

# Merging Partial Behavioral Models

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# Preliminaries

- Behavioral Model
  - Model describing behavioral aspect of a software system
  - Examples
    - State-based : Labeled Transition System (LTS)
    - Scenario-based : Message Sequence Charts (MSC)
- Complete Behavioral Model
  - Describes all possible behaviors of a software system

# Motivation

- Construction of complete models is a complex task
- Partial behavioral models
  - Specified by different users with different viewpoints
  - Covering different components of a system
  - Multiple descriptions of the same component
  - Scenario based partial descriptions (MSCs)

# State-based specifications

- State-based models are more amenable to verification
- Synthesis of state-based model from partial scenario specifications (LTS from MSC)
- LTS models are inherently absolute (disallow all transitions not explicitly shown in the model)
- But model absoluteness is limiting (partial scenarios)
- Requires state-based models which can explicitly model unknown(partial) behaviors
  - Modal Transition Systems (MTS), Partial LTS
  - MTS from MSC

# Model Merging

- Analysis effective in state-based models describing complete behavior of system
- Justifies merging of partial models (merging MTSs for different scenarios)
- Problem
  - How to merge MTSs describing different, yet overlapping aspects of a system
  - How to combine MTSs of the same aspect specified by different users with different viewpoints

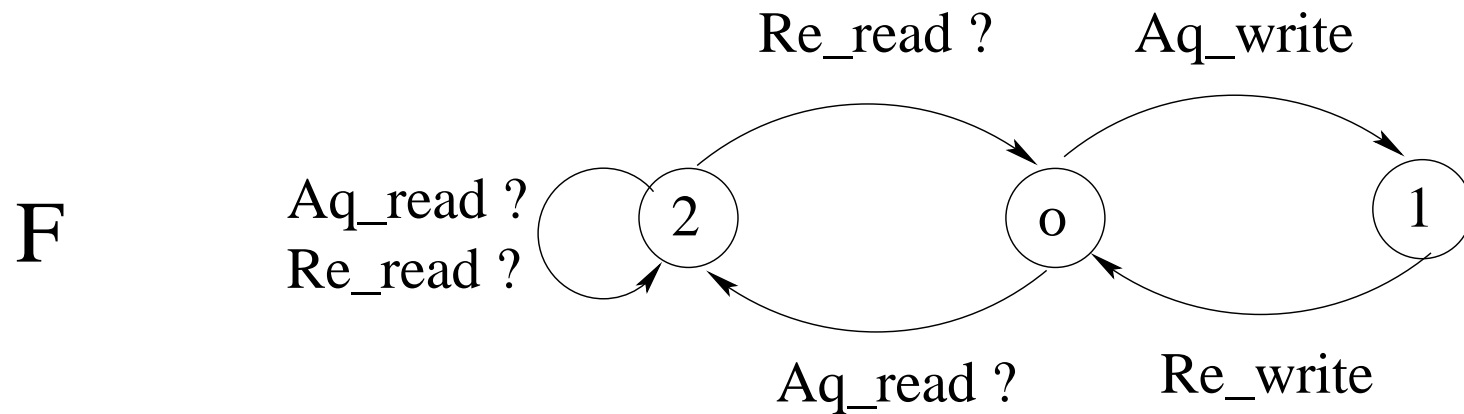
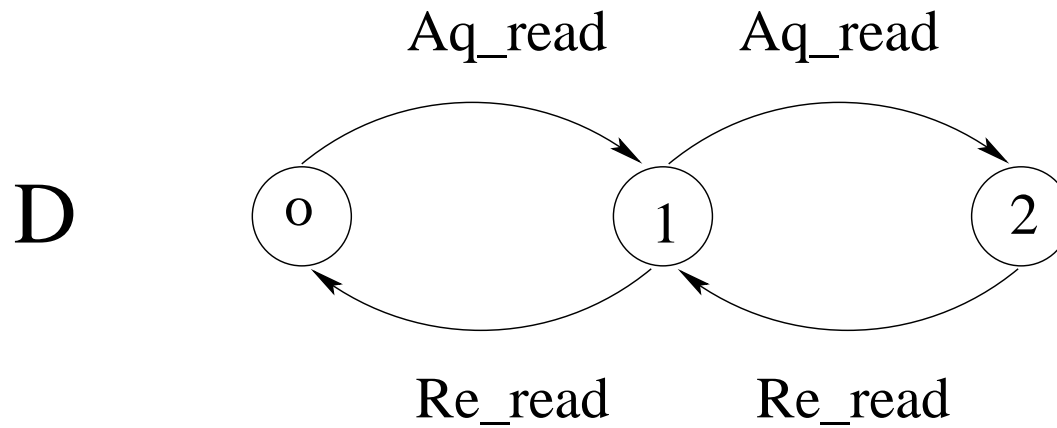
# Labeled Transition System

- $Act$  be a universal set of observable actions
- $Act_\tau = Act \cup \{\tau\}$  where  $\tau$  is internal action
- Labeled Transition System(LTS)  
 $P = \langle S, L, \Delta, s_0 \rangle$  where
  - $S$  is a set of states
  - $L \subseteq Act_\tau$  is a set of action labels
  - $\Delta \subseteq (S \times L \times S)$  is a transition relation
  - $s_0 \in S$  is the initial state
- $\alpha P = L \setminus \{\tau\}$  denotes observable action set

# Modal Transition System

- Modal Transition System(MTS) extends LTS with an additional set of uncertain transitions
- MTS  $M = \langle S, L, \Delta^r, \Delta^p, s_0 \rangle$ ,  $\Delta^r \subseteq \Delta^p$
- $\Delta^r$  represents required transitions and  $\Delta^p \setminus \Delta^r$  represents maybe transitions
- LTS is a special type of MTS

# Example





# MTS Semantics

- MTS  $M = \langle S, L, \Delta^r, \Delta^p, s_0 \rangle$
- $M \xrightarrow[r]{l} M'$  if  $M' = \langle S, L, \Delta^r, \Delta^p, s'_0 \rangle$  and  $(s_0, l, s'_0) \in \Delta^r$
- $M \xrightarrow[m]{l} M'$  if  $M' = \langle S, L, \Delta^r, \Delta^p, s'_0 \rangle$  and  $(s_0, l, s'_0) \in \Delta^p \setminus \Delta^r$
- $M$  proscribes  $l$  ( $M \not\xrightarrow{l}$ ) if  $M$  cannot transit on  $l$

# Semantics Contd.

- $\omega = \omega_1 \cdots \omega_k \in Act_\tau$
- $(M \xrightarrow[r]{\omega} N) \Rightarrow$ 
  - $\exists M_0, \dots, M_k; M_0 = M, M_k = N$
  - $\forall i, (M_i \xrightarrow[r]{\omega_{i+1}} M_{i+1}), 0 \leq i < k$
- $M \Longrightarrow_r^l M'$  denotes  $M \xrightarrow[r]{\tau^* l \tau^*} M'$

# Semantics Contd.

- $(M \xrightarrow{\omega}_m N) \Rightarrow$ 
  - $\exists M_0, \dots, M_k; M_0 = M, M_k = N$
  - $\forall i, (M_i \xrightarrow{p^{\omega_{i+1}}} M_{i+1}), 0 \leq i < k$
  - $\exists M_j, (M_j \xrightarrow{m^{\omega_{j+1}}} M_{j+1}), 0 \leq j < k$
- $M \xRightarrow{l}_m M'$  denotes
  - $\exists M'', M \xrightarrow{m^{\tau^* l}} M''$
  - $M'' \xrightarrow{r^{\tau^*}} M'$

# MTS Refinement

- Refinement of a MTS results in a more concrete model than the original
- Some knowledge over maybe behavior is gained
- “More defined than” relation
- Intuitively, refinement converts some maybe transitions to required ones and some other maybe transitions are removed completely

# Refinement Definition

- $\rho$  be universe of MTSs
- $M \preceq N$  when  $\alpha M = \alpha N$  and
  - $(M, N)$  contained in some refinement relation  $R \subseteq \rho \times \rho$
  - $\forall l \in Act_\tau,$ 
    1.  $(M \xrightarrow[r]{l} M') \Rightarrow ((\exists N', N \xrightarrow[r]{l} N') \wedge (M', N') \in R)$
    2.  $(N \xrightarrow[p]{l} N') \Rightarrow ((\exists M', M \xrightarrow[p]{l} M') \wedge (M', N') \in R)$

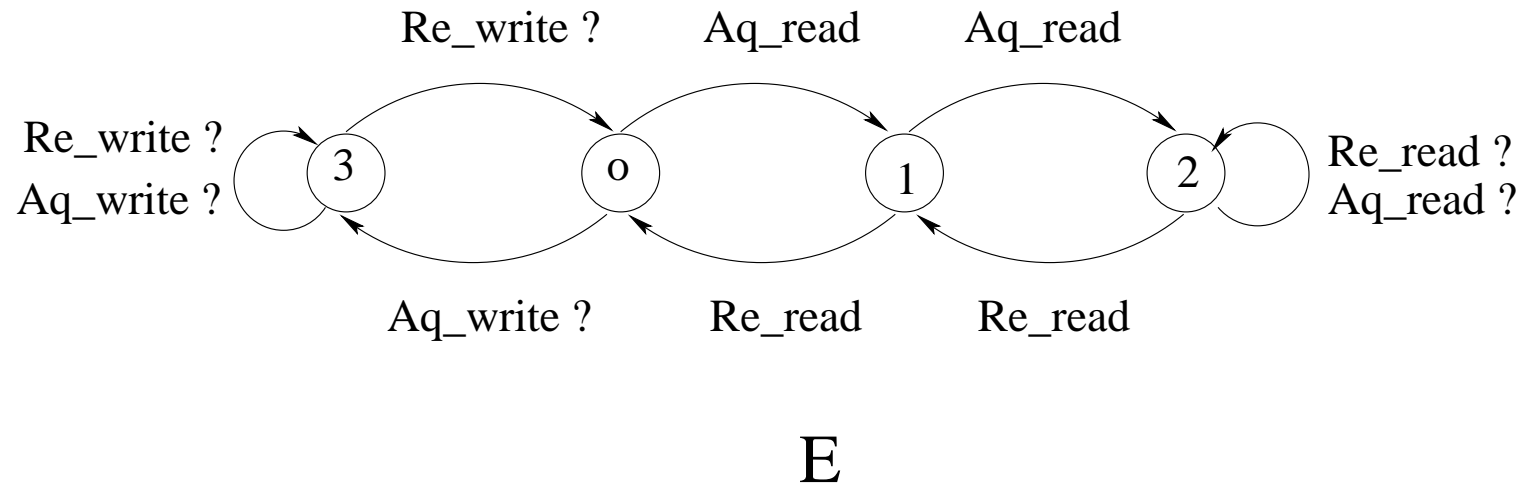
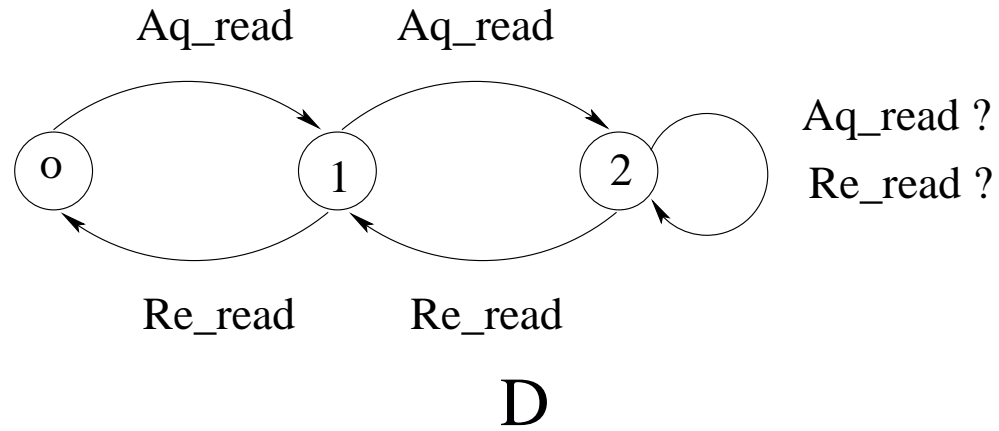
# Label Hiding

- Refinement requires alphabets of models to be same
- Hiding makes set of actions of a model unobservable to environment
- All transitions labeled with the hidden action are replaced with  $\tau$
- $M@_X$  denotes MTS with label set  $X$ 
  - All labels not in  $X$  are replaced with  $\tau$

# Observational Refinement (OR)

- $M \preceq_o N$  when  $\alpha M = \alpha N$  and
  - $(M, N)$  is contained in some refinement relation  $R \subseteq \rho \times \rho$
  - $\forall l \in Act,$ 
    - $(M \Longrightarrow_r^l M') \Rightarrow ((\exists N', N \Longrightarrow_r^l N') \wedge (M', N') \in R)$
    - $(N \Longrightarrow_p^l N') \Rightarrow ((\exists M', M \Longrightarrow_p^l M') \wedge (M', N') \in R)$

# Example

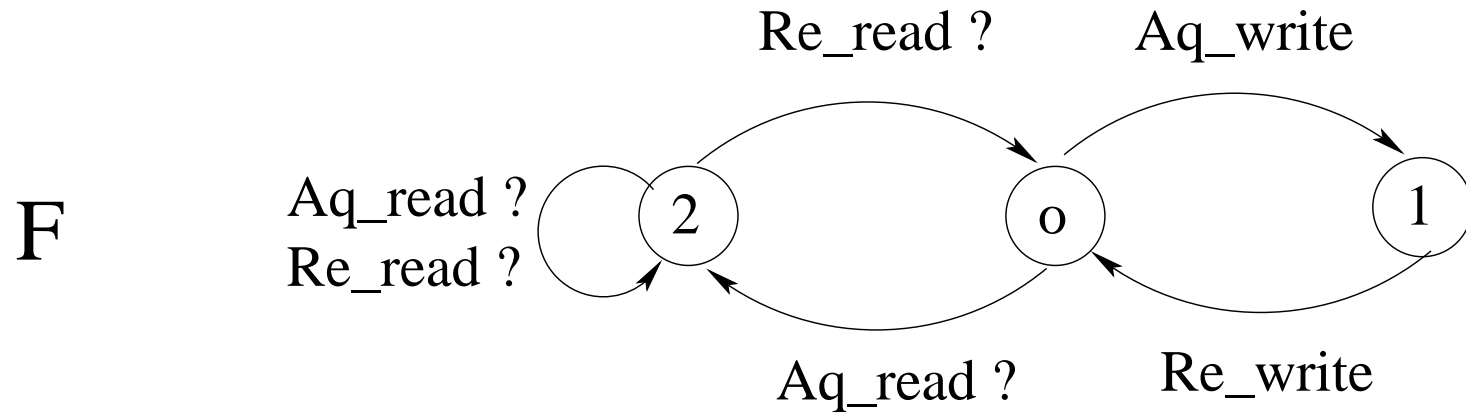
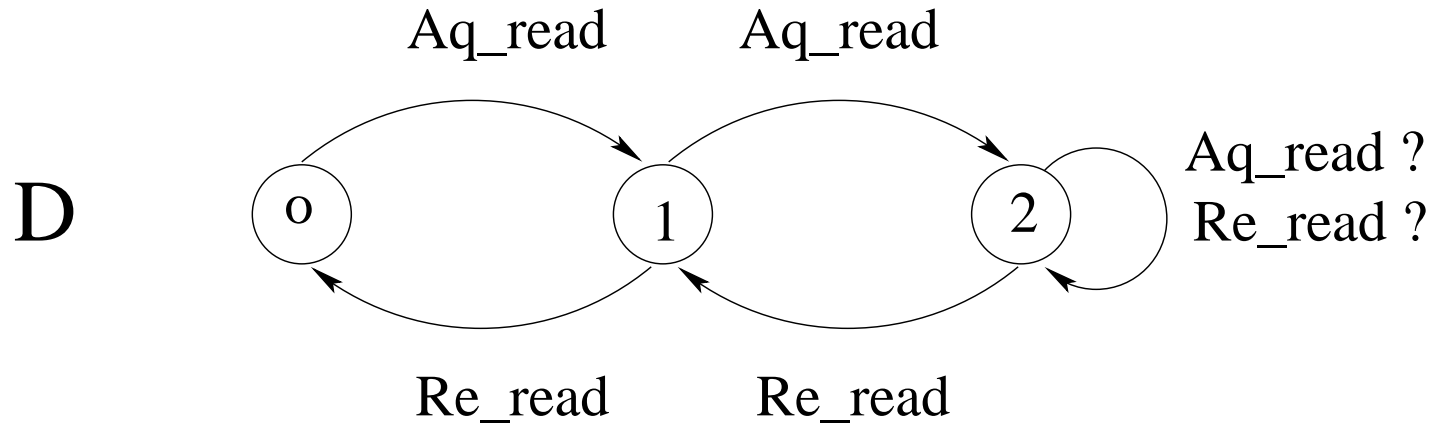




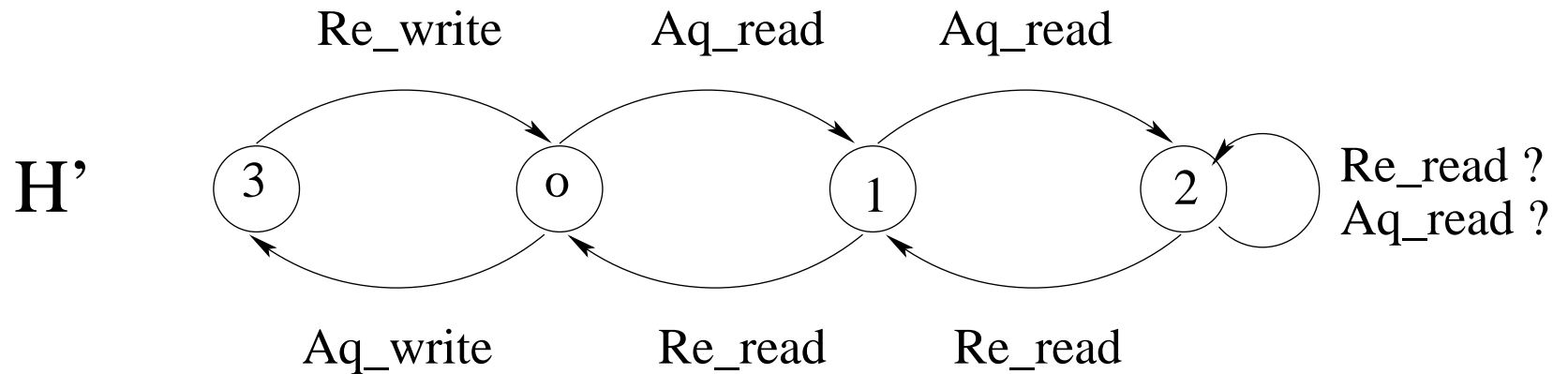
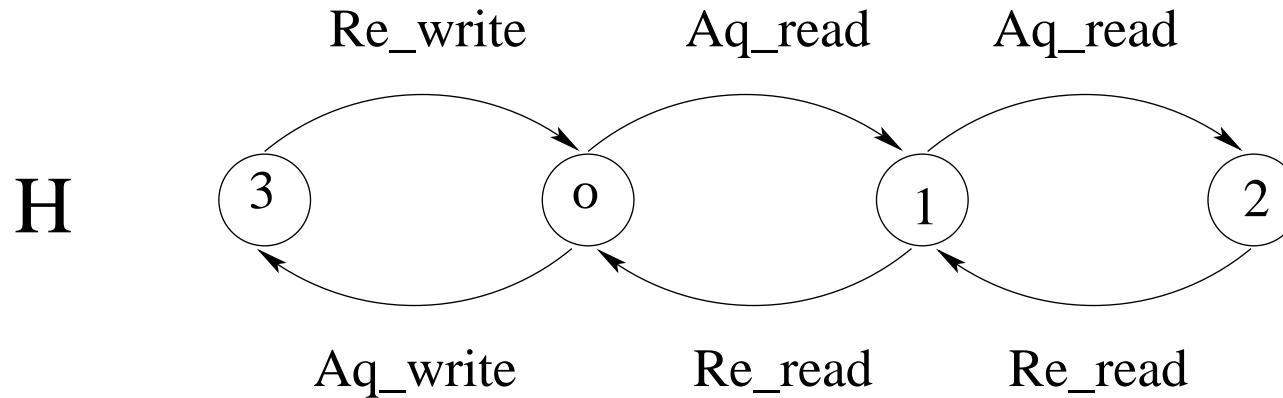
# MTS Merging

- Knowledge from two partial models(MTS) used to generate a unified MTS
- Merging is about finding a common refinement of the two models
- Models being merged can have different action labels
- $P$  is a common observational refinement of  $M$  and  $N$  if  $(\alpha P \supseteq (\alpha M \cup \alpha N))$ ,  $(M \preceq_o P@ \alpha M)$  and  $(N \preceq_o P@ \alpha N)$

# Example

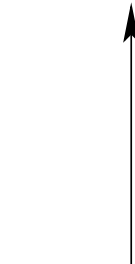


# Example Contd.



# Example Contd.

$H@X$



$D, H'@X$



$F@X$

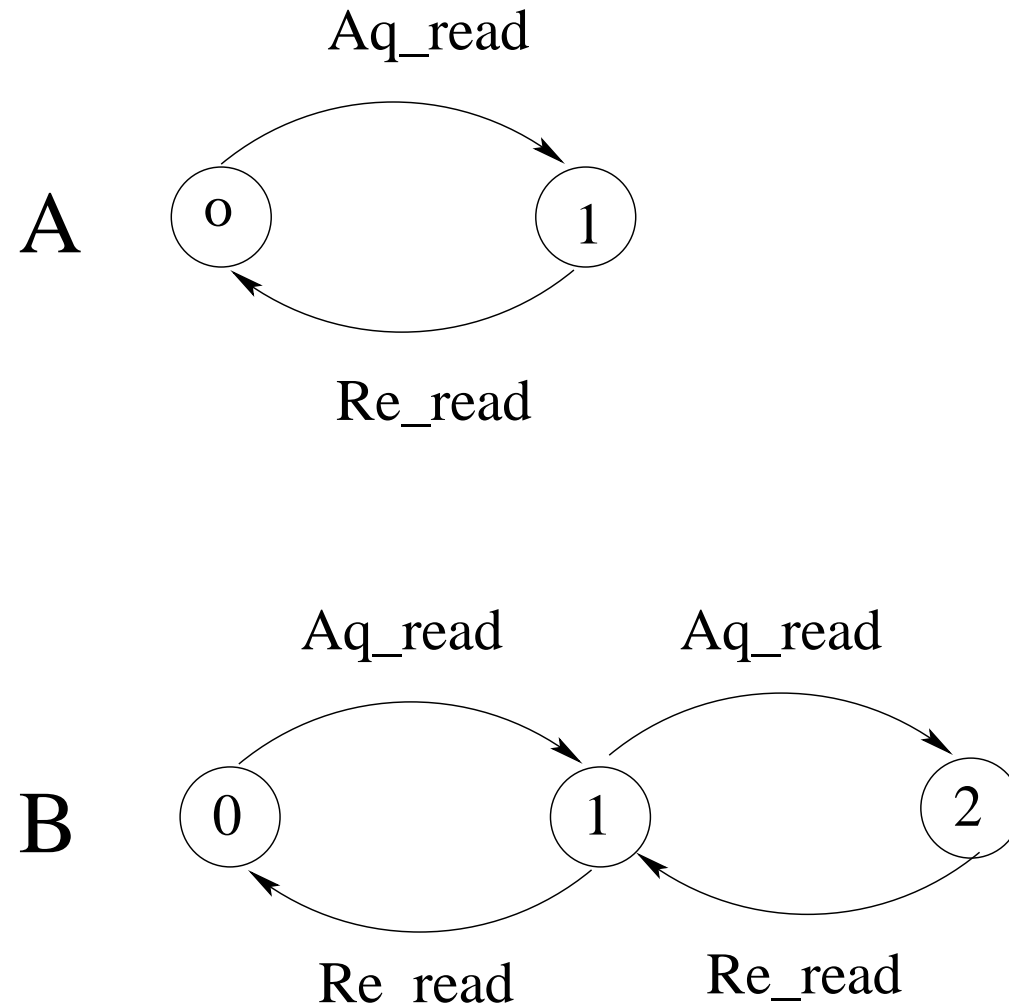
# Least Common Refinement

- $H$  and  $H'$  are both common refinements of  $D$  and  $F$
- $H'$  is the preferred common refinement;  $H$  proscribes three or more readers which is not required
- $P$  is the least common refinement(LCR) of  $M$  and  $N$  if  $P$  is a common refinement of  $M$  and  $N$ ,  $\alpha P = (\alpha M \cup \alpha N)$ , and for any common refinement  $Q$  of  $M$  and  $N$ ,  $P \preceq_o Q @ \alpha P$
- But common refinement or LCR need not exist for two MTSs

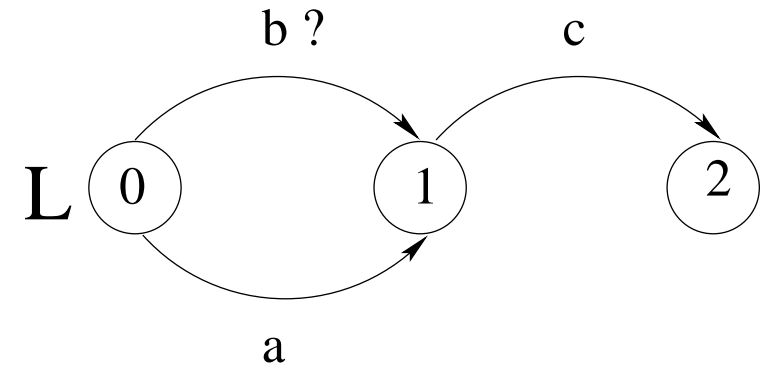
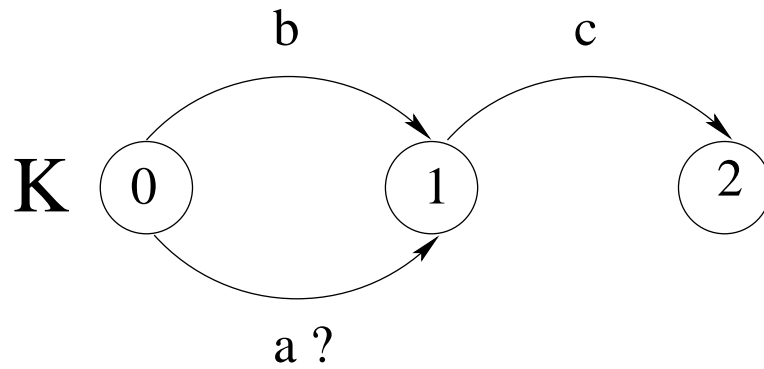
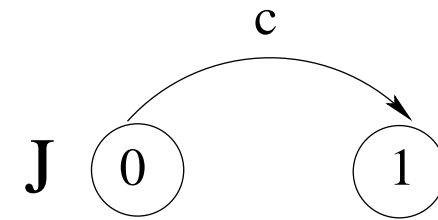
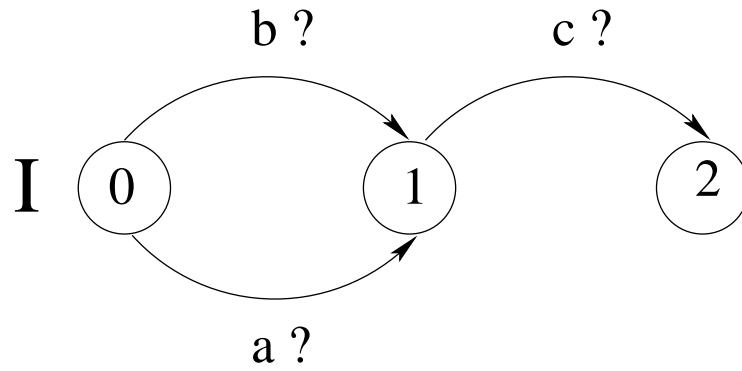
# Model Consistency

- Two MTSs  $M$  and  $N$  are consistent if and only if there exists an MTS  $P$  such that  $P$  is a common refinement of  $M$  and  $N$
- Consistency does not guarantee the existence of LCR
- An MTS  $P$  is minimal common refinement (MCR) of  $M$  and  $N$  if  $P$  is a common refinement of  $M$  and  $N$ ,  $\alpha P = (\alpha M \cup \alpha N)$ , and there is no MTS  $Q \neq P$  such that  $Q$  is a common refinement of  $M$  and  $N$  and  $Q @ \alpha P \preceq_o P$

# Example

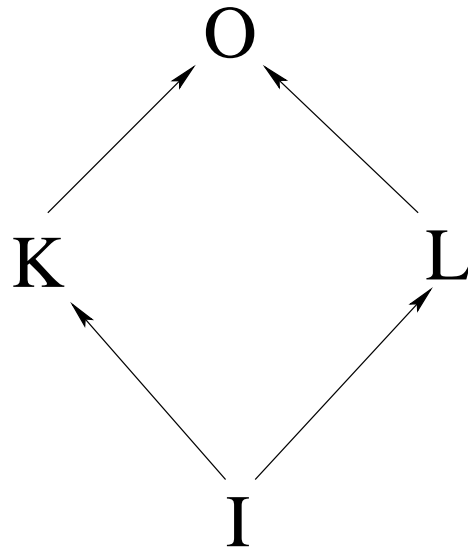
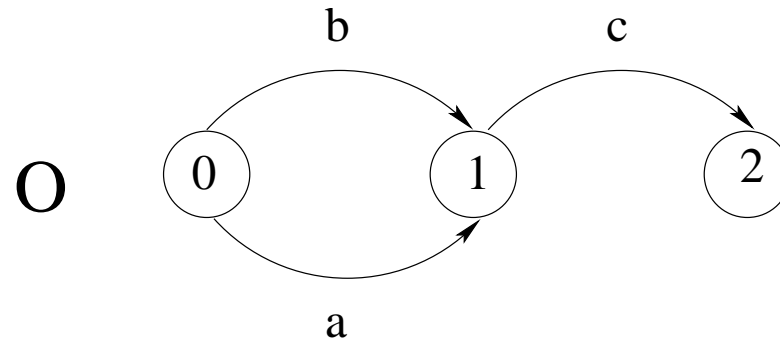


# Example

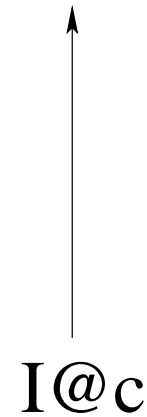




# Example Contd.



J, K@c, L@c, O@c

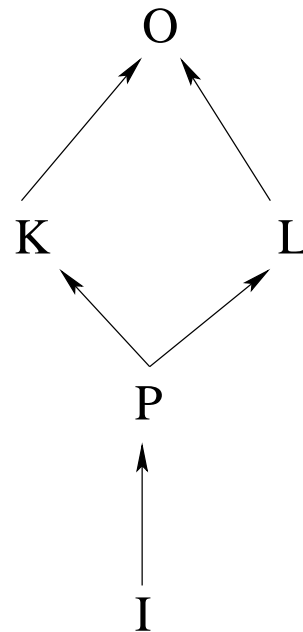
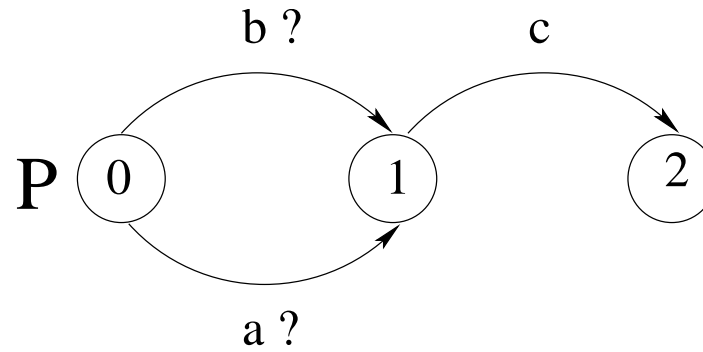


# Greatest Lower Bound

- Merging consistent models with no LCR will result in any one of the MCRs
- A better approach would be to find the greatest lower bound(glb) of all MCRs
- The user can then build one of the MCRs using this glb model
- glb is unique with respect to observational equivalence

- glb always exists
- glb itself might not be a common refinement of the models being merged
- Let  $M$  and  $N$  be consistent.  $Q$  is a lower bound of all MCRs if  $\alpha Q = (\alpha M \cup \alpha N)$  and for any MCR  $P$ , it holds that  $Q \preceq_o P$ .  $Q$  is a glb if for any other lower bound  $Q'$ , it holds that  $Q' \preceq_o Q$
- If  $P$  is a LCR, then  $P$  is also the glb of all MCRs of  $M$  and  $N$

# Example



# Algorithms

- Consistency checking between two partial models
- Constructing LCR if it exists
- Supporting construction of MCRs using glb
- $+_u$  Operator
  - Used for consistency checking
  - Gives a upper bound for all MCRs
- $+_l$  Operator
  - Gives a lower bound (approximate glb)
  - Used to construct the LCR or one of the MCRs

# $+_u$ Operator

- **TD**  $\forall l \notin \alpha N (M \xrightarrow[r]{l} M') \Rightarrow (M +_u N \xrightarrow[r]{l} M' +_u N)$
- **TM**  $\forall l \notin \tau (M \xrightarrow[r]{l} M') \wedge (N \xrightarrow[m]{l} N') \Rightarrow (M +_u N \xrightarrow[r]{l} M' +_u N')$
- **MD**  $\forall l \notin \alpha N (M \xrightarrow[m]{l} M') \Rightarrow (M +_u N \xrightarrow[r]{l} M' +_u N)$
- **TT**  $\forall l \notin \tau (M \xrightarrow[r]{l} M') \wedge (N \xrightarrow[r]{l} N') \Rightarrow (M +_u N \xrightarrow[r]{l} M' +_u N')$
- **MM**  $\forall l \notin \tau (M \xrightarrow[m]{l} M') \wedge (N \xrightarrow[m]{l} N') \Rightarrow (M +_u N \xrightarrow[m]{l} M' +_u N')$

# Disagreement states

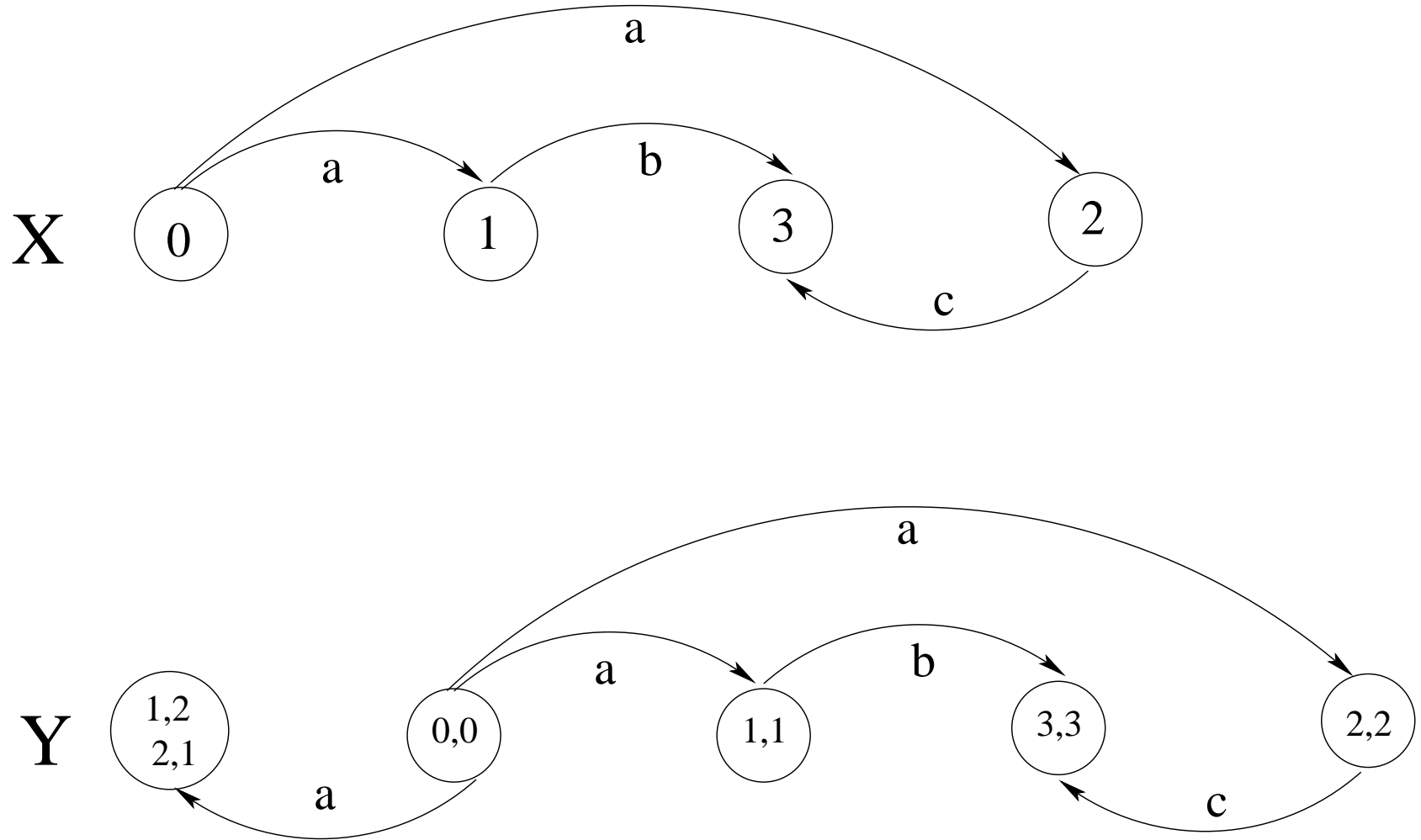
- $M = \langle S_M, L_M, \Delta_M^r, \Delta_M^p, s_{0_M} \rangle$
- $N = \langle S_N, L_N, \Delta_N^r, \Delta_N^p, s_{0_N} \rangle$
- $(m, n) \in (S_M \times S_N)$  is a disagreement state if  $\exists l \in (\alpha M \cap \alpha N)$  such that
- $M_m \xrightarrow[r]{l}$  and  $N_n \not\xrightarrow{l}$  or
- $M_m \not\xrightarrow{l}$  and  $N_n \xrightarrow[r]{l}$
- Consistent models ensure disagreement states can progress using unobservable actions

# Determinacy Condition

- $C = \langle S_M \times S_N, L_C, \Delta_C^r, \Delta_C^p, (s_{0_m}, s_{0_n}) \rangle$
- Determinacy condition holds if  $\forall (m, n) \in C$  and all  $l \in L_M \cap L_N$  it is not the case that  $M_m$  and  $N_n$  are non-deterministic on  $l$
- Consistent  $M$  and  $N$ ,  $(M +_u N)$  satisfying determinacy  $\Rightarrow$ 
  - $M +_u N$  is a common observational refinement
  - For every  $Q$  that is a MCR,  
 $Q @ \alpha (M +_u N) \preceq_o (M +_u N)$



# Example



# Consistency Checker

## Algorithm : Consistency check

- INPUT: MTSs  $M$  and  $N$
- OUTPUT: If  $M$  and  $N$  not consistent, return one of the disagreement states else return null

- Build  $M +_u N$  marking disagreement states
- For each marked state  $(m, n)$
- If  $N_n \not\rightarrow^l$
- If  $\forall \omega \in (Act_\tau \setminus \alpha M)^*$ ,  $N_n \not\rightarrow^{\omega l}$
- Return  $(m, n)$
- Else if  $M_m \not\rightarrow^l$  **\*\*Similar as above\*\***
- Else return null

# $+_l$ Operator

- **DM**  $\forall l \notin \alpha M (N \xrightarrow[l]{m} N') \Rightarrow ((M +_l N) \xrightarrow[l]{m} M +_l N')$
- **MD**  $\forall l \notin \alpha N (M \xrightarrow[l]{m} M') \Rightarrow ((M +_l N) \xrightarrow[l]{m} M' +_l N)$
- If  $M$  and  $N$  are consistent and  $(M +_l N)$  satisfies the determinacy condition, then for any MCR  $Q$  of  $M$  and  $N$ ,  $(M +_l N) \preceq_o Q @ \alpha(M +_l N)$
- $(M +_l N)$  approximates the glb of  $M$  and  $N$

# DM and MD rules

- To obtain exact glb, DM and MD rules should convert some maybe transitions into required transitions
- If all are converted we get  $M +_u N$
- If none are converted we get  $M +_l N$
- If DM and MD rules are never applied then  $+_u \equiv +_l$  and they produce LCR

# Elaboration

- Refinement of lower bound obtained using  $+_l$  into a MCR
- **Algorithm : Elaboration**
- INPUT: MTSs  $M$  and  $N$ ; consistent and satisfy determinacy
- OUTPUT: MTS  $P$  which is the required MCR/LCR

- Build  $P = M +_l N$  marking disagreement states
- For each marked state  $(m,n)$  if  $N \not\rightarrow^l$
- Build  $T = \{\omega \in (\alpha N \setminus \alpha M)^* : \exists N', (N_n \xRightarrow{\omega}_m N'_n) \wedge (N'_n \xRightarrow{l}_m N''_n)\}$
- User chooses  $\omega' \in T$  (If  $|T| = 1$  we get LCR)
- Replace maybe transitions with required ones;  
 $(M_m +_l N_n) \xRightarrow{\omega'}_r (M_m +_l N'_n)$
- Else if  $M_m \not\rightarrow^l$  \*\*similar as above\*\*

# Complexity Analysis

- $S_M$  and  $S_N$  are states of  $M$  and  $N$
- $T_M$  and  $T_N$  are transitions;  $T_i$  is  $O(S_i \times L_i)$
- Potential size of state space of common refinement is  $S = O(|S_M| \times |S_N|)$
- Consistency check is similar to weak bisimulation  $O(L \times S \times T)$
- Computing  $+_u$  and  $+_l$  does not increase this complexity
- Use BFS for computing  $T$  in the elaboration algorithm



# References

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- “Synthesis of Behavioral Models from Scenarios”, Sebastian Uchitel, Jeff Kramer and Jeff Magee, IEEE Transactions on Software Engineering, Feb 2003
- “From Scenarios to Timed Automata: Building Specifications from User Requirements”, Stephane Some, Rachida Dssouli and Jean Vaucher, Asia Pacific Software Engineering Conference, 1995