

**FINAL EXAMINATION**  
**Networked Life (NETS 112)**  
**December 15, 2015**  
**Prof. Michael Kearns**

*This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen.*

*If you run out of room on a page, you may use the back, but be sure to indicate you have done so.*

**Name:** \_\_\_\_\_

**Penn ID:** \_\_\_\_\_

**Problem 1:** \_\_\_\_\_/10

**Problem 2:** \_\_\_\_\_/20

**Problem 3:** \_\_\_\_\_/15

**Problem 4:** \_\_\_\_\_/10

**Problem 5:** \_\_\_\_\_/20

**Problem 6:** \_\_\_\_\_/15

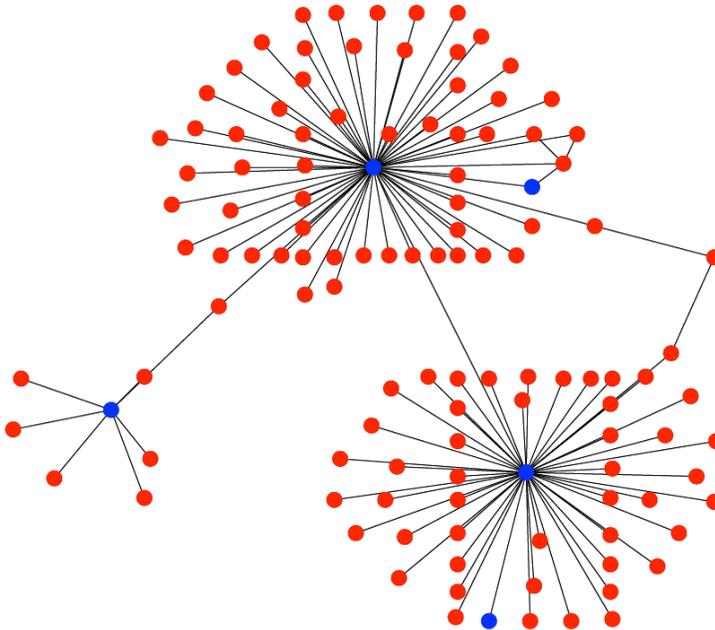
**Problem 7:** \_\_\_\_\_/10

**TOTAL:** \_\_\_\_\_/100

**Problem 1 (10 points)** For each of the following statements, simply write “TRUE” or “FALSE”.

- a. Social welfare is always maximized at Nash equilibrium.
- b. Preferential attachment is an example of a “rich get richer” process.
- c. It is possible for a network to have a high clustering coefficient, yet not contain distinct communities of vertices.
- d. If the players in a game repeatedly best-respond to the actions of the other players, they will eventually converge to a Nash equilibrium.
- e. If an Internet router is too busy to process an arriving packet, it will notify the sender to try again later.
- f. A cycle in which all players purchase immunization is an equilibrium for our network formation game experiment.
- g. The technology underlying Skype is vastly different than that for the traditional land-line telephone network.
- h. If we add roads to a network of roads, the average commute time at equilibrium can only improve.
- i. Human subjects playing the Ultimatum Game tend to deviate significantly from the theoretical equilibrium.
- j. Playing 17 is a better strategy than playing 0 when playing the Beauty Contest Game against a population of human opponents.

**Problem 2 (20 points)** The image below visualizes the final network collectively built in our behavioral experiment on a network formation game with attack and immunization. The blue vertices have purchased immunization, and the red vertices have not. Which vertex purchased each edge is not indicated, but in general the spokes bought the edges to the hubs.

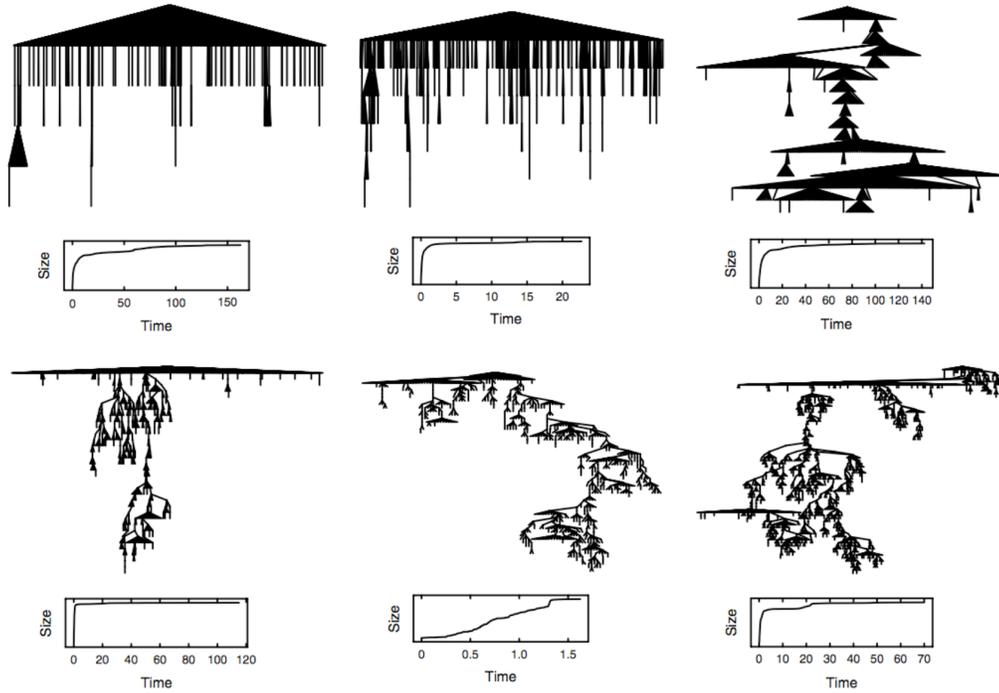


- a. Is this network a Nash equilibrium? If not, clearly indicate on the image a player/vertex who is not best-responding to the actions of the other players, and describe what a better choice for this player would be and why.
  
- b. Clearly label on the image all of the players who are receiving the lowest payoff; below do your best to calculate and explain that lowest payoff.

c. Exactly how many of the immunized players are best-responding to the actions of all the other players? Explain your answer below.

d. Suppose we change the behavior of the adversary, so that he now simply picks *any* non-immunized vertex  $v$  at random to attack; as before, this attack then spreads and kills all vertices in  $v$ 's connected component after the removal of the immunized vertices. Clearly identify a vertex in the figure who was best-responding in the original game, but would not be under this new attack model. Explain your answer below.

**Problem 3 (15 points)** The following image is from one of the required readings.



- a. As clearly and precisely as you can, describe what the tree-like structures are visualizing. What data source lies behind them? Exactly what process are they visualizing?

b. What do the authors suggest might account for the structural differences between the first two shallow trees and the deeper remaining four?

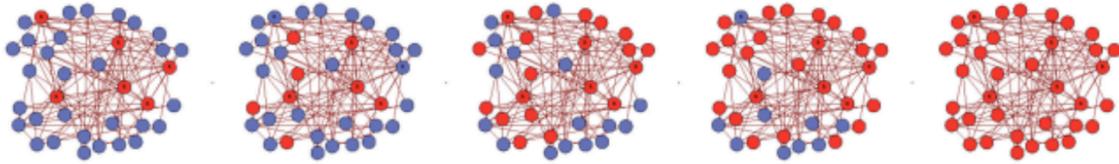
c. Do the authors conclude that the shallow or deep structures are more typical in their data set? What explanations do they provide for their conclusion?

**Problem 4 (10 points)** For a 2-player game in which all payoffs are positive, we define the Price of Anarchy to be the ratio  $x/y$ , where  $x$  is the maximum possible value for the sum of the payoffs to the two players, and  $y$  is the value of the sum of the payoffs for the two players in a Nash equilibrium.

- a. Write down the numerical payoff matrix for a 2-player, 2-action game in which the Price of Anarchy is exactly equal to 1. Explain your reasoning, and clearly indicate the Nash equilibrium of your game.

- b. Write down the numerical payoff matrix for a 2-player, 2-action game in which the Price of Anarchy is as large as you can make it. Explain your reasoning, and clearly indicate the Nash equilibrium of your game

**Problem 5 (20 points)** The following image was discussed in lecture, and is taken from the required reading “Experiments in Social Computation”.



- a. Precisely describe the actions and payoffs to the players in this game. What is the main strategic tension that these payoffs create?

- b. Name the generative model that produced the network shown, and describe how the different types of players were arranged in the network.

c. Describe what phenomenon the sequence of images is illustrating, and give at least one reason why it perhaps surprising.

d. Can the outcome shown be explained by players simply choosing to agree with the majority of their neighbors? Why or why not?



**Problem 7 (10 points)** Consider the Erdos-Renyi model of network formation. Suppose that at some point during the process of adding random edges, there exist two connected components, each of size approximately  $N/2$ . As precisely as you can, explain why it is extremely unlikely these two components will remain disconnected from each other as we add further random edges.