Compositional Real-Time Scheduling Framework

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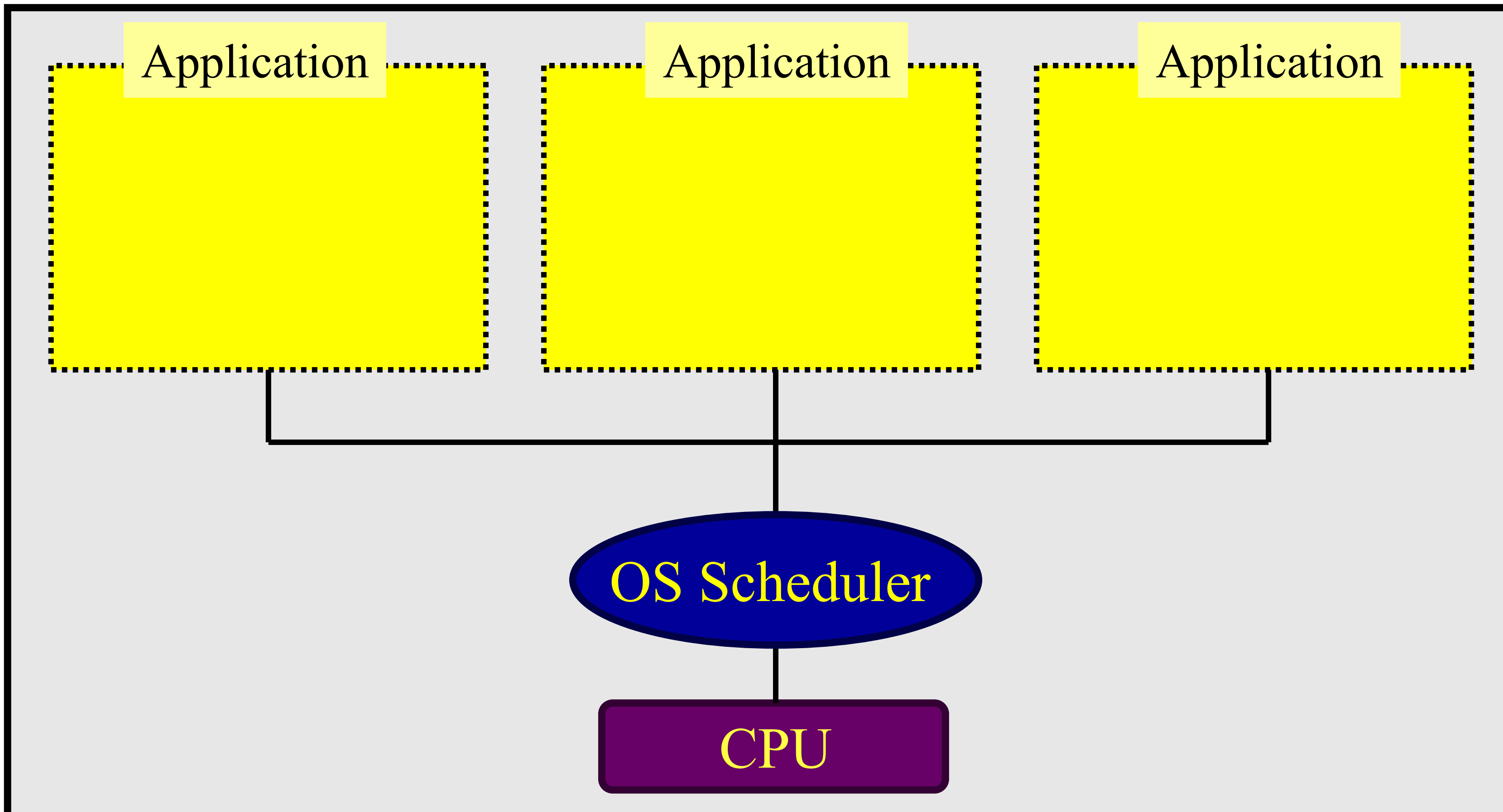
Outline

- Compositional scheduling framework
  - Scheduling component model
  - Periodic resource model
    - Schedulability analysis
    - Utilization bound
    - Component timing abstraction

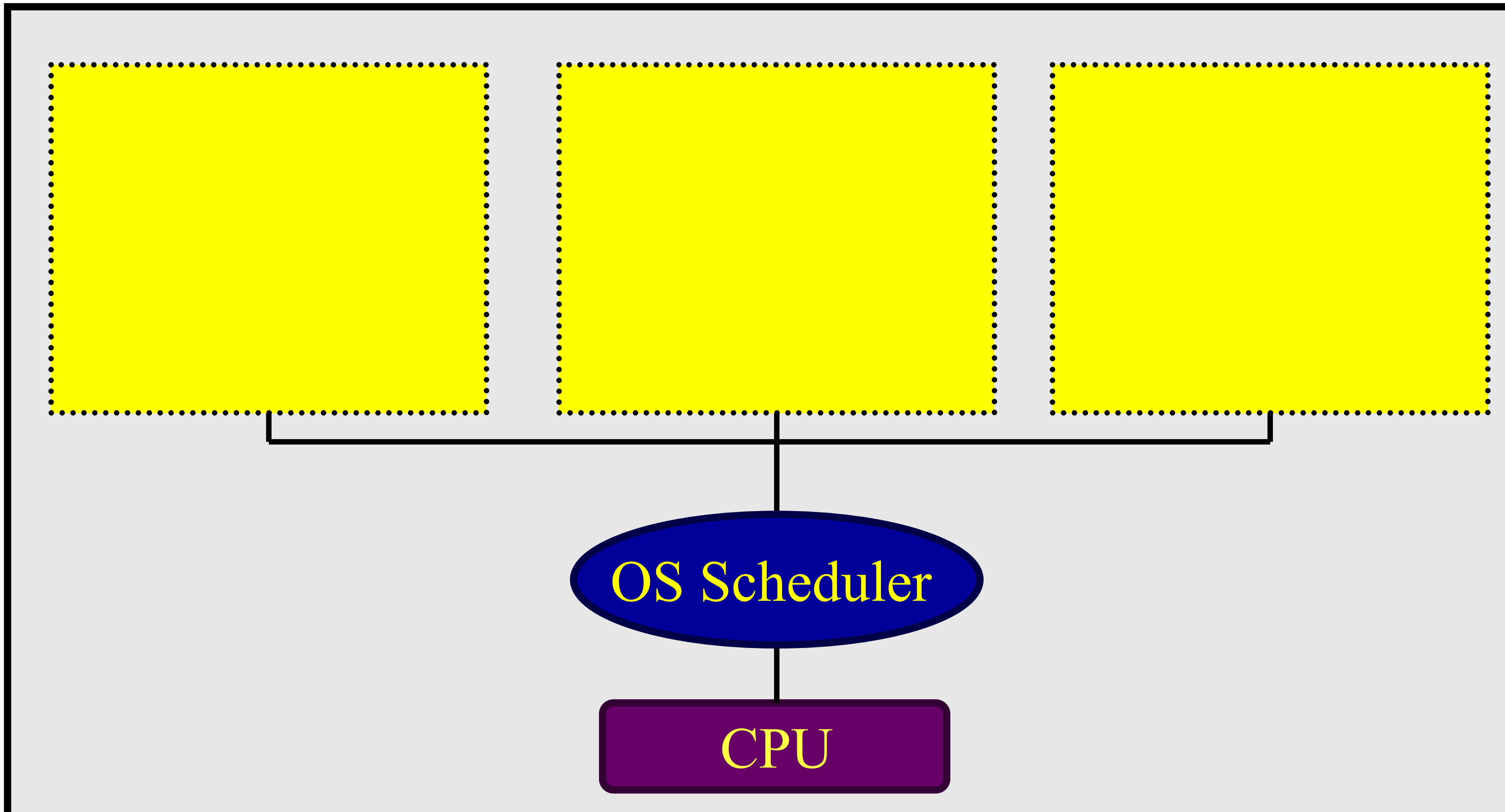
Traditional Scheduling Framework

- *Single real-time task in a single application*

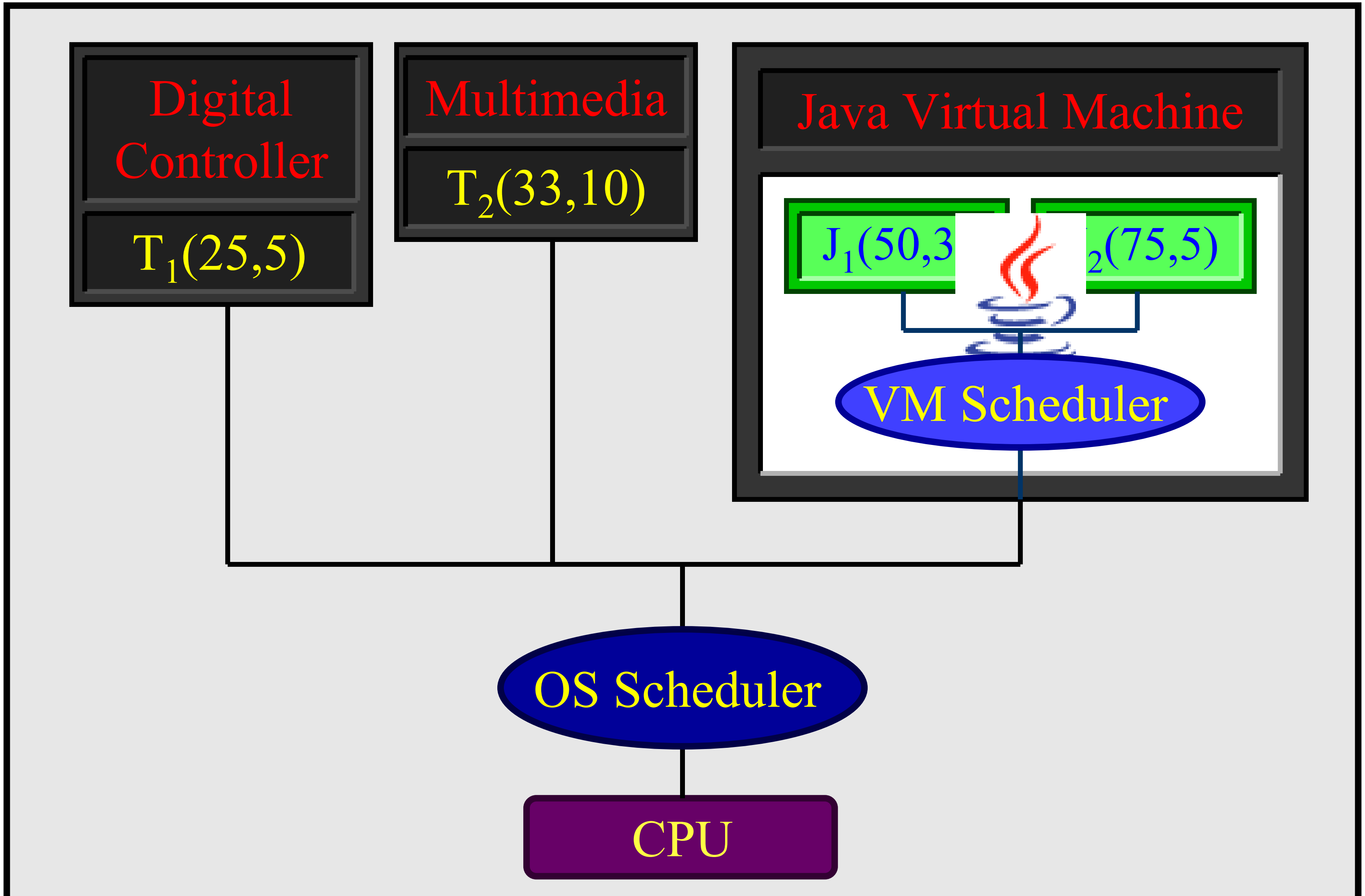


Hierarchical Scheduling Framework (HFS)

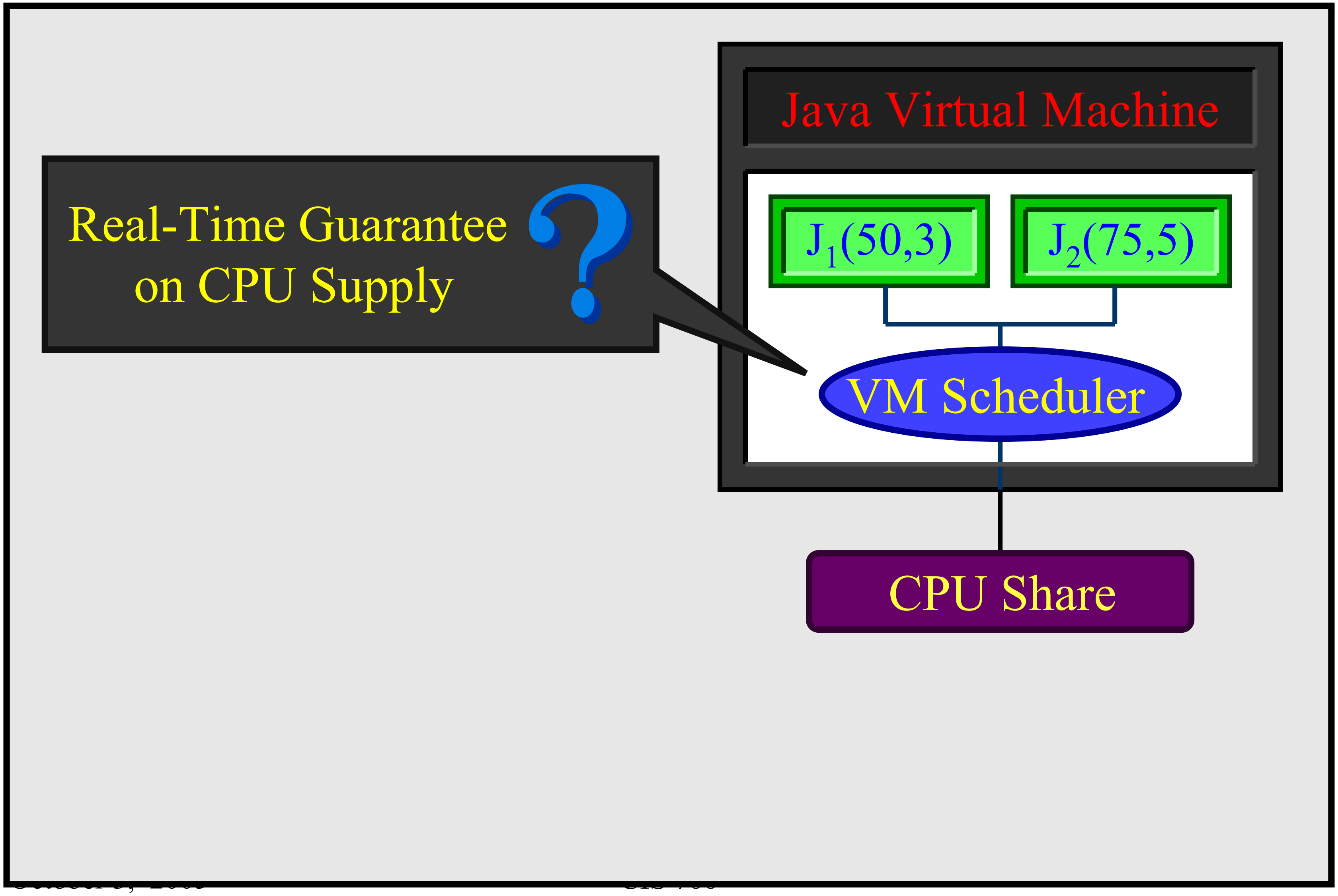
- Multiple real-time tasks with a scheduler in a single application, forming a hierarchy of scheduling



Compositional Scheduling Framework



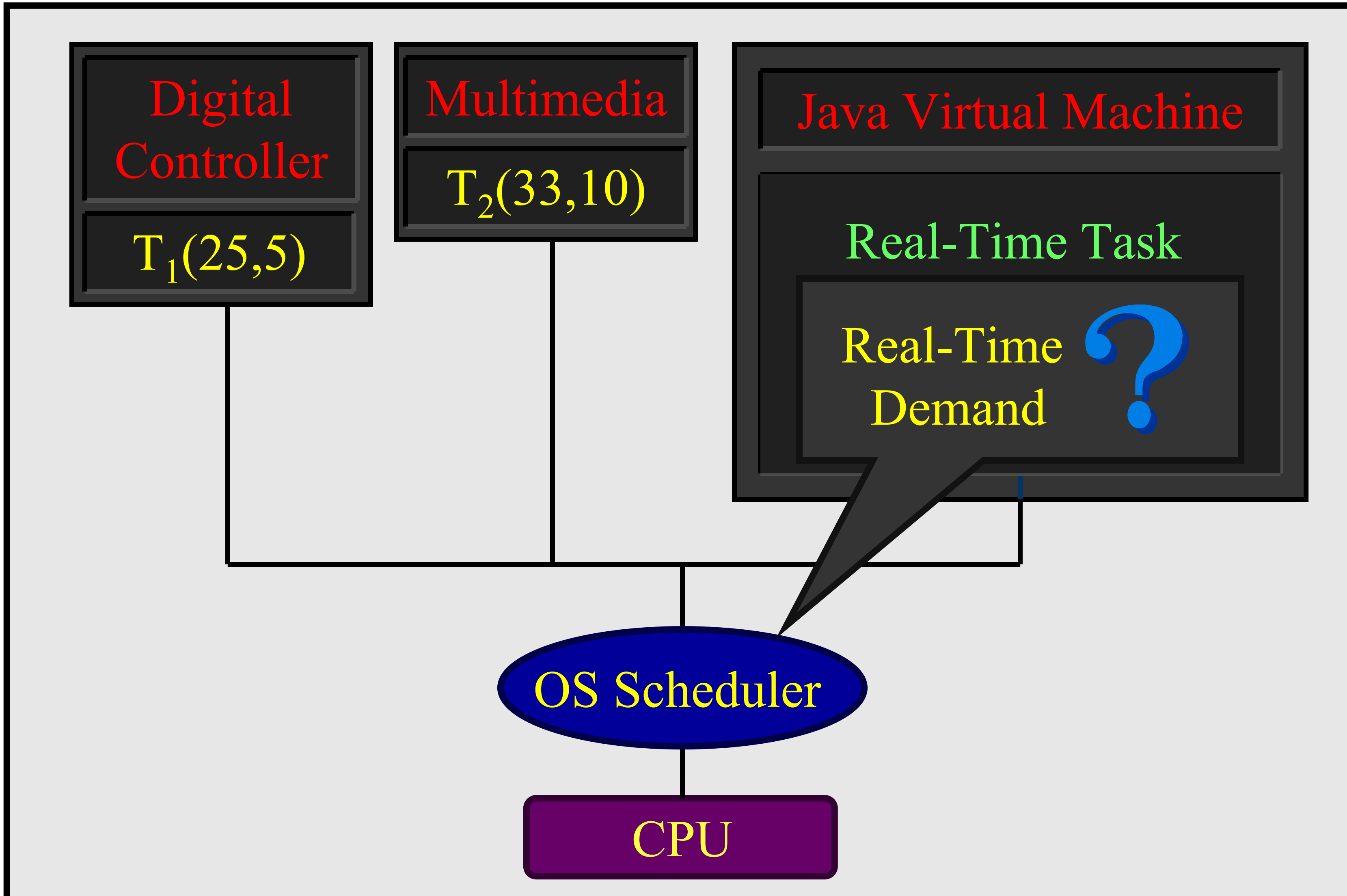
VM Scheduler’s Viewpoint



Problems & Approach I

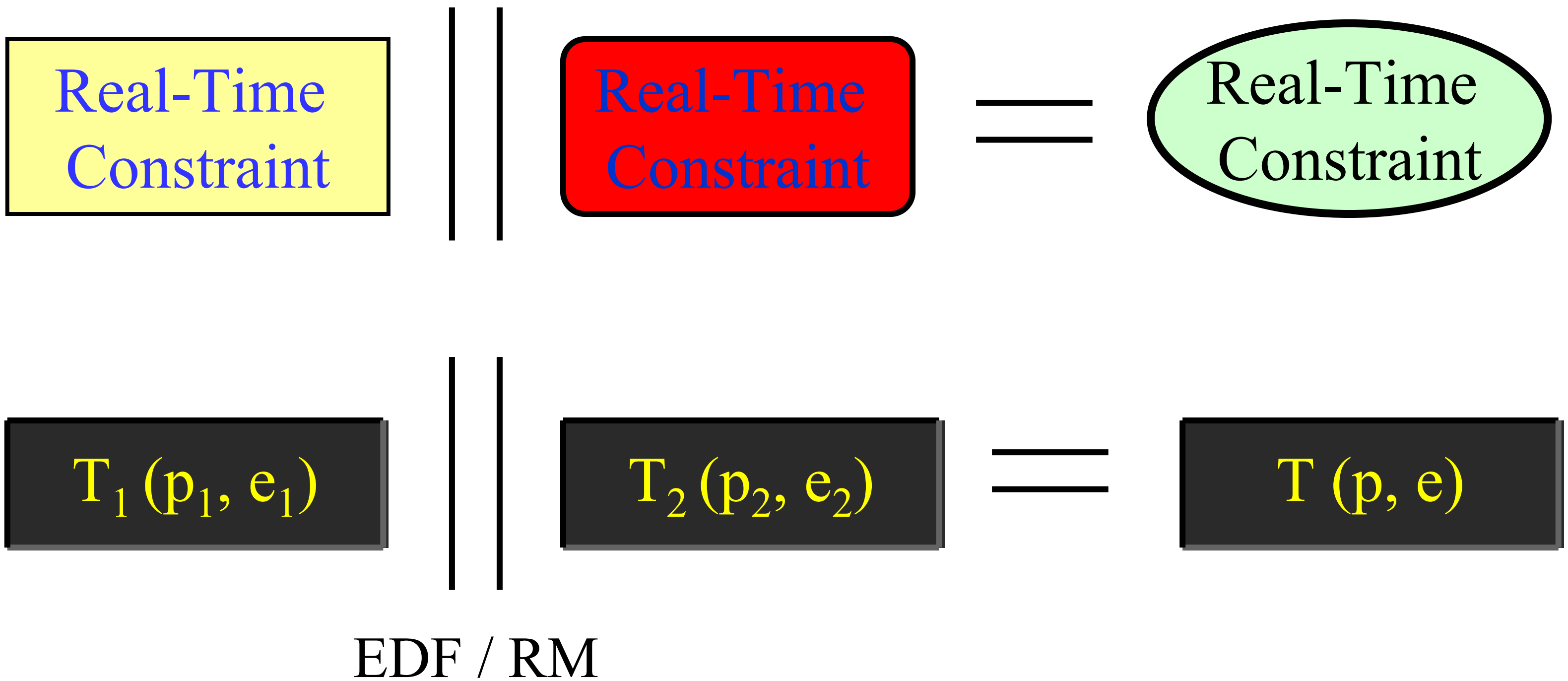
- Resource supply modeling
  - Characterize temporal property of resource allocations
    - we propose a periodic resource model
  - Analyze schedulability with a new resource model

OS Scheduler’s Viewpoint



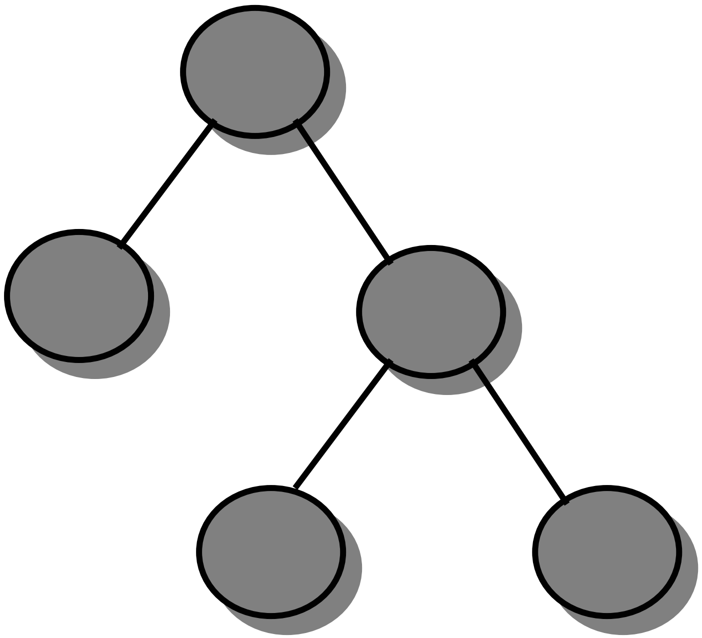
Problems & Approach II

- Real-time demand composition
  - Combine multiple real-time requirements into a single real-time requirement



Compositional Real-time Scheduling Framework

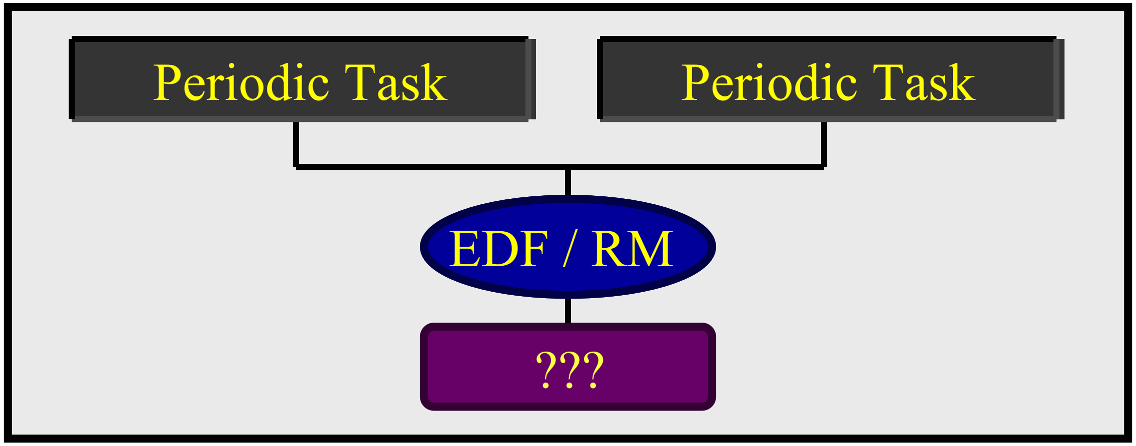
- Goal
  - to support compositionality for timeliness aspect
  - to achieve system-level schedulability analysis using the results of component-level schedulability analysis



- Scheduling component modeling

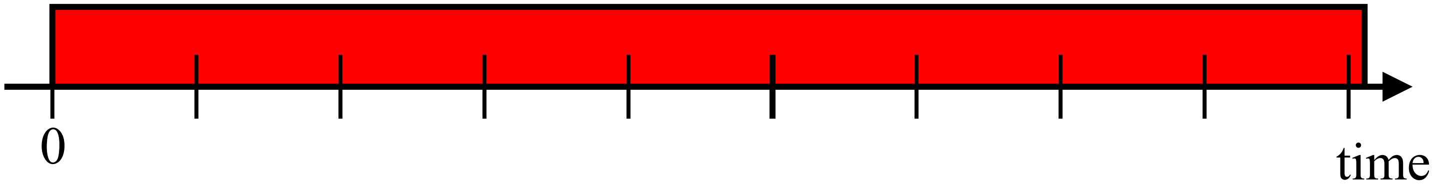
Scheduling Component Modeling

- Scheduling
  - assigns resources to workloads by scheduling algorithms
- Scheduling Component Model : C(W,R,A)
  - W : workload model
  - R : resource model
  - A : scheduling algorithm

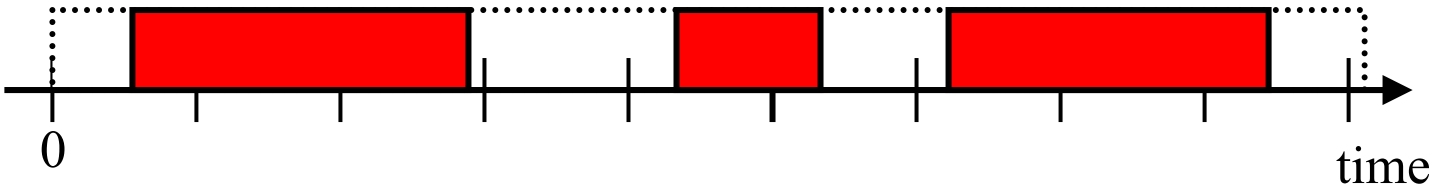


Resource Modeling

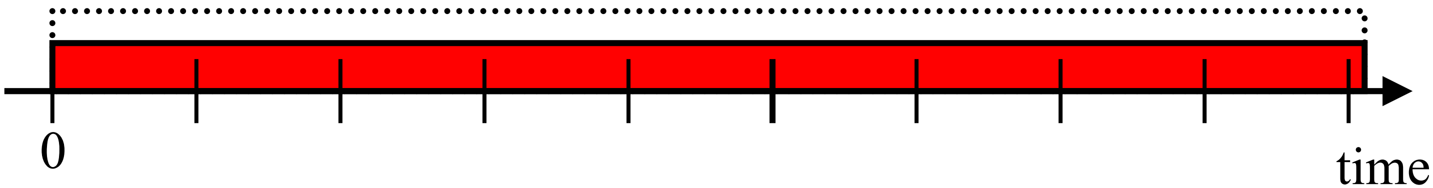
- Dedicated resource : always available at full capacity



- Shared resource : not a dedicated resource
  - Time-sharing : available at some times

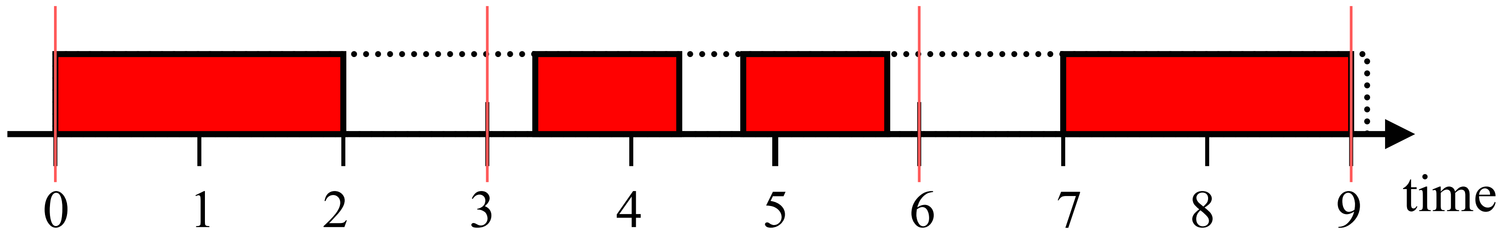


  - Non-time-sharing : available at fractional capacity



Resource Modeling

- Time-sharing resources
  - Bounded-delay resource model [Mok et al., '01] characterizes a time-sharing resource w.r.t. a non-time-sharing resource
  - Periodic resource model $\Gamma(\Pi,\Theta)$ [Shin & Lee, RTSS '03] characterizes periodic resource allocations



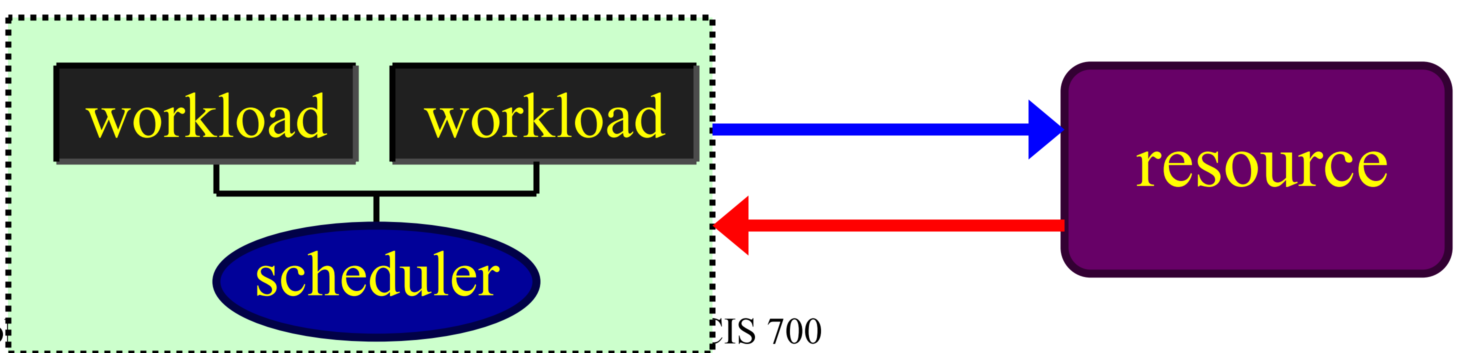
Schedulability Analysis

- A workload set is schedulable under a scheduling algorithm with available resources if its real-time requirements are satisfiable
- Schedulability analysis determines whether

resource demand,
which a workload set requires under a scheduling algorithm

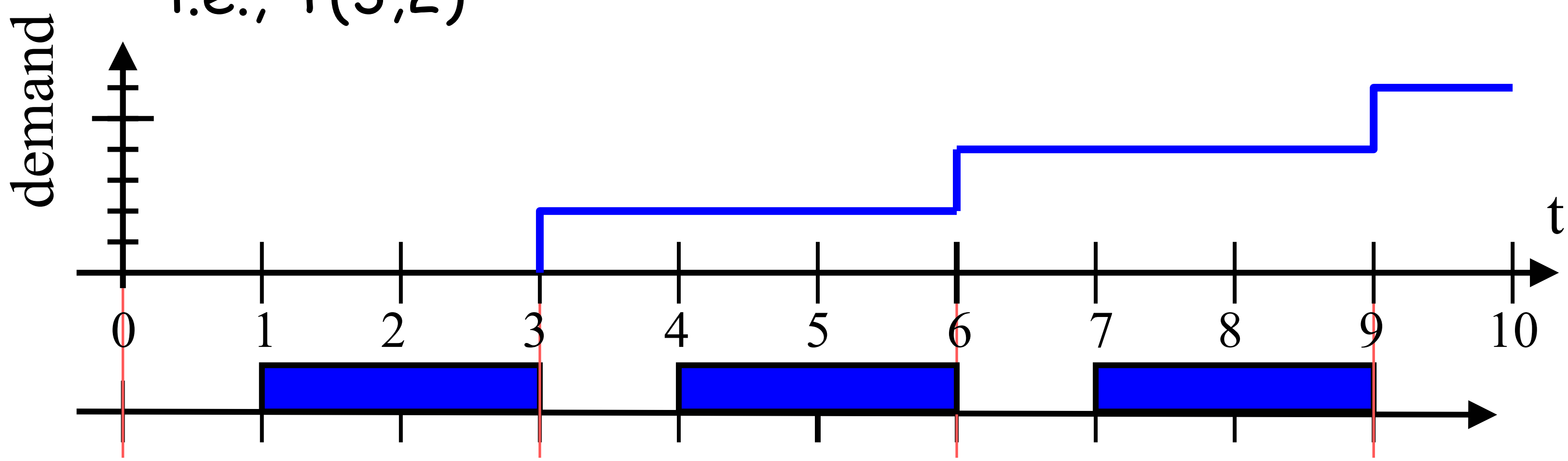
$\leq$

resource supply,
which available resources provide



Resource Demand Bound

- Resource demand bound during an interval of length t
  - dbf(W,A,t) computes the maximum possible resource demand that W requires under algorithm A during a time interval of length t
- Periodic task model T(p,e) [Liu & Layland, '73]
  - i.e., T(3,2)

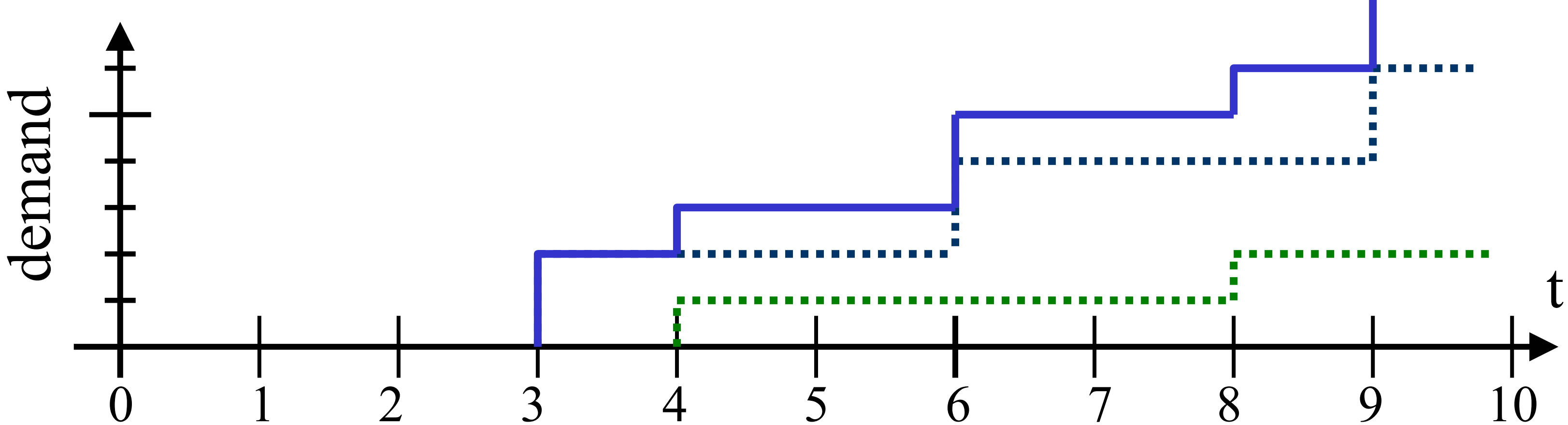


Demand Bound Function - EDF

- For a periodic workload set *W* = {Ti(pi,ei)},
  - dbf (*W*,*A*,*t*) for EDF algorithm [Baruah et al.,'90]

$$\mathrm{dbf}\,(\mathrm{W},\mathrm{EDF},\mathrm{t}) = \sum_{T_i \in W} \left\lfloor \frac{t}{p_i} \right\rfloor \cdot e_i$$

  - Example: W = {T_1(3,2), T_2(4,1)}

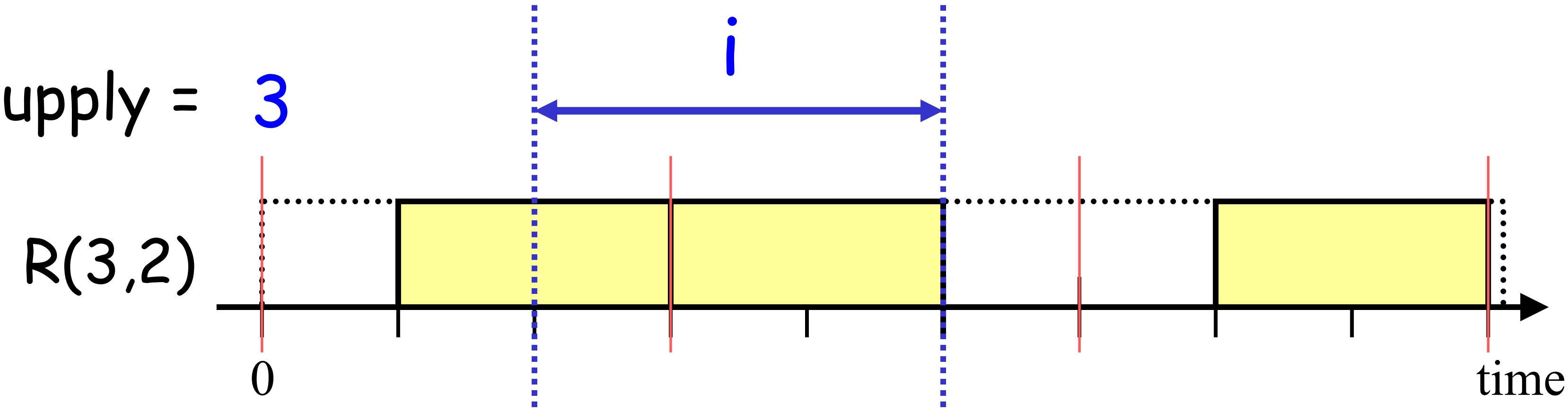


Resource Supply Bound

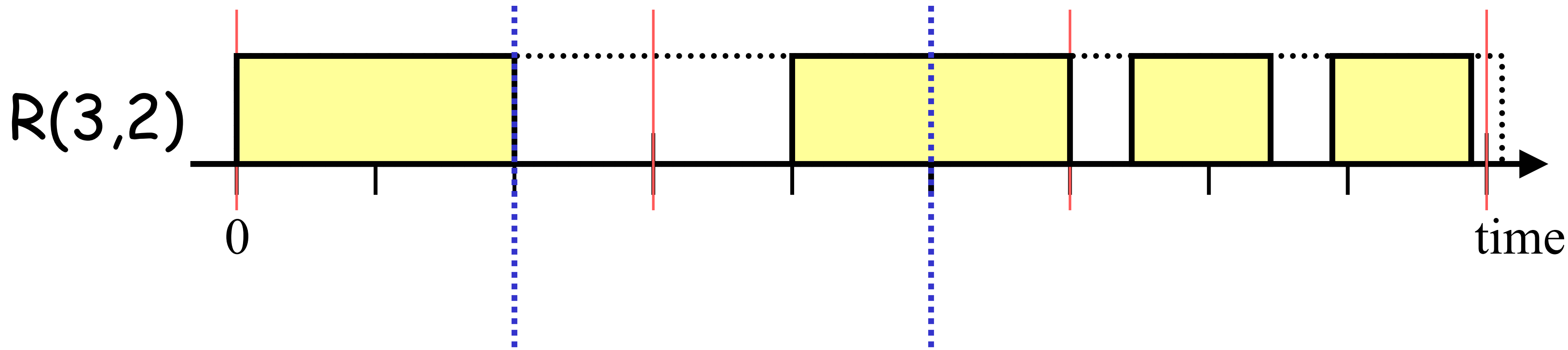
- Resource supply during an interval of length t
  - $sbf_R(t)$: the minimum possible resource supply by resource R over all intervals of length t
- For a single periodic resource model, i.e., $\Gamma(3,2)$
  - we can identify the worst-case resource allocation

Resource Supply Bound

- supply = 3



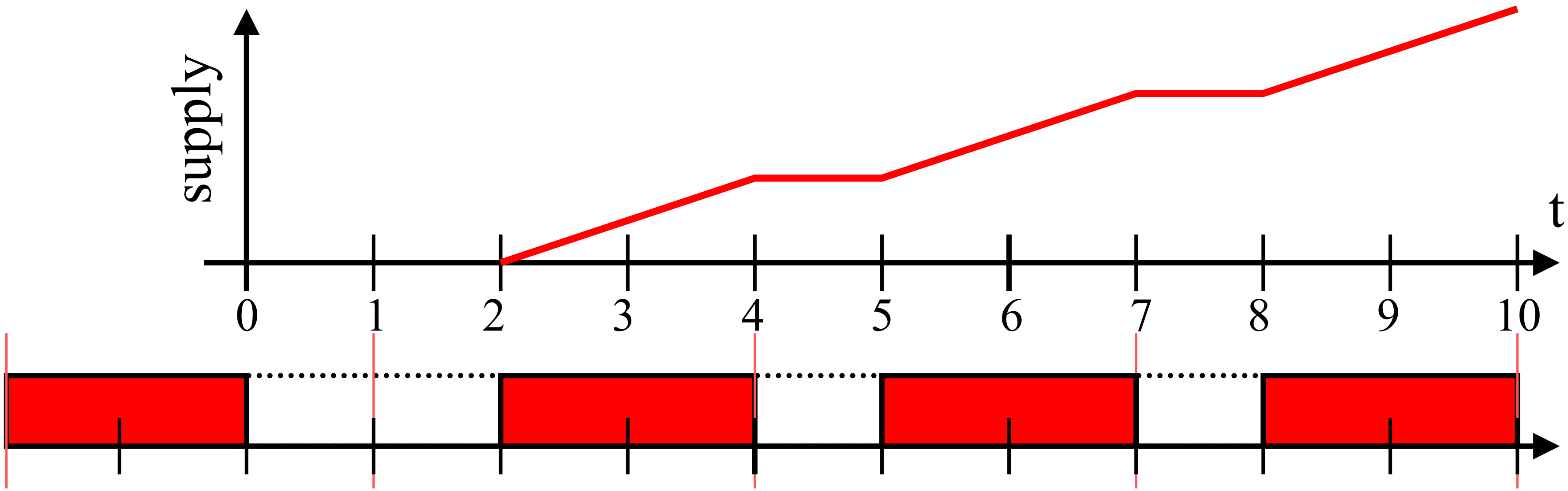
- supply = 1



- $sbf_R(i)$ = **1**

Resource Supply Bound

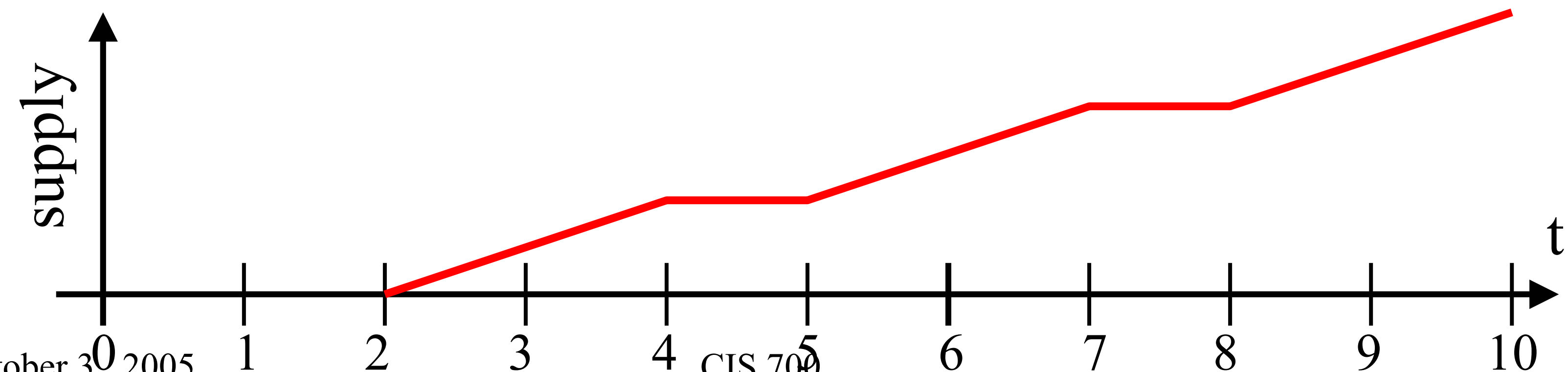
- Resource supply during an interval of length t
  - $sbf_R(t)$: the minimum possible resource supply by resource R over all intervals of length t
- For a single periodic resource model, i.e., Γ(3,2)
  - we can identify the worst-case resource allocation



Supply Bound Function

- Resource supply during an interval of length t
  - $sbf_{\Gamma}(t)$: the minimum possible resource supply by resource R over all intervals of length t
- For a single periodic resource model $\Gamma(\Pi,\Theta)$

$$\mathrm{sbf}_{\Gamma}(t) = \begin{cases} t-(k+1)(\Pi-\Theta) & \text{if } t \in [(k+1)\Pi-2\Theta, (k+1)\Pi-\Theta] \\ (k-1)\Theta & \text{otherwise} \end{cases}$$



Schedulability Conditions (EDF)

- A workload set W is schedulable over a resource model R under EDF if and only if for all interval i of length t

$$dbf_w(i) \le t$$

[BHR90]

Resource demand in an interval

Resource supply during the interval (from a dedicated resource)

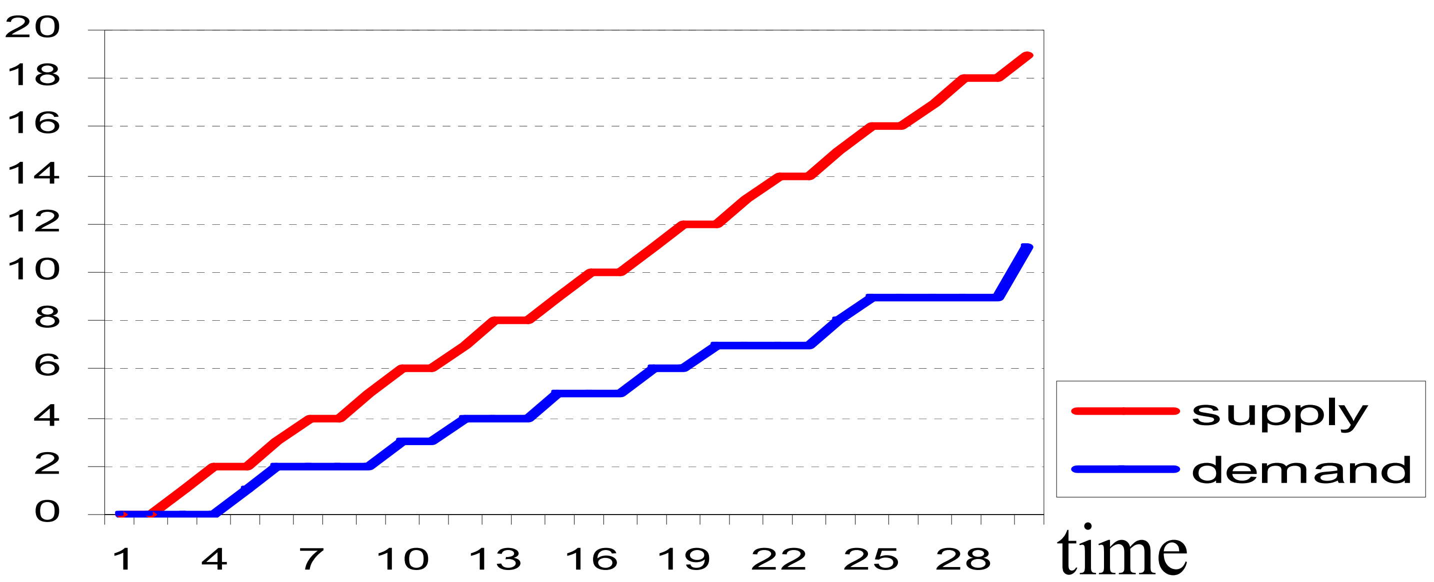
$$dbf_w(i) \le$$

  - $sbf_R(i)$: the minimum resource supply by resource R during an interval i
  - $dbf_w(i)$: the resource demand of workload W during an interval i

Schedulability Condition - EDF

- A periodic workload set W is schedulable under EDF over a periodic resource model $\Gamma(\Pi,\Theta)$ if and only if

$$\forall t > 0 \quad \mathrm{dbf}(W, EDF, t) \le \mathrm{sbf}_{\Gamma}(t)$$



Schedulability Condition - RM

- A periodic workload set W is schedulable under EDF over a periodic resource model $\Gamma(\Pi,\Theta)$ if and only if

$$\forall t > 0 \ \ \forall T_i \in W \qquad \mathrm{dbf}(W, RM, t, i) \le \mathrm{sbf}_\Gamma(t)$$

- For a periodic workload set *W* = {$T_i(p_i,e_i)$},
  - dbf (*W*,*A*,*t*,*i*) for RM algorithm [Lehoczky et al., '89]

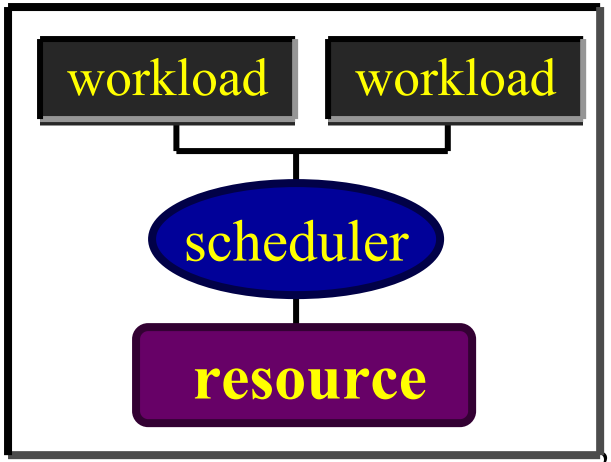
$$\mathrm{dbf}\ (W, RM, t, i) = e_i + \sum_{T_k \in HP(T_i)} \left\lceil \frac{t}{p_k} \right\rceil \cdot e_k$$

Utilization Bounds

- For a periodic workload T(p,e), utilization $U_T = e/p$
- For a periodic workload set W, utilization U_W is $\sum_{T_i \in W} \frac{e_i}{p_i}$

- Utilization bound (UB) of a resource model R
  - given a scheduling algorithm A and a resource model R, $UB_{R,A}$ is a number s. t. a workload set W is schedulable if

$$U_W \leq UB_{R,A}$$

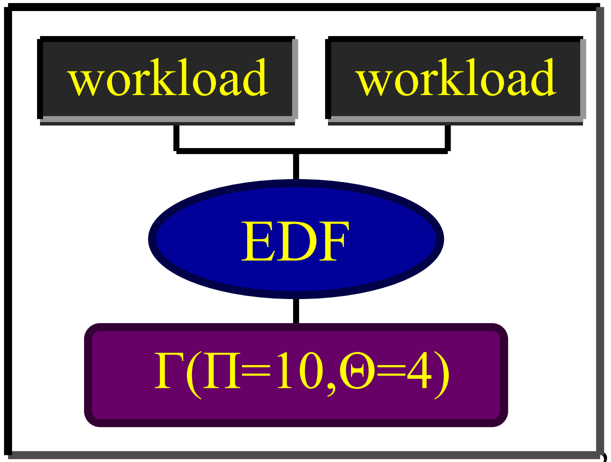


Utilization Bounds

- Example:
  - Consider a periodic resource $\Gamma(\Pi, \Theta)$, where Π = 10 and Θ = 4, and suppose $UB_{\Gamma, EDF}$ = 0.4.
  - Then, a set of periodic task W is schedulable if

$$U_W \leq 0.4$$

  - W = {T1(20,3), T2(50,5)} s.t. U_W = 0.25, is schedulable



Utilization Bound - EDF

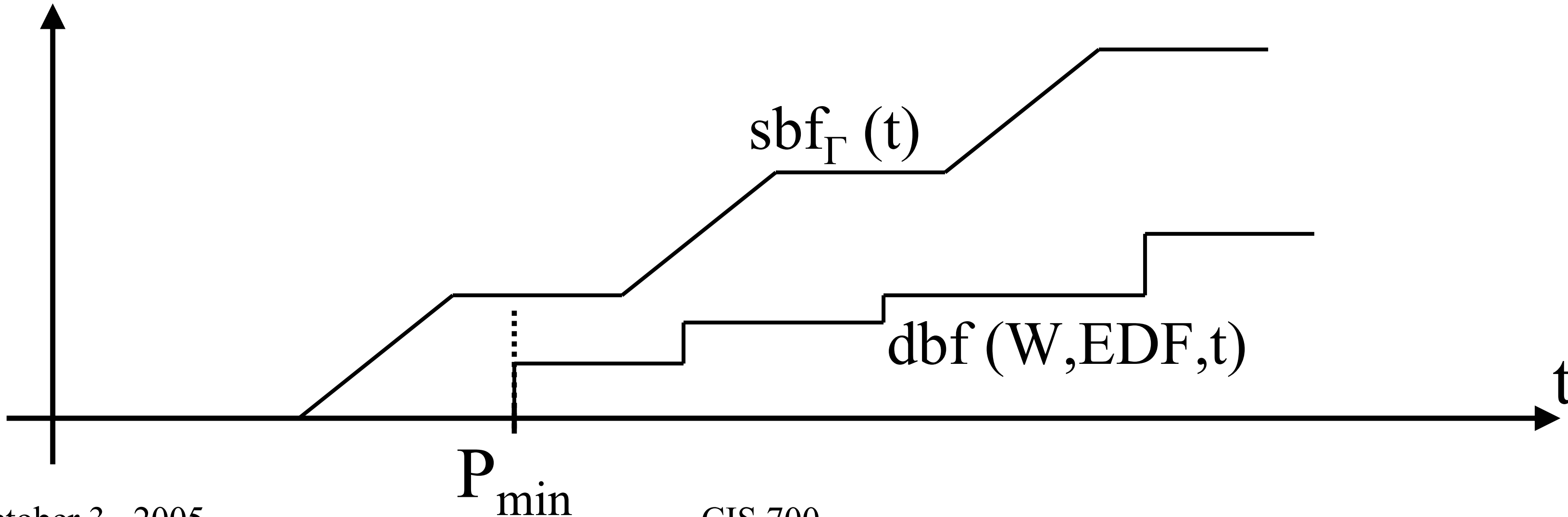
- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, where A = EDF, its utilization bound is

$$\mathrm{UB}_{\Gamma,\,\mathrm{EDF}}(\mathrm{P}_{\mathrm{min}}) = \frac{k \times U_\Gamma}{k + 2(1 - U_\Gamma)}$$

- P_{min} is the minimum task period (deadline) in W.
- k represents the relationship between resource period Π and the minimum task period P_{min}, $k \approx P_{min} / \Pi$

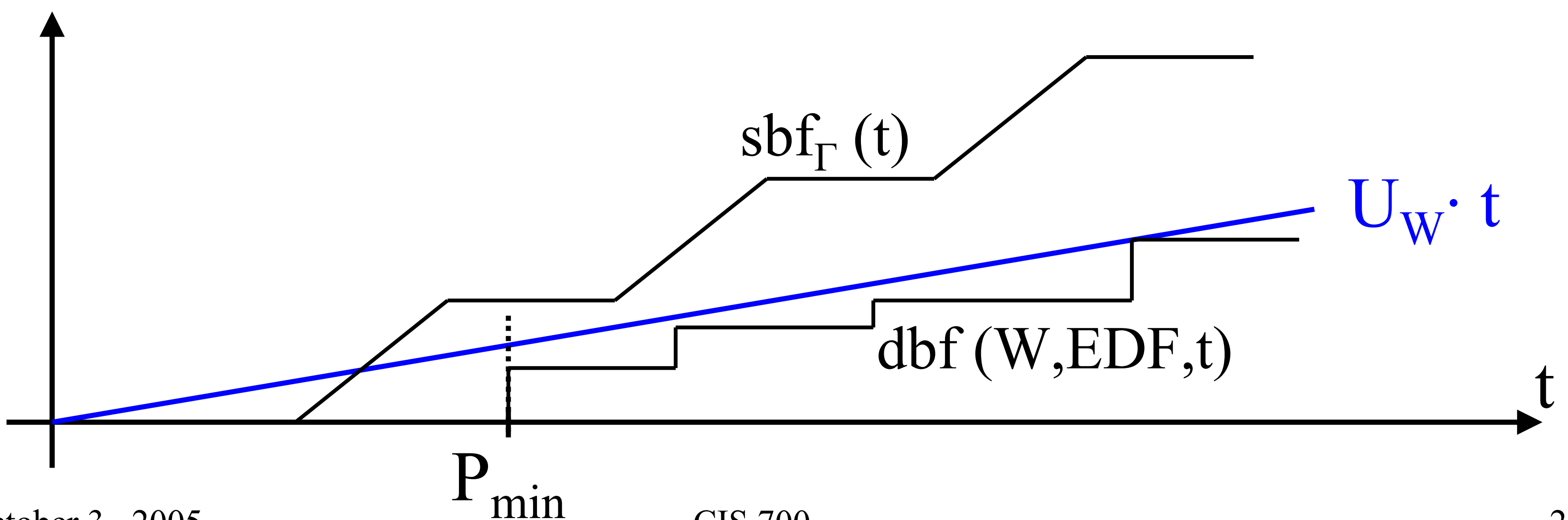
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - C is schedulable iff $dbf(W, EDF, t) \leq sbf_{\Gamma}(t)$



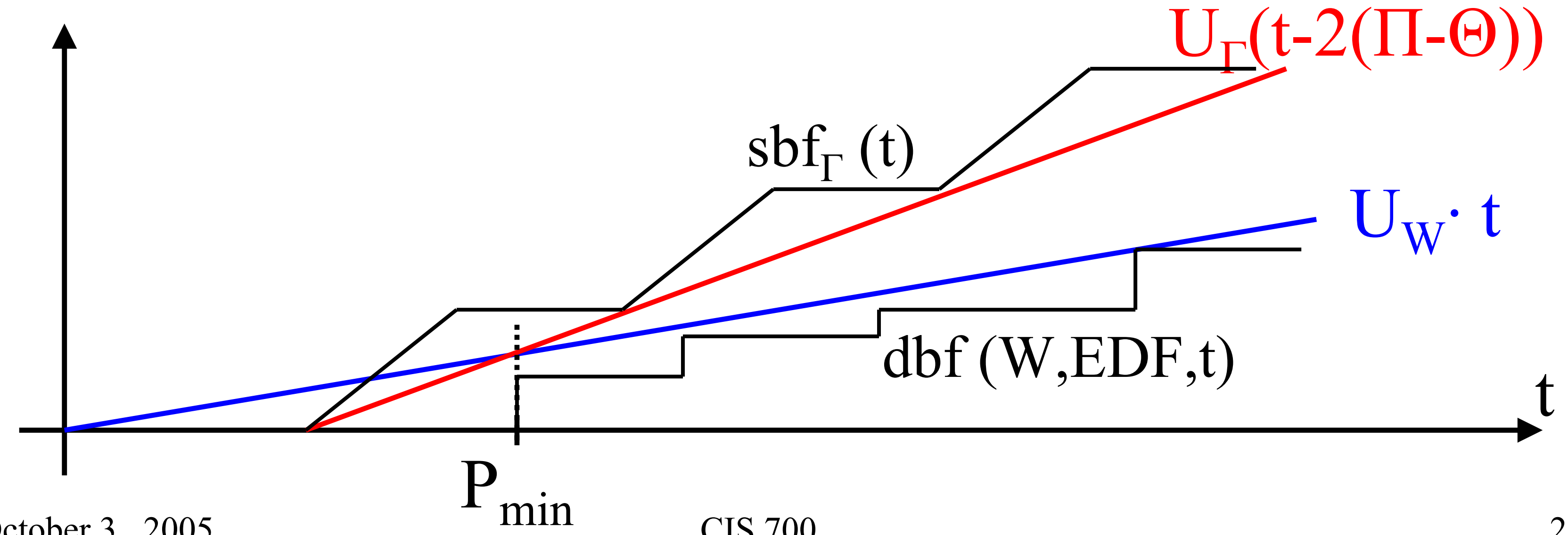
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - C is schedulable iff $dbf(W,EDF,t) \leq sbf_{\Gamma}(t)$
  - $dbf(W,EDF,t) \leq U_W \cdot t$



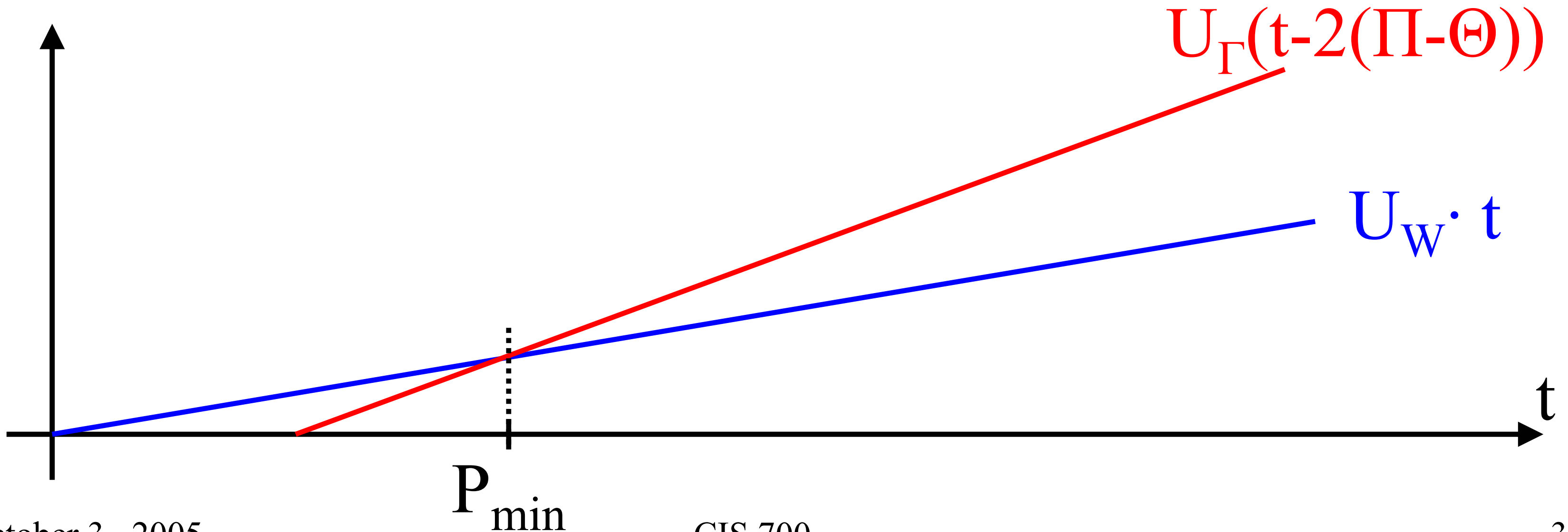
EDF Utilization Bound - Intuition

- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - C is schedulable iff dbf $(W, EDF, t) \le sbf_{\Gamma}(t)$
  - dbf $(W, EDF, t) \le U_W \cdot t$
  - $U_\Gamma(t-2(\Pi-\Theta)) \le sbf_\Gamma(t)$



EDF Utilization Bound - Intuition

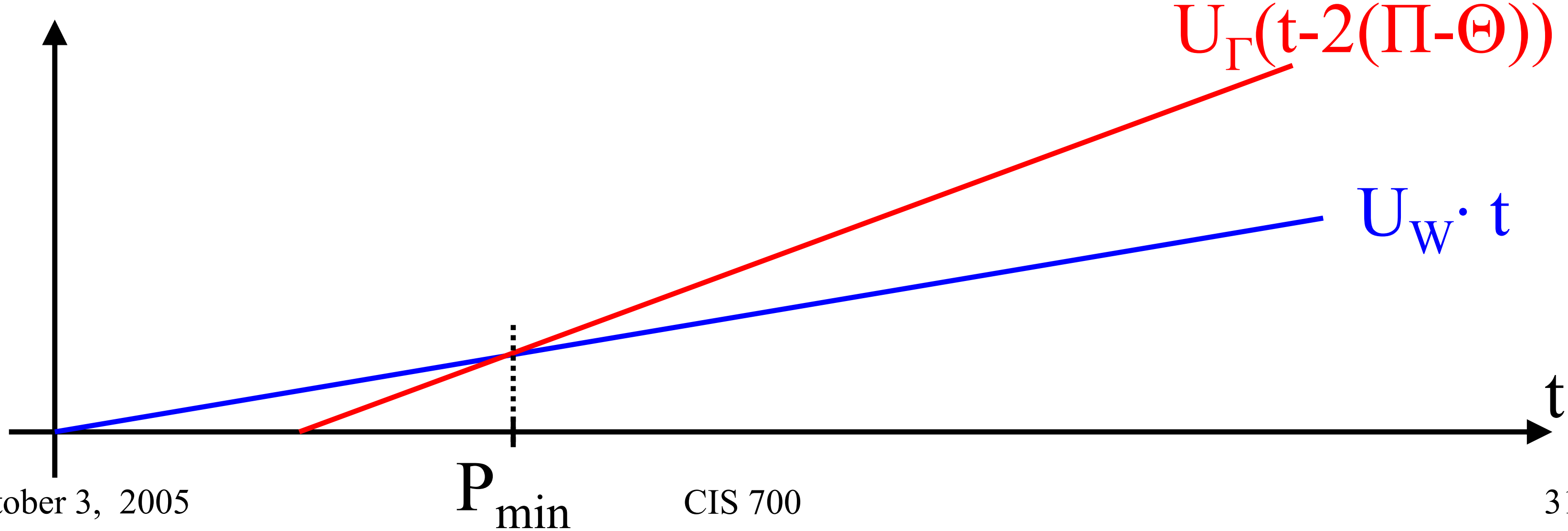
- Observation for a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - C is schedulable iff $dbf(W, EDF, t) \leq sbf_{\Gamma}(t)$
  - $dbf(W, EDF, t) \leq U_W \cdot t$
  - $U_{\Gamma}(t-2(\Pi-\Theta)) \leq sbf_{\Gamma}(t)$
  - Therefore, C is schedulable if $U_W \cdot t \leq U_{\Gamma}(t-2(\Pi-\Theta))$



EDF Utilization Bound - Intuition

- For a component $C(W, \Gamma(\Pi, \Theta), EDF)$
  - for all $t > P_{min}$, if $U_W \cdot t \leq U_\Gamma(t-2(\Pi-\Theta))$
    then C is schedulable.

$$U_W \leq U_\Gamma(t-2(\Pi-\Theta)) / t$$



Utilization Bound - RM

- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, where A = RM, its utilization bound is
  - [Saewong, Rajkumar, Lehoczky, Klein, '02]

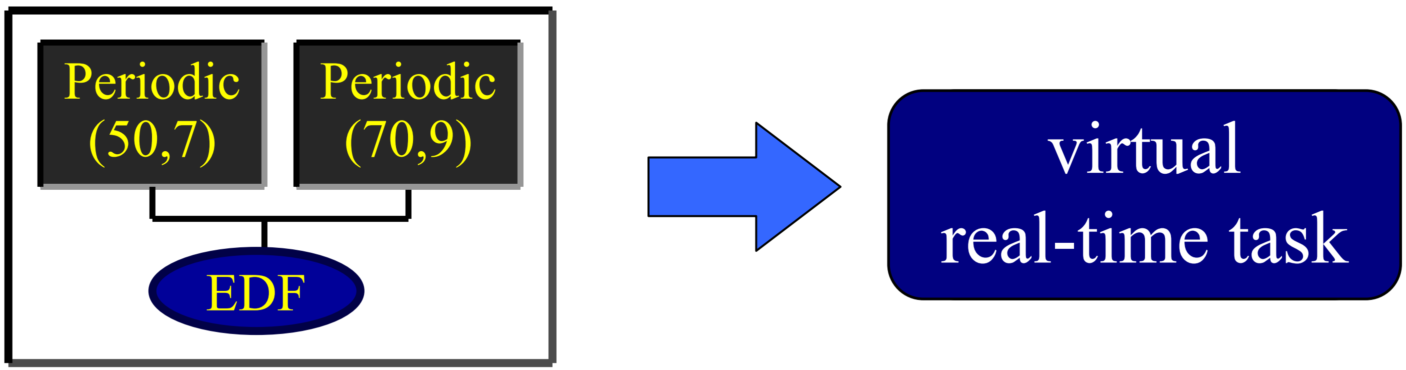
$$\mathrm{UB}_{\Gamma,\,\mathrm{RM}}(n) = n\left(\left(\frac{3-U_\Gamma}{3-2\times U_\Gamma}\right)^{\frac{1}{n}} - 1\right)$$

  - We generalize this earlier result, where $k \approx P_{min} / \Pi$.

$$\mathrm{UB}_{\Gamma,\,\mathrm{RM}}(n, \mathrm{P}_{\min}) = U_\Gamma \times n\left(\left(\frac{2k+2(1-U_\Gamma)}{k+2(1-U_\Gamma)}\right)^{\frac{1}{n}} - 1\right)$$

Component Abstraction

- Component timing abstraction
  - To specify the collective real-time demands of a component as a timing interface



Component Abstraction

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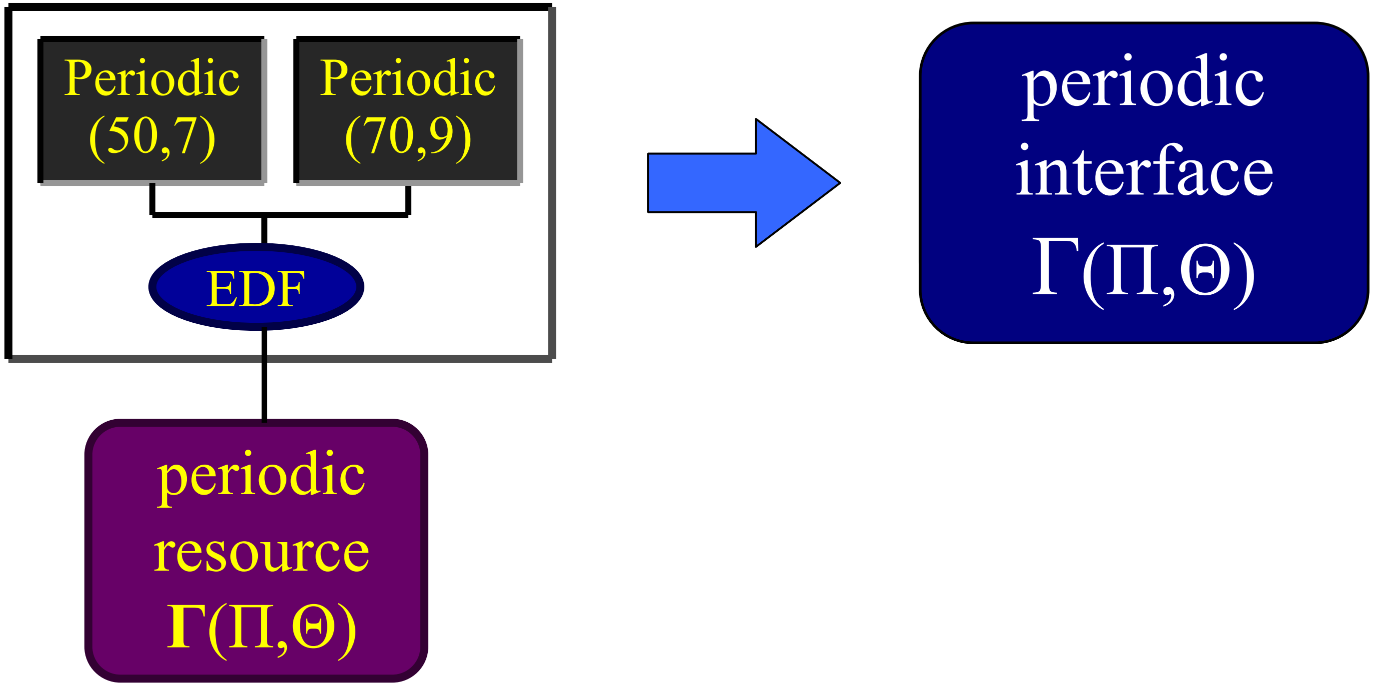
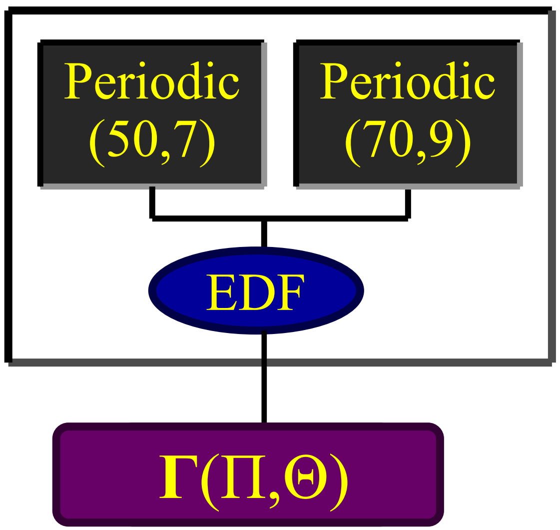
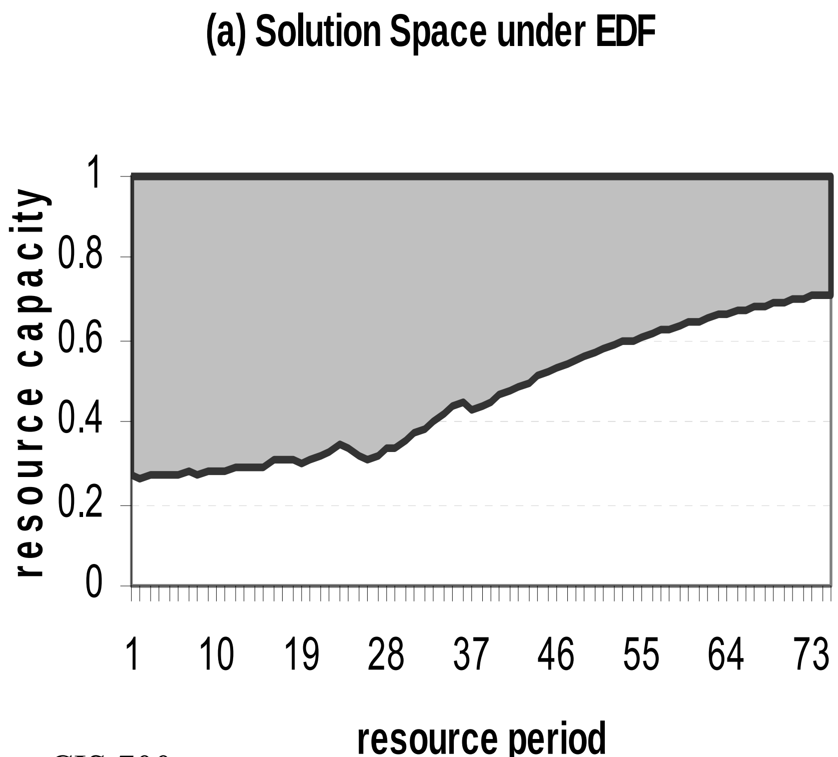


Component Abstraction (Example)

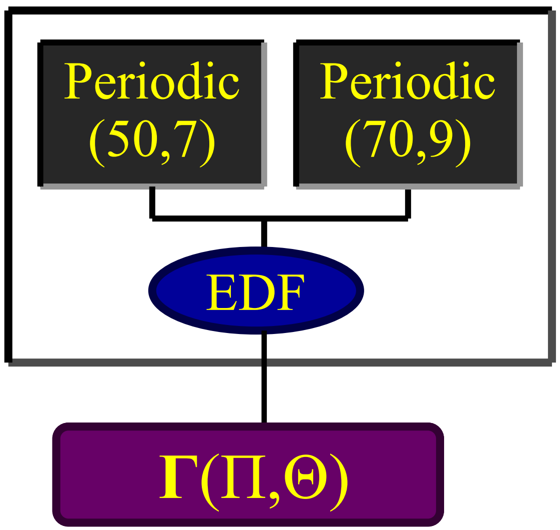
- In this example, a solution space of a periodic resource $\Gamma(\Pi,\Theta)$ that makes $C(W, \Gamma(\Pi, \Theta), EDF)$ schedulable is

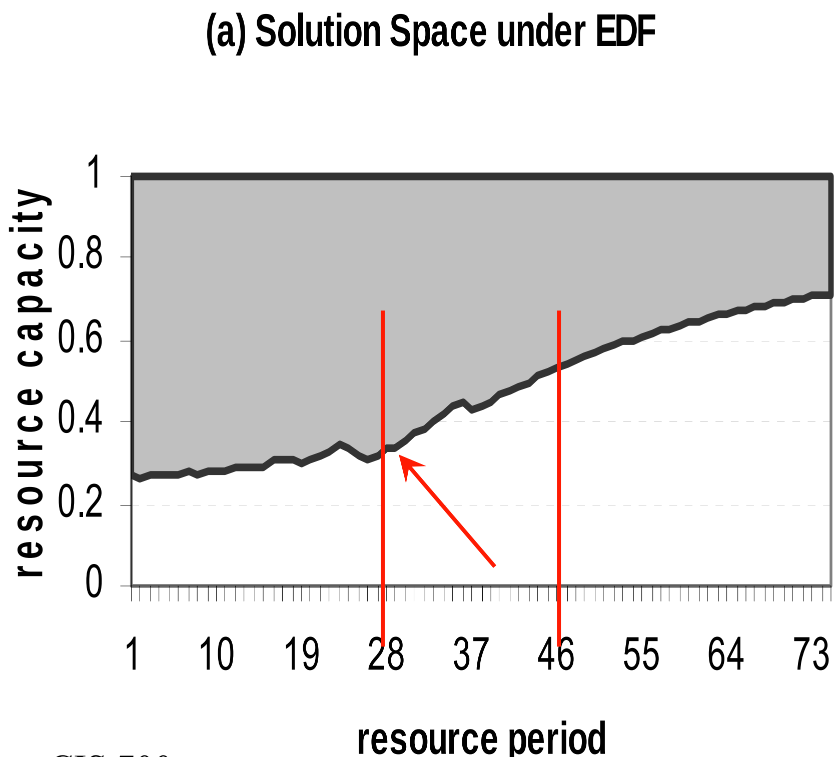




Component Abstraction (Example)

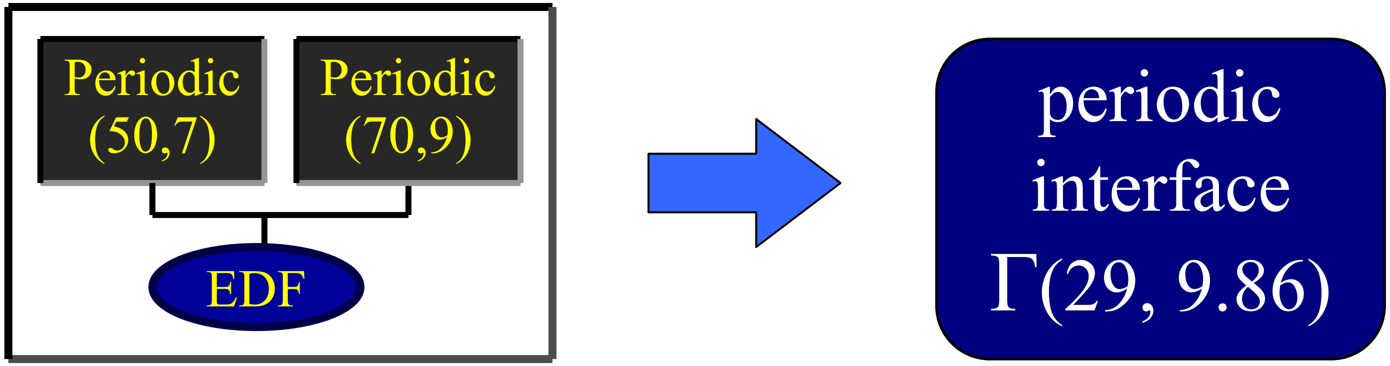
- An approach to pick one solution out of the solution space
  - Given a range of Π, we can pick $\Gamma(\Pi,\Theta)$ such that U_Γ is minimized. (for example, $28 \le \Pi \le 46$)



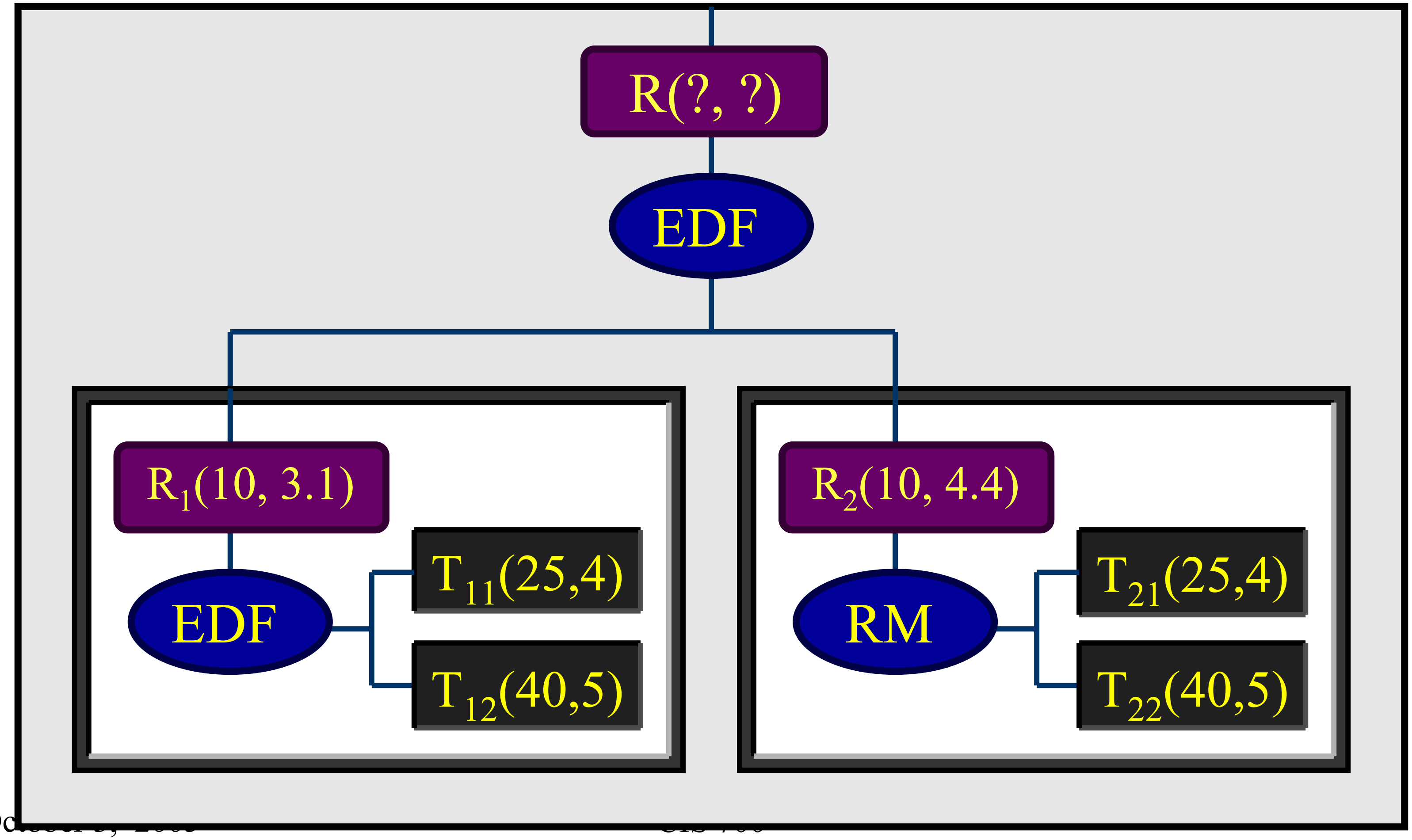


Component Timing Abstraction

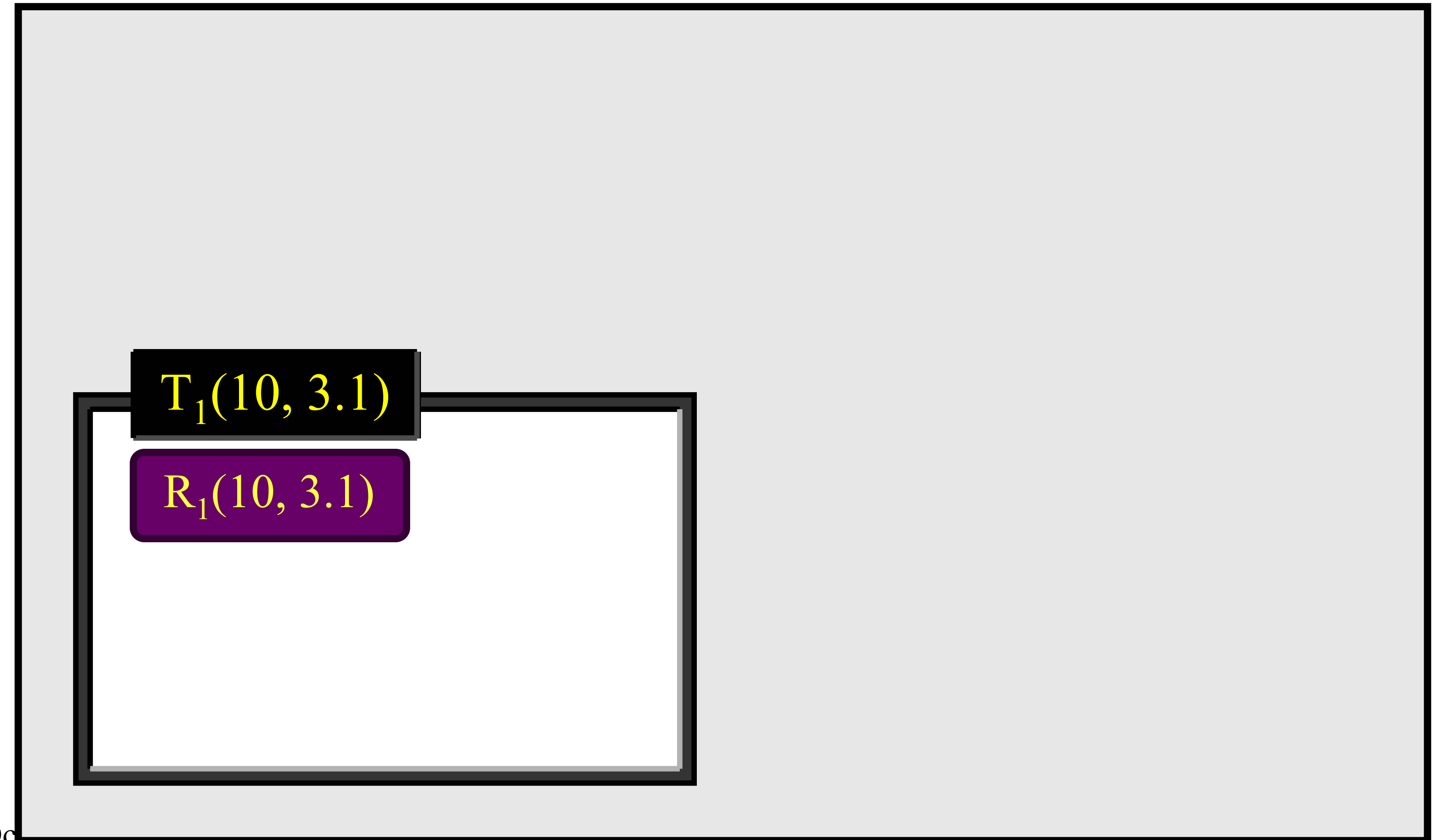
- Component timing abstraction
  - To abstract the collective real-time demands of a component as a timing interface



Compositional Real-Time Guarantees

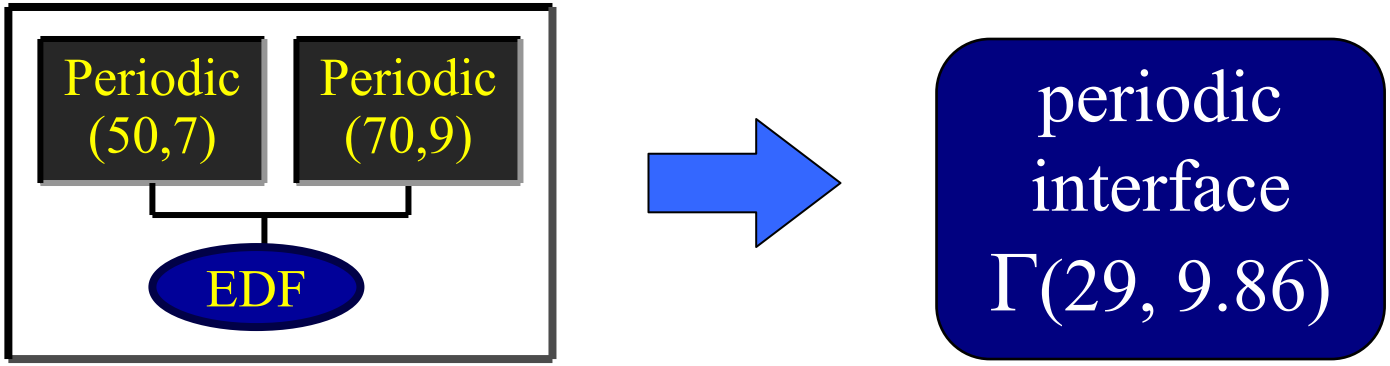


Compositional Real-Time Guarantees



Abstraction Overhead

- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, its abstraction overhead (O_Γ) is $\frac{U_\Gamma}{U_W} - 1$



$U_W = 0.27$ $\quad$ $U_\Gamma = 0.34$

Abstraction Overhead Bound

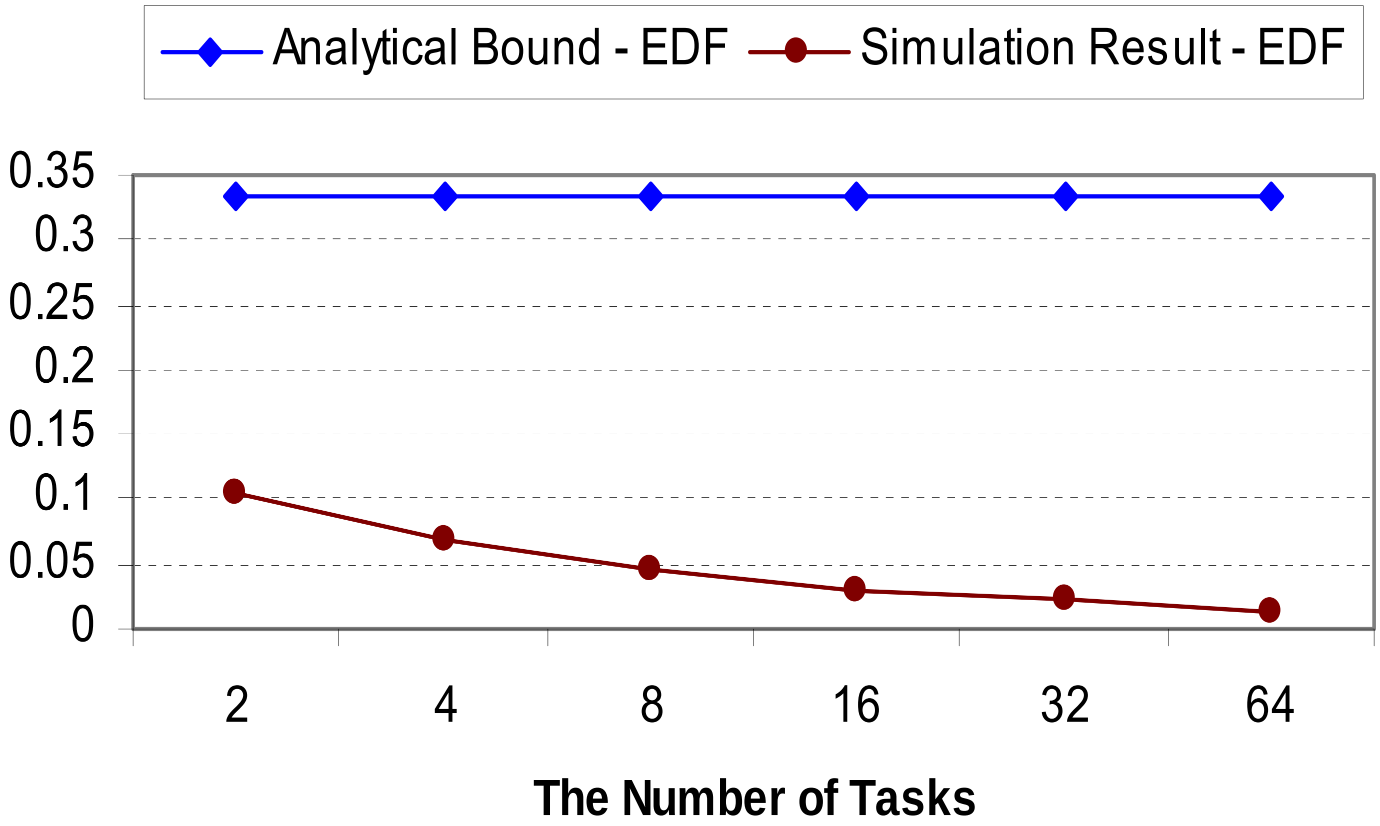
- For a scheduling component $C(W, \Gamma(\Pi, \Theta), A)$, its abstraction overhead (O_Γ) is
  - A = EDF $$O_{\Gamma,\,\mathrm{EDF}} \leq \frac{2 \times (1 - U_W)}{k + 2 \times U_W}$$
  - A = RM $$O_{\Gamma,\,\mathrm{RM}} \leq \frac{1}{\log\left(\frac{2k + 2(1 - U_W)}{k + 2(1 - U_W)}\right)} - 1$$

Abstraction Overhead

- Simulation Results
  - with periodic workloads and periodic resource under EDF/RM
  - the number of tasks n : 2, 4, 8, 16, 32, 64
  - the workload utilization $U(W)$: 0.2~0.7
  - the resource period : represented by k

Abstraction Overhead

- k= 2, U(W) = 0.4



Summary

- Compositional real-time scheduling framework
  - with the periodic model [Shin & Lee, RTSS '03]

  1. resource modeling
     – utilization bounds (EDF/RM)
  2. schedulability analysis
     • exact schedulability conditions (EDF/RM)
  3. component timing abstraction and composition
     • overhead evaluation
       – upper-bounds and simulation results

Future Work

- Extending our framework for handling
  - Soft real-time workload models
  - non-periodic workload models
  - task dependency

References

- Insik Shin & Insup Lee,
  "Periodic Resource Model for Compositional Real-Time Gaurantees", the Best Paper of RTSS 2003.
- Insik Shin & Insup Lee,
  "Compositional Real-time Scheduling Framework", RTSS 2004.

THANK YOU