

## What's Crypto Good for?

- Data Confidentiality
  - o Keep data and communication secret
  - o Privacy of personal financial/health records, etc.
  - o Military and commercial relevance
  - o Encryption / decryption
- Data Integrity and Authenticity
  - o Protect reliability of data against tampering
  - Can we be sure of the source and content of information?
  - o Hashes
  - o Also related: repudiation









- A method of encryption and decryption is called a cipher.
- Generally there are two related functions: one for encryption and other for decryption.
- Some cryptographic methods rely on the secrecy of the algorithms.
- Such methods are mostly of historical interest these days.
- All modern algorithms use a key to control encryption and decryption.
- Encryption key may be different from decryption key.



































# **Simplified Math Facts**

- Public key cryptography is based on the mathematical concept of multiplicative inverse.
- Multiplicative inverses are two numbers that when multiplied equals one (e.g., 7 x 1/7 = 1)
- In modular mathematics, two whole numbers are inverses if they multiplies to 1 (e.g., 3 x 7 mod 10 = 1)
- Use modular inverse pairs to create public and private keys.
- Example
  - o Message is 4
  - To scramble it, use 4 X 3 mod 10 = 2
  - To recover it, use 2 x 7 mod 10 = 4
- The security of public key systems depends on the difficulty of calculating inverses.

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- Pick large random primes p,q.
- Let p\*q = n and φ=(p-1)(q-1).
- Choose a random number e such that: 1<e<φ and gcd(e, φ)=1. (relative primes)
- Calculate the unique number d such that 1<d< $\phi$  and d\*e = 1 (mod  $\phi$  ). (d is inverse of e)

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- The public key is {e,n} and the private key is {d,n}.
- The factors p and q may be kept private or destroyed.

#### Why Does it Work?

- It is secure because it is difficult to find φ or d using only e and n. Finding d is equivalent in difficulty to factoring n as p\*q.
- It is feasible to encrypt and decrypt because:
  - o It is possible to find large primes.
  - o It is possible to find relative primes and their inverses.
  - o Modular exponentiation is feasible.





#### More on RSA

- Introduced by Rivest, Shamir, and Adleman in 1979
- Foundations in number theory and computational difficulty of factoring
- Not mathematically proven to be unbreakable, but has withstood attacks and analysis
  - Ideally, we would like to prove a theorem saying "if intruder does not know the key, then it cannot construct plaintext from the ciphertext by executing a polynomial-time algorithm"
- Public and private keys are derived from secretly chosen large prime numbers
- Plaintext is viewed as a large binary number and encryption is exponentiation in modulo arithmetic
- Intruder will have to factor large numbers (and there are no known polynomial-time algorithms for this)

o 2002's major result: polynomial-time test to check if a number is prime CIS 505. Spring 2007 Crypto 33

#### More on RSA

- RSA has been implemented in hardware.
- In hardware, RSA is about 1000 times slower than DES.
- In software, it is about 100 times slower.
- These numbers may change, but RSA can never approach the speed of symmetric algorithms.
- RSA encryption goes faster if e is chosen appropriately.
- Security of RSA depends on the problem of factoring large numbers. Though it has never been proven that one needs to factor n to calculate m from c and e!
- Most public key systems use at least 1,024-bit key.
- The RSA algorithm is patented in the US, but not in any other country.
- The US patent expires on September 20, 2000!

## 3. Digital Signatures

- A digital signature is a protocol the produces the same effect as a real signature.
  - o It is a mark that only sender can make
  - o Other people can easily recognize it as belonging to the sender.
- Digital signatures must be:
  - Unforgeable: If P signs message M with signature S(P,M), it is impossible for someone else to produce the pair [M, S(P,M)].
  - o Authentic: R receiving the pair [M, S(P,M)] can check that the signature is really from P.

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How can Alice sign a digital document ?

- Let S(A,M) be the message M tagged with Alice's signature
- No forgery possible: If Alice signs M then nobody else can generate S(A,M)
- Authenticity check: If you get the message S(A,M) you should be able to verify that this is really created by Alice
- No alteration: Once Alice sends S(A,M), nobody (including Alice) can tamper this message
- No reuse: Alice cannot duplicate S(A,M) at a later time

## **Digital Signatures: Symmetric Key**

- Under private key encryption system, the secrecy of the key guarantees the authenticity of the message as well as its secrecy.
- It does not prevent forgery, however.
- There is no protection against repudiation (denial of sending a message).
- An arbitrator (a trusted third party) is needed to prevent forgery.

#### **Digital Signatures - Public Key**

- Public key encryption systems are ideally suited to digital signatures.
- Reverse of public key encryption/decryption.
- To sign a message, use your private key to encrypt the message.
- Send this signature together with the message.
- The receiver can verify the signature using your public key.
- Only you could have signed the message since your private key belongs to you and only you.
- The receiver saves the message and signature and anyone else can verify should you claim forgery.









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- How to assure integrity
  - o Alice makes a message digest from a plaintext message.
  - o Alice signs the message digest and sends the signed digest and plaintext to Bob
  - o Bob re-computes the message digest from the plaintext.
  - o Bob decrypts the signed digest with Alice's public key.
  - Bob verifies that message is authentic if the message digest he computed is identical to the decrypted digest signed by Alice.

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# 6. Public Key Distribution

- Every user has his/her own public key and private key.
- Public keys are all published in a database.
- Sender and receiver agree on a cryptosystem.
- Sender gets receiver's public key from the db.
- · Sender encrypts the message and sends it.
- Receiver decrypts it using his/her private key.
- What can be a problem?

















## **PKI: Risks**

- Certificates only as trustworthy as their CAs
  o Root CA is a single point of failure
- PKI only as secure as private signing keys
- DNS not necessarily unique
- Server certificates authenticate DNS addresses, not site contents
- CA may not be authority on certificate contents
  o i.e., DNS name in server certificates

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Lecture materials are drawn from many sources,

**Ack and References** 

- including Matt Blaze, Micah Sherr
- <u>Cryptography: Theory and Practice</u>. Douglas R. Stinson.
- Applied Cryptography. Bruce Schneier
- <u>Handbook of Applied Cryptography</u>. Alfred J. Menezes, Paul C. van Oorschot and Scott A. Vanstone. Available free online at <u>ttp://www.cacr.math.uwaterloo.ca/hac/</u>