? If so, briefly describe why. If not, plot a simple internal network for Comcast and Sprint to illustrate a counter example.

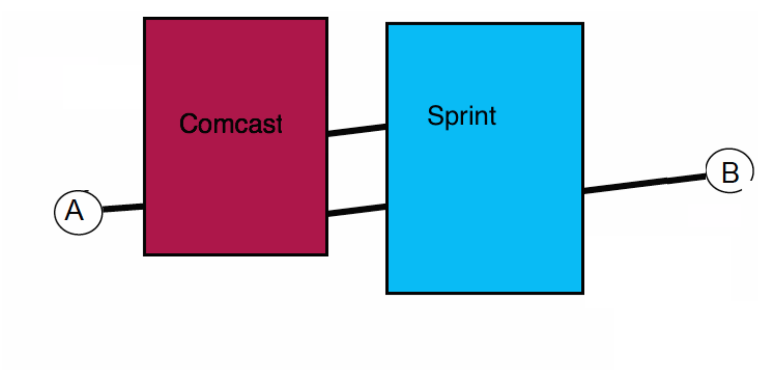

Figure 7: Online Packet Routing