FINAL EXAMINATION
Networked Life (NETS 112)
December 18, 2014
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: ____

Penn ID: ____

Problem 1: _________ **/10**

Problem 2: _________ **/10**

Problem 3: _________ **/12**

Problem 4: _________ **/12**

Problem 5: _________ **/10**

Problem 6: _________ **/10**

Problem 7: _________ **/12**

Problem 8: _________ **/12**

Problem 9: _________ **/12**

Total: ______________ **/100**

Problem 1 (10 points) Indicate whether the following statements are *True* or *False*.

(a) In the bipartite networked trading model, making the network larger by adding more vertices and edges will always reduce wealth inequality.

(b) Every game with a finite number of players and actions has a mixed strategy Nash equilibrium which assigns non-zero probability to all the actions of all the players.

(c) In the Dodd's et al. paper "An Experimental Study of Search in Global Social Networks", the most frequent reason cited for sending messages to people later in completed chains was similarity in occupation to the target.

(d) The "medals" awarded in our own graph coloring experimental assignment were spread fairly evenly throughout the class.

(e) In the laboratory experiments on biased voting in networks, allowing the players to build the network themselves dramatically degraded collective performance.

(f) It is possible to create a network with 1 billion vertices in which every vertex has degree at most 3 and every pair of vertices has distance at most 3.

(g) The Ultimatum Game demonstrates what behavioral economists refer to as Inequality Amplification.

(h) For the majority of graphs in our experimental graph clustering assignment, the maximum score achieved was above 0.2.

(i) The geographic distances covered by successive router hops keeps increasing as a packet travels from its source to its destination.

(j) Networks in which there is a strong community structure, in the sense of our online clustering assignment, should generally also tend to have high clustering coefficient.

Problem 2 (10 points) Consider the recent online experiments in graph clustering or finding communities. You will have noticed that for some graphs, a much higher score was possible than for others. Clearly describe a probabilistic (randomized) model for generating networks that has a small number of parameters or "knobs", and which is capable of generating networks in which the maximum score possible is essentially any desired value, depending on how the parameters are chosen.

Problem 3 (12 points) Consider a house with N housemates and a shared Wi-Fi network. Each housemate would like to download their favorite content over the network. If housemate i attempts to download b_i bits of content during a specified time period, then the common speed or rate r at which downloads occur for everyone is $r = 1/(b_1 + b_2 + \ldots + b_N)$, and the payoff or utility to housemate i is then $r \times b_i$. There is no limit on how large b_i may be.

(a) (3 points) Clearly describe the equilibrium of this game. Explain or justify your answer.

(b) (3 points) Clearly describe the maximum social welfare solution – that is, ignoring equilibrium considerations, the choices of actions for the housemates that would maximize their total utility.

(c) (3 points) Based on your answers to (a) and (b), what is the Price of Anarchy for this game?

(d) (3 points) Describe a simple economic or technological mechanism which could cause the housemates to choose the maximum social welfare solution over the equilibrium solution.

Problem 4 (12 points). Consider networks in which there are two types of individuals/vertices. Red vertices are happy if at least half of their neighbors are also Red, and otherwise are unhappy. Blue vertices are happy if at least half of their neighbors are also Blue, and otherwise are unhappy.

(a) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is maximized.

(b) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and the total happiness is minimized.

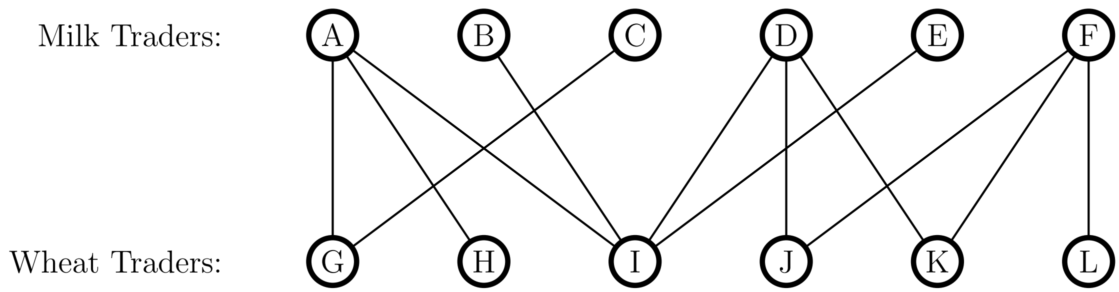
(c) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and only the Red vertices are happy.

(d) (3 points) Draw a connected network in which there are 5 Red vertices and 5 Blue vertices, and there are exactly 3 happy Red vertices, and exactly 3 happy Blue vertices.

Problem 5 (10 points) Consider the online experimental assignments on graph coloring, competitive contagion, and network clustering. Each of these assignments had very particular instructions and scoring/grading rules. Carefully compare and contrast these instructions and rules across the three assignments, and discuss what types of behavior or outcomes they were designed to incentivize. Where appropriate, you may want to discuss notions of equilibrium and competition between subjects. Your answer should demonstrate both knowledge of the rules themselves, as well as similarities and distinctions between the different assignments.

Problem 6 (10 points) This problem considers the bipartite networked trading model.

(a) (5 points) What are the equilibrium prices and trades for the network shown below?



(b) (5 points) What is the fewest number of edges you need to add in order for the equilibrium in this network to have no variation in wealth? Clearly list or indicate the added edges.

Problem 7 (12 points) Imagine a driving app that allows a user to specify their desired origin and destination, and then suggests a route to the user in a way that minimizes the collective driving time for all uses. Is it necessarily in a particular driver's best interests to:

(a) (4 points) Use the app at all? Justify your answer.

(b) (4 points) Truthfully report their desired origin and destination? Justify your answer.

(c) (4 points) Follow the route recommended by the app? Justify your answer.

Problem 8 (12 points) Consider the following two-player game. There are two separate islands – Island 1, whose size or area is S_1, and Island 2, whose size is S_2. The two players – call them Red and Blue – can invade only one island, and must decide which one to invade. If one player invades Island i, and the other does not, the payoff to the invader is S_i. If both players choose to invade Island i, they split the territory and each receive payoff $S_i/2$. For each setting of S_1 and S_2 below, carefully describe the Nash equilibria of this game, and the payoffs to the players at equilibrium.

(a) (4 points) $S_1 = 15, S_2 = 15$

(b) (4 points) $S_1 = 21, S_2 = 9$

(c) (4 points) $S_1 = 19, S_2 = 11$

Problem 9 (12 points) Consider our online competitive contagion assignment. Suppose that for some graph G, the empirical distribution of seed pairs chosen by the participants is P. We say that P is an equilibrium if every seed pair (i, j) appearing in P (i.e. with non-zero probability) receives the same average payoff (call it x) against P, and there is no seed pair (i, j) not appearing in P that receives a payoff greater than x against P.

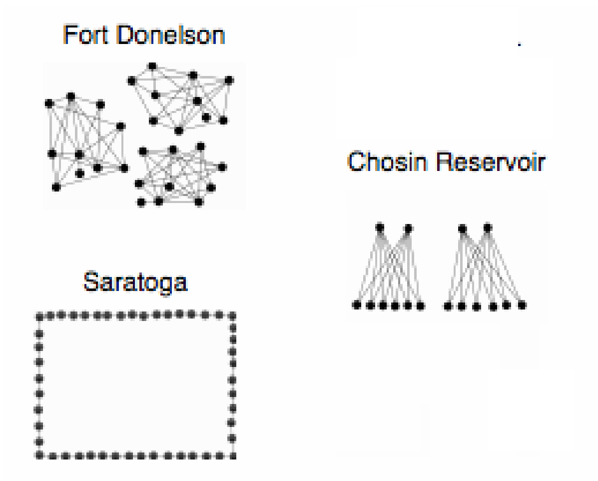


Figure 1: Competitive Contagion Networks

Consider the networks in Figure 1. For each property, mention all the networks that satisfy that property.

(a) (4 points) There is more than one equilibrium distribution.

(b) (4 points) There is an equilibrium distribution where more than one player would play the same seed pair.

(c) (4 points) Any equilibrium distribution requires all the players play the same seed pair.