Compositional Real-Time Scheduling Framework

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Outline

• Compositional scheduling framework
  - Scheduling component model
  - Periodic resource model
    • Schedulability analysis
    • Utilization bound
    • Component timing abstraction
Traditional Scheduling Framework

- Single real-time task in a single application
Hierarchical Scheduling Framework (HFS)

- Multiple real-time tasks with a scheduler in a single application, forming a hierarchy of scheduling
Compositional Scheduling Framework

Digital Controller
T₁(25,5)

Multimedia
T₂(33,10)

Java Virtual Machine

OS Scheduler

CPU

Multimedia Controller

VM Scheduler

J₁(50,3)
J₂(75,5)
VM Scheduler’s Viewpoint

Real-Time Guarantee on CPU Supply

Java Virtual Machine

VM Scheduler

CPU Share

J₁(50,3)  J₂(75,5)
Problems & Approach I

• Resource supply modeling
  - Characterize temporal property of resource allocations
    • we propose a periodic resource model
  - Analyze schedulability with a new resource model
OS Scheduler’s Viewpoint

- Digital Controller
  - T₁(25,5)

- Multimedia
  - T₂(33,10)

- Java Virtual Machine

- Real-Time Task
  - Real-Time Demand

OS Scheduler

CPU
Problems & Approach II

- **Real-time demand composition**
  - Combine multiple real-time requirements into a single real-time requirement

\[ T_1(p_1, e_1) \quad T_2(p_2, e_2) = T(p, e) \]

EDF / RM
Compositional Real-time Scheduling Framework

• **Goal**
  - to support *compositionality* for *timeliness* aspect
  - to achieve system-level schedulability analysis using the results of component-level schedulability analysis

• Scheduling component modeling
Scheduling Component Modeling

- Scheduling
  - assigns resources to workloads by scheduling algorithms

- Scheduling Component Model: $C(W,R,A)$
  - $W$: workload model
  - $R$: resource model
  - $A$: scheduling algorithm
Resource Modeling

- **Dedicated resource**: always available at full capacity

- **Shared resource**: not a dedicated resource
  - **Time-sharing**: available at some times
  - **Non-time-sharing**: available at fractional capacity
• **Time-sharing resources**
  - Bounded-delay resource model [Mok et al., ’01] characterizes a time-sharing resource w.r.t. a non-time-sharing resource
  - Periodic resource model $\Gamma(\Pi, \Theta)$ [Shin & Lee, RTSS ’03] characterizes periodic resource allocations
Schedulability Analysis

- A workload set is **schedulable** under a scheduling algorithm with available resources if its real-time requirements are satisfiable
- **Schedulability analysis** determines whether

  ![Diagram]

  resource demand, which a workload set requires under a scheduling algorithm

  ≤

  resource supply, which available resources provide
Resource Demand Bound

- Resource demand bound during an interval of length $t$
  - $dbf(W,A,t)$ computes the maximum possible resource demand that $W$ requires under algorithm $A$ during a time interval of length $t$

- Periodic task model $T(p,e)$ [Liu & Layland, ’73]
  - i.e., $T(3,2)$
Demand Bound Function - EDF

- For a periodic workload set $W = \{T_i(p_i,e_i)\}$,
  - $\text{dbf} (W,A,t)$ for EDF algorithm [Baruah et al.,'90]

$$
\text{dbf} (W,\text{EDF},t) = \sum_{T_i \in W} \left\lfloor \frac{t}{p_i} \right\rfloor e_i
$$

- Example: $W = \{T_1(3,2), T_2(4,1)\}$
Resource Supply Bound

• Resource supply during an interval of length $t$
  – $sbf_R(t)$: the minimum possible resource supply by resource $R$ over all intervals of length $t$

• For a single periodic resource model, i.e., $\Gamma(3,2)$
  – we can identify the worst-case resource allocation
Resource Supply Bound

- supply = 3
- supply = 1
- $sbf_{R(i)} = 1$
Resource Supply Bound

- Resource supply during an interval of length $t$
  - $sbf_R(t)$: the minimum possible resource supply by resource $R$ over all intervals of length $t$

- For a single periodic resource model, i.e., $\Gamma(3,2)$
  - we can identify the worst-case resource allocation
Supply Bound Function

- Resource supply during an interval of length $t$
  - $\text{sbf}_\Gamma(t)$: the minimum possible resource supply by resource $R$ over all intervals of length $t$

- For a single periodic resource model $\Gamma(\Pi, \Theta)$

$$\text{sbf}_\Gamma(t) = \begin{cases} 
  t - (k+1)(\Pi - \Theta) & \text{if } t \in [(k+1)\Pi - 2\Theta, (k+1)\Pi - \Theta] \\
  (k-1)\Theta & \text{otherwise}
\end{cases}$$

![Graph showing supply bound function](graph.png)
Schedulability Conditions (EDF)

- A workload set $W$ is schedulable over a resource model $R$ under EDF if and only if for all interval $i$ of length $t$

\[ dbf_w(i) \leq t \]  

$\textbf{Resource demand}$ in an interval

\[ dbf_w(i) \leq sbf_R(i) - sbf_R(i) : \text{the minimum resource supply by resource } R \text{ during an interval } i \]

- $sbf_R(i)$: the **minimum resource supply** by resource $R$ during an interval $i$

- $dbf_w(i)$: the **resource demand** of workload $W$ during an interval $i$

[BHR90]
Schedulability Condition - EDF

- A periodic workload set $W$ is schedulable under EDF over a periodic resource model $\Gamma(\Pi, \Theta)$ if and only if

$$\forall t > 0 \quad dbf(W, EDF, t) \leq sbf_\Gamma(t)$$
Schedulability Condition - RM

- A periodic workload set \( W \) is schedulable under EDF over a periodic resource model \( \Gamma(\Pi, \Theta) \) if and only if

\[
\forall t > 0 \ \forall T_i \in W \quad \text{dbf}(W, \text{RM}, t, i) \leq \text{sbfr}(t)
\]

- For a periodic workload set \( W = \{T_i(p_i, e_i)\} \),

\[
\text{dbf}(W, A, t, i) \quad \text{for RM algorithm [Lehoczky et al., '89]}
\]

\[
\text{dbf}(W, \text{RM}, t, i) = e_i + \sum_{T_k \in HP(T_i)} \left\lceil \frac{t}{p_k} \right\rceil \cdot e_k
\]
Utilization Bounds

- For a periodic workload $T(p,e)$, utilization $U_T = e/p$
- For a periodic workload set $W$, utilization $U_W$ is

\[ \sum_{i \in W} \frac{e_i}{p_i} \]

- Utilization bound (UB) of a resource model $R$
  - given a scheduling algorithm $A$ and a resource model $R$, $UB_{R,A}$ is a number s. t. a workload set $W$ is schedulable if

\[ U_W \leq UB_{R,A} \]
Utilization Bounds

• Example:
  - Consider a periodic resource $\Gamma(\Pi, \Theta)$, where $\Pi = 10$ and $\Theta = 4$, and suppose $UB_{\Gamma, EDF} = 0.4$.
  - Then, a set of periodic task $W$ is schedulable if
  \[
  U_W \leq 0.4
  \]
  - $W = \{T1(20,3), T2(50,5)\}$ s.t. $U_W = 0.25$, is schedulable

\[
\begin{array}{c}
\text{workload} \quad \text{workload} \\
\downarrow \quad \downarrow \\
\text{EDF} \\
\downarrow \\
\Gamma(\Pi=10,\Theta=4)
\end{array}
\]
Utilization Bound - EDF

• For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, where $A = EDF$, its utilization bound is

$$UB_{\Gamma, EDF}(P_{\text{min}}) = \frac{k \times U_{\Gamma}}{k + 2(1 - U_{\Gamma})}$$

• $P_{\text{min}}$ is the minimum task period (deadline) in $W$.
• $k$ represents the relationship between resource period $\Pi$ and the minimum task period $P_{\text{min}}$, $k \approx P_{\text{min}} / \Pi$
EDF Utilization Bound - Intuition

- Observation for a component \( C(W, \Gamma(\Pi, \Theta), EDF) \)
  - \( C \) is schedulable iff \( \text{dbf}(W, EDF, t) \leq \text{sbf}_\Gamma(t) \)
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), \text{EDF})$
  - $C$ is schedulable iff $\text{dbf}(W, \text{EDF}, t) \leq \text{sbf}_\Gamma(t)$
  - $\text{dbf}(W, \text{EDF}, t) \leq U_W \cdot t$
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - $C$ is schedulable iff $\text{dbf} (W, EDF, t) \leq \text{sbf}_\Gamma (t)$
  - $\text{dbf} (W, EDF, t) \leq U_W \cdot t$
  - $U_\Gamma (t-2(\Pi-\Theta)) \leq \text{sbf}_\Gamma (t)$
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - $C$ is schedulable iff $\text{dbf}(W, EDF, t) \leq sbf_{\Gamma}(t)$
  - $\text{dbf}(W, EDF, t) \leq U_W \cdot t$
  - $U_{\Gamma}(t-2(\Pi-\Theta)) \leq sbf_{\Gamma}(t)$
  - Therefore, $C$ is schedulable if $U_W \cdot t \leq U_{\Gamma}(t-2(\Pi-\Theta))$
EDF Utilization Bound - Intuition

- For a component $C(W, \Gamma(\Pi, \Theta), \text{EDF})$
  - for all $t > P_{\text{min}}$, if $U_W \cdot t \leq U_{\Gamma}(t-2(\Pi-\Theta))$
    then $C$ is schedulable.

\[
U_W \leq \frac{U_{\Gamma}(t-2(\Pi-\Theta))}{t}
\]
Utilization Bound - RM

- For a scheduling component $C(W, \Pi(\Pi, \Theta), A)$, where $A = RM$, its utilization bound is
  - [Saewong, Rajkumar, Lehoczky, Klein, ’02]
    \[
    UB_{\Gamma, RM}(n) = n \left( \frac{3 - U_{\Gamma}}{3 - 2 \times U_{\Gamma}} \right)^{\frac{1}{n}} - 1
    \]
  - We generalize this earlier result, where $k \approx P_{\text{min}}/\Pi$.
    \[
    UB_{\Gamma, RM}(n, P_{\text{min}}) = U_{\Gamma} \times n \left( \frac{2k + 2(1 - U_{\Gamma})}{k + 2(1 - U_{\Gamma})} \right)^{\frac{1}{n}} - 1
    \]
Component Abstraction

- Component timing abstraction
  - To specify the collective real-time demands of a component as a timing interface

```
Periodic (50,7)  Periodic (70,9)
         |          |
          EDF     virtual real-time task
```
Component Abstraction

- Component timing abstraction
  - To specify the collective real-time demands of a component as a timing interface

```
periodic resource \( \Gamma(\Pi,\Theta) \)
```

```
Periodic (50,7)  Periodic (70,9)
```

EDF

```
periodic interface \( \Gamma(\Pi,\Theta) \)
```
Component Abstraction (Example)

- In this example, a solution space of a periodic resource $\tau(\Pi,\Theta)$ that makes $C(W, \Gamma(\Pi, \Theta), EDF)$ schedulable is

![Diagram showing solution space under EDF](image-url)
Component Abstraction (Example)

- An approach to pick one solution out of the solution space
  - Given a range of $\Pi$, we can pick $\Gamma(\Pi, \Theta)$ such that $U_{\Pi}$ is minimized. (for example, $28 \leq \Pi \leq 46$)

![Diagram showing solution space under EDF](image)
Component Timing Abstraction

- Component timing abstraction
  - To abstract the collective real-time demands of a component as a timing interface

![Diagram showing periodic interface and timing abstraction]

Periodic (50,7) Periodic (70,9)

EDF

periodic interface
\( \Gamma(29, 9.86) \)
Compositional Real-Time Guarantees

R(?, ?)

EDF

R_1(10, 3.1)

EDF

T_{11}(25,4)

T_{12}(40,5)

R_2(10, 4.4)

RM

T_{21}(25,4)

T_{22}(40,5)
Compositional Real-Time Guarantees
Abstraction Overhead

- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, its abstraction overhead ($O_{\Gamma}$) is
  \[ \frac{U_{\Gamma}}{U_{W}} - 1 \]

\[ U_{W} = 0.27 \]

\[ U_{\Gamma} = 0.34 \]

Periodic (50,7) → EDF → Periodic (70,9) → periodic interface $\Gamma(29, 9.86)$
Abstraction Overhead Bound

- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, its abstraction overhead ($O_\Gamma$) is

\[- A = \text{EDF} \quad O_{\Gamma, \text{EDF}} \leq \frac{2 \times (1 - U_W)}{k + 2 \times U_W}\]

\[- A = \text{RM} \quad O_{\Gamma, \text{RM}} \leq \frac{1}{\log \left( \frac{2k + 2(1 - U_W)}{k + 2(1 - U_W)} \right)} - 1\]
Abstraction Overhead

• Simulation Results
  - with periodic workloads and periodic resource under EDF/RM
  - the number of tasks $n : 2, 4, 8, 16, 32, 64$
  - the workload utilization $U(W) : 0.2\sim0.7$
  - the resource period : represented by $k$
Abstraction Overhead

- \( k = 2, U(W) = 0.4 \)
Summary

• Compositional real-time scheduling framework
  - with the periodic model [Shin & Lee, RTSS ’03]
  1. resource modeling
     - utilization bounds (EDF/RM)
  2. schedulability analysis
     • exact schedulability conditions (EDF/RM)
  3. component timing abstraction and composition
     • overhead evaluation
       - upper-bounds and simulation results
Future Work

- Extending our framework for handling
  - Soft real-time workload models
  - non-periodic workload models
  - task dependency
References

• Insik Shin & Insup Lee,

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THANK YOU