

1. Write the number of the left-hand item next to the item on the right that corresponds to it.

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|-------------------------------|---|
| 1. Stanford prison experiment | model of how the rich get richer                |
| 2. Friendster                 | models random pairwise conflicts                |
| 3. neuron                     | describes North American city sizes             |
| 4. router                     | obtained by selfish routing                     |
| 5. tipping                    | component of the web graph                      |
| 6. small worlds               | improved by the power of weak ties              |
| 7. job-hunting                | phenomenon found by Travers and Milgram         |
| 8. social capacity            | clearance of goods                              |
| 9. independent set            | model of random graphs                          |
| 10. planar graphs             | based on shared randomization                   |
| 11. preferential attachment   | Arrow's impossibility theorem                   |
| 12. Erdos-Renyi               | complement of clique                            |
| 13. Zipf's law                | vertex in a biological network                  |
| 14. tendrils                  | model of a random web surfer                    |
| 15. PageRank algorithm        | sudden amplification of the incremental         |
| 16. bag of words              | model used in information retrieval             |
| 17. correlated equilibrium    | subject of the four-color theorem               |
| 18. social choice theory      | explored power of context                       |
| 19. evolutionary game theory  | online social network                           |
| 20. market equilibrium        | the magic number 150                            |
| 21. Price of Anarchy          | vertex in a technological network               |
| 22. differential pricing      | measure in a mathematical collaboration network |
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QuickTime™ and a  
TIFF (LZW) decompressor  
are needed to see this picture.

2. Consider the diagram above, whose x-axis value indicates the number of individuals currently attending some activity (like a seminar or volleyball game), and whose y-axis value indicates how many will attend the activity the next time, given the x-axis value.

- A. If 55 people attend the first time, what will be the eventual (or limiting) number attending?
- B. If 25 people attend the first time, what will be the eventual number?

C. Are there any other points of equilibrium? If yes, where are they?

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3. Consider what we referred to in class and the readings as the "market for lemons".

A. [One paragraph.] Briefly describe the dynamics of the market for lemons. Indicate whether the price and volume of lemons is expected to increase or decrease with time, and explain why. Be sure to highlight any aspects of the market for lemons that distinguish it from markets with different behavior.

B. [Short phrase.] Suggest at least one real-world market for a product or service (other than lemons and insurance) that might exhibit the same behavior, and indicate why.

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Choice	Median choice													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	45	49	52	55	56	55	46	-59	-88	-105	-117	-127	-135	-142
2	48	53	58	62	65	66	61	-27	-52	-67	-77	-86	-92	-98
3	48	54	60	66	70	74	72	1	-20	-32	-41	-48	-53	-58
4	43	51	58	65	71	77	80	26	8	-2	-9	-14	-19	-22
5	35	44	52	60	69	77	83	46	32	25	19	15	12	10
6	23	33	42	52	62	72	82	62	53	47	43	41	39	38
7	7	18	28	40	51	64	78	75	69	66	64	63	62	62
8	-13	-1	11	23	37	51	69	83	81	80	80	80	81	82
9	-37	-24	-11	3	18	35	57	88	89	91	92	94	96	98
10	-65	-51	-37	-21	-4	15	40	89	94	98	101	104	107	110
11	-97	-82	-66	-49	-31	-9	20	85	94	100	105	110	114	119
12	-133	-117	-100	-82	-61	-37	-5	78	91	99	106	112	118	123
13	-173	-156	-137	-118	-96	-69	-33	67	83	94	103	110	117	123
14	-217	-198	-179	-158	-134	-105	-65	52	72	85	95	104	112	120

4. Consider Table 1.1., which is reproduced from the class readings on behavioral game theory, and describes the payoffs to a player for each of their possible choices as a function of the median choice of the population. The group of players each simultaneously chooses a "choice" 1-14 (the rows) and gets a payoff based on which number, 1-14, ended up being closest to the median choice of the whole group.

- A. List all the Nash equilibria of this game.
  
- B. [Short Phrase.] Suppose that a large fraction of the players in the game come from a culture in which the number 5 is considered lucky, and choose to play it in the first round of repeated play. What is the outcome likely to be?
  
- C. [Short Phrase.] Suppose we were to draw a network in which each vertex was a player in this game, and there is an edge between players  $i$  and  $j$  if there is some joint action in which the action of player  $i$  can influence the payoff to player  $j$ , and vice versa. Briefly describe the structure of this network.

5. In this problem we will examine networks that embody a simplified version of the Interdependent Security (IDS) games that we studied in class.

Consider an undirected network. We will refer to the degree of player  $i$  as  $d(i)$ . Each player in the network must decide whether to invest in some security mechanism (e.g. airline baggage screening, computer users updating their anti-virus software, etc.). We assume that there is some  $k$ , either 0 or a positive integer, with the following property: for any player  $i$ , player  $i$  will invest if and only if at least  $d(i) - k$  neighbors are investing.

- A. [Short Phrase.] Let  $k=1$ . what happens to a node of degree 1?
- B. [At most 5 sentences.] Assume that all players are initially not investing, but may update their behavior in response to changes in their neighbor's decisions. Suppose the network is a tree, and that  $k = 1$ . What will the final result (equilibrium) be? Why?
- C. Consider the following computational problem: given as input such a network and the value of  $k$ , find the smallest set of vertices whose subsidization causes "tipping" to full investment – that is, the smallest set  $S$  of players such that if we somehow force those players to invest, all the remaining players will have the financial incentive to invest. For the network given below, find this smallest set  $S$  when  $k = 2$ .
- [NETWORK NEEDED!]
- D. When  $k = 0$ , the problem of finding the minimum number who could be subsidized/forced to invest is equivalent to one of the standard optimization problems on graphs studied in class. Name this problem, and explain

briefly why they are equivalent.

6. Consider economic networks like those we studied in class. Each vertex is either a buyer or a seller. Buyers begin with \$1 and sellers with 1 unit of wheat. Buyers have utility only for wheat, and thus want to exchange their cash for wheat; sellers have the opposite incentive. All edges in the network connect a buyer to a seller, so the network is bipartite.

At equilibrium in such networks, every seller sells exactly one unit of wheat (and thus has neither excess supply or demand); every buyer spends exactly \$1 (and thus neither exceeds nor underutilizes their budget); and every buyer purchases wheat only from sellers offering the lowest price available to that buyer. Note that we are *not* allowing price discrimination in this definition; **a seller must sell only at a single price**, not different prices to different buyers.

- A. Give the smallest connected network possible in which at equilibrium, there are at least two sellers charging different prices for wheat. Draw the network clearly, indicating buyers with a B and sellers with an S. Annotate each seller with the price they charge at equilibrium.
- B. Suppose that such a buyer-seller network has the property that every vertex has the exact same number of neighbors. What is the ratio of buyers to sellers?
- C. Is the scenario mentioned in b sufficient to guarantee that all sellers will charge the same price at equilibrium? If you answer yes, briefly argue why this is so, and say what the common equilibrium price is. If you answer no, provide a counterexample.

D. True or false: In a buyer-seller network, the seller with the highest degree will always be the one who charges the highest price at equilibrium.

7. Draw one line from each item on the left to each item in the middle, and draw one line from each item in the middle to each item on the right, to connect the concepts that correspond.

$(E,V)$  such that  $V$  can be split into subsets  $V_1$  and  $V_2$  and for each  $\{a,b\}$  in  $E$ ,  $a$  in  $V_1$  and  $b$  in  $V_2$

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Bipartite graph

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Describes a networked economy of buyers and sellers, each of whom only can trade with a certain number of those in the other class

$(E,V)$  such that for each  $v$  in  $V$ ,  $v=a$  or  $v=b$  for some  $\{a,b\}$  in  $E$

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Connected graph

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A Friendster-like network in which people are not added until the accept friendship, people cannot unfriend one another, and people cannot close their accounts or otherwise leave

$(E,V)$  such that for each  $v$  in  $V$ ,  $v$  appears  $\text{choose}(N,2)$  times in the elements of  $E$

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Complete graph

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Describes a Web site where each page is only one click away from each other page

X and Y axes are scaled logarithmically

/

Log-log plot

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Provides a way to detect heavy-tailed distributions in the underlying data

$U_1 = P_1 - \alpha_1(\max(P_2 - P_1, 0)) - \beta_1(\max(P_1 - P_2, 0))$

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Inequality aversion

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Models the guilt and envy of participants, can explain some "irrational" economic behavior

n by n grid of vertices, all connected within distance p, q additional connections with probability that goes as  $1/d^r$

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Kleinberg's model

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Explains one way that it may be possible to find short paths using only local navigation

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8. The following question refers to Malcolm Gladwell's book *The Tipping Point* and his discussion of how the structure of certain networks matters.

- A. [Three words.] Write down the three types of influential people who are described in "The Law of the Few."
- B. [One word.] If you were given just an undirected, unweighted graph of an acquaintance network, which of these three types of people would be easiest to identify?
- C. [Two short phrases.] What are *two* features you might look at, or look for, to determine which people are in this category?
- D. [Four words.] Consider making each undirected link into a pair of directed links, and then allowing these directed links to have weights. So, for instance, person A might have a link with weight 10 to person B, while person B might have a link leading back to A that only has weight 10. There are two categories of people left. First, write down the two remaining categories. For each one, imagine you are assigning a meaning to the weights of the graph. What should these weights should represent, in each case, if we want to capture how the influence of these types of people

works? One word is sufficient for the meaning of the weights in each case.

9.

[One sentence.] Define "clustering." Be precise enough so that someone who knows the basics of graph theory could tell, based on your definition, which of two networks is more highly clustered.

[One sentence.] Define "small diameter." Be precise enough so that someone who knows the basics of graph theory could tell, based on your definition, which of two networks has smaller diameter.