Competitive Contagion Scoring Review

• Let $P$ be the population distribution of seed choices on graph $G$
• For every seed set $s$ that appears with non-zero probability in $P$, we will compute its \textit{expected payoff with respect to $P$}: 
  – average of $\text{pay}(s,s')$ over many trials and many draws of $s'$ from $P$
  – enough draws/trials to distinguish/rank expected payoffs accurately
• We will then rank the $s$ that appear in $P$ by their expected payoffs
• If you played $s$ on $G$, you will receive a number of points equal to the \textit{number of other players} you \textit{strictly beat} in expected payoff
• Example: Suppose $s_1$, $s_2$ and $s_3$ appear in $P$, and have expected payoffs and population counts as follows:
  – $s_1$: payoff 0.57, count 11; $s_2$: payoff 0.48, count 71; $s_3$: payoff 0.31, count 18
  – if you play $s_1$, your score is $71+18=89$; if $s_2$, your score is 18; if $s_3$, your score is 0
• If everyone plays the same thing, nobody receives any points
• You must submit seeds for \textit{all} graphs in order to receive any credit
• Your overall score/grade for the assignment is the sum of your scores over all graphs, which will then be curved
• In general, there is no right/best choice for seeds: depends on $P$!
Questions Worth Pondering

• What does it mean for the population distribution $P$ to be an equilibrium?
• If $P$ is an equilibrium what can we say about different players’ payoffs?
• If $P$ is an equilibrium and $G$ is connected, what can we say about payoffs?
• What if $G$ is not connected?
Experimental Agenda

• Human-subject experiments at the intersection of CS, economics, sociology, “network science”
• Subjects simultaneously participate in groups of ~ 36 people
• Subjects sit at networked workstations
• Each subject controls some simple property of a single vertex in some underlying network
• Subjects have only local views of the activity: state of their own and neighboring vertices
• Subjects have (real) financial incentive to solve their “piece” of a collective (global) task
• Simple example: graph coloring (social differentiation)
  – choose a color for your vertex from fixed set
  – paid iff your color differs from all neighbors when time expires
  – max welfare solutions = proper colorings
• Across many experiments, have deliberately varied network structure and task/game
  – networks: inspired by models from network science (small worlds, preferential attachment, etc.)
  – tasks: chosen for diversity (cooperative vs. competitive) and (centralized) computational difficulty
• Goals:
  – structure/tasks → performance/behavior
  – individual & collective modeling → prediction
  – computational and equilibrium theories
Experiments to Date

- **Graph Coloring**
  - **player controls**: color of vertex; number of choices = chromatic number
  - **payoffs**: $2 if different color from all neighbors, else 0
  - **max welfare states**: optimal colorings
  - **centralized computation**: hard even if approximations are allowed

- **Consensus**
  - **player controls**: color of vertex from 9 choices
  - **payoffs**: $2 if same color as all neighbors, else 0
  - **max welfare states**: global consensus of color
  - **centralized computation**: trivial

- **Independent Set**
  - **player controls**: decision to be a “King” or a “Pawn”; variant with King side payments allowed
  - **payoffs**: $1/minute for Solo King; $0.50/minute for Pawn; 0 for Conflicted King; continuous accumulation
  - **max welfare states**: maximum independent sets
  - **centralized computation**: hard even if approximations are allowed

- **Exchange Economy**
  - **player controls**: limit orders offering to exchange goods
  - **payoffs**: proportional to the amount of the other good obtained
  - **max welfare states**: market clearing equilibrium
  - **centralized computation**: at the limit of tractability (LP used as a subroutine)

- **Biased Voting**
  - **player controls**: choice of one of two colors
  - **payoffs**: only under global agreement; different players prefer different colors
  - **max welfare states**: all red and all blue
  - **centralized computation**: trivial

- **Networked Bargaining**
  - **player controls**: offers on each edge to split a cash amount; may have hidden deal limits and “transaction costs”
  - **payoffs**: on each edge, a bargaining game --- payoffs only if agreement
  - **max welfare states**: all deals/edges closed
  - **centralized computation**: nontrivial, possibly difficult

- **Voting with Network Formation**
  - **player controls**: edge purchases and choice of one of two colors
  - **payoffs**: only under global agreement; different players prefer different colors
  - **max welfare states**: ???
  - **centralized computation**: ???
Coloring and Consensus
“first neighborhood” view
Small Worlds Family

Simple Cycle

5-Chord Cycle

20-Chord Cycle

Leader Cycle

Preferential Attachment, $\nu = 2$

Preferential Attachment, $\nu = 3$
Art by Consensus
Sample Findings

- Generally strong collective performance
  - nearly all problems globally solved in a couple minutes or less
- Systematic effects of structure on performance and behavior:
  - rewiring harms coloring performance in "clique chain" family
  - rewiring helps consensus performance in clique chain family
- Preferential attachment much harder than small worlds for coloring
  - natural heuristics can give reverse order of difficulty
- Providing more global views of activity:
  - helps coloring performance in small world family
  - harms coloring performance in preferential attachment
- Coloring problems solved more rapidly than consensus
  - easier to get people to disagree than agree

![Graph showing running time vs. rewiring probability]

![Bar chart showing average experiment duration for information views]
Biased Voting in Networks
Biased Voting in Networks

- Cosmetically similar to consensus, with a crucial strategic difference
- Deliberately introduce a tension between:
  - individual preferences
  - desire for collective unity
- Only two color choices; challenge comes from competing incentives
- If everyone converges to same color, everyone gets some payoff
- But different players have different preferences
  - each player has payoffs for their preferred and non-preferred color
  - e.g. $1.50 red/$0.50 blue vs. $0.50 red/$1.50 blue
  - can have symmetric and asymmetric payoffs
- High-level experimental design:
  - choice of network structures
  - arrangement of types (red/blue prefs) & strengths of incentives
  - most interesting to coordinate network structure and types
Democratic Primary Games

Zak Xavier

Game progress: 67%
Game status: Voter Game in progress
Elapsed time:
If unanimity is reached, your payoff will be
$0.75 for red, $1.25 for blue

Your color: blue red
Minority Power: Preferential Attachment
Summary of Findings

- 55/81 experiments reached global consensus in 1 minute allowed
  - mean of successful ~ 44s

- Effects of network structure:
  - Cohesion harder than Minority Power: 31/54 Cohesion, 24/27 Minority Power
  - all 24 successful Minority Powers converge to minority preference!
  - Cohesion P.A. (20/27) easier than Cohesion E-R
  - overall, P.A. easier than E-R (contrast w/coloring)
  - within Cohesion, increased inter-group communication helps
    - some notable exceptions...

- Effects of incentives:
  - asymmetric beats weak symmetric beats strong symmetric
  - the value of “extremists”
Effects of “Personality”

- **Wealth**: $p = 0.54$
- **Early Color Changes**: $p = 1.51 \times 10^{-10}$
- **Stubbornness**: $p = 1.34 \times 10^{-46}$

fraction < value
Behavioral Modeling

model: play color c with probability \( \sim \) payoff(c) \( \times \) fraction in neighborhood playing c
Lessons Learned, 2005-2011

- At least for $n=36$, human subjects remarkably good
  - diverse set of collective tasks
  - diverse set of network topologies
  - efficiency $\sim 90\%$ across all tasks/topologies
- Network structure matters; interaction with task
  - contrast with emphasis on topology alone
- Importance of subject variability and style/personality
- Most recently: endogenized creation of the network
  - network formation games
  - challenging computationally (best response) and analytically
Edge Purchases: Strategic Tensions

• Buy edges or not?
• For information or influence?
• Early in the game or late?
• To high degree or low degree players?
• Nearby or far away?
Experimental Design

- **Session A**: 99 experiments
  - 63 “unseeded” with varying payoffs, imbalances, asymmetries
  - 36 seeded with Minority Power settings
- **Session B**: 72 experiments
  - mixture of unseeded and variety of seeded (cliques, torus)
- **A**: 47/99 solved (47%): 25/63 unseeded, MP 22/36
- **B**: 27/72 solved (38%)
- **Session C**: 72 experiments
  - final networks from “hard” settings in Session A
  - permitted 0 or 1 edge purchases per player
  - started with both initial and final incentives from Session A
- **C**: 25/72 (35%); All: 99/243 (41%)
- Subjects seem to build difficult networks!