Propagating Euclidean Calibration on a Large Number of Cameras

Joao P. Barreto - GRASP Lab./University of Pennsylvania

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1 Theoretical BackGround

This section introduces the theoretical ideas that supporting the algorithms. Assume K cameras. If $\mathbf{X}$ is a point in the world the respective image on the i camera is $\mathbf{x_i}$ provided by

$$\lambda_i \mathbf{x_i} = \mathbf{P_i X}, i = 1 \ldots K \tag{1}$$

Matrix $\mathbf{P_i}$ is the 3×4 projection matrix of camera i which can be splited in the following manner

$$\mathbf{P_i} = [\mathbf{A_i} | \mathbf{a_i}] \tag{2}$$

Notice that $\mathbf{A_i}$ is a full rank 3×3 matrix and $\mathbf{a_i}$ is a 3×1 vector.

1.1 Reconstruction of 3D Points

Assume that a certain point $\mathbf{X}$ in the scene is visible in K_r views ($K_r > 1$) and both the image points and the calibration matrices are known. Multiplying both term of equation 1 by $\hat{\mathbf{x}}_\mathbf{i}$ (the corresponding skew symmetric matrix) yields

$$\hat{\mathbf{x}}_\mathbf{i} \mathbf{P_i X} = 0, i = 1 \ldots K_r \tag{3}$$

Considering now the K_r views comes that

$$\underbrace{\begin{bmatrix} \hat{\mathbf{x}}_{\mathbf{1}}\mathbf{P_1} \\ \hat{\mathbf{x}}_{\mathbf{2}}\mathbf{P_2} \\ \vdots \\ \hat{\mathbf{x}}_{\mathbf{K_r}}\mathbf{P_{K_r}} \end{bmatrix}}_{\mathbf{R}} \mathbf{X} = 0 \tag{4}$$

Since $\mathbf{X}$ must lie in the null space of the $3K_r \times 4$ matrix $\mathbf{R}$ we can compute it using SVD decomposition.

1.2 Computation of the Depth from K_d views

Consider that a certain point is visible in K_d views ($K_d > 1$) and that the corresponding projection matrices $\mathbf{P_i}, i = 1 \dots K_d$ are known. If the first camera is the reference camera comes from equations 1 and 2 that

$$\begin{aligned} & \lambda_1 \mathbf{x_1} && = \mathbf{A_1}\mathbf{X} + \mathbf{a_1} \\ \leftrightarrow \quad & \mathbf{X} && = \mathbf{A_1}^{-1}(\lambda_1 \mathbf{x_1} - \mathbf{a_1}) \end{aligned} \tag{5}$$

Replacing $\mathbf{X}$ in the projection equation of one of the other cameras yields

$$\begin{aligned} & \lambda_i \mathbf{x_i} && = \mathbf{A_i}\mathbf{X} + \mathbf{a_i} \\ \leftrightarrow \quad & \lambda_i \mathbf{x_i} && = \lambda_1 \mathbf{A_i}\mathbf{A_1}^{-1}\mathbf{x_1} + (\mathbf{a_i} - \mathbf{A_i}\mathbf{A_1}^{-1}\mathbf{a_1}) \end{aligned} \tag{6}$$

Equation 7 is derived by multiplying both members of equation 6 by $\hat{\mathbf{x}}_{\mathbf{i}}$. Notice that $\alpha_1 = \lambda_1^{-1}$.

$$\hat{\mathbf{x}}_{\mathbf{i}}\mathbf{A_i}\mathbf{A_1}^{-1}\mathbf{x_1} + \alpha_1 \hat{\mathbf{x}}_{\mathbf{i}}(\mathbf{a_i} - \mathbf{A_i}\mathbf{A_1}^{-1}\mathbf{a_1}) = 0 \tag{7}$$

Consider a certain point simultaneously viewed by K_d camera. Assuming camera 1 as the reference camera we can build for each point

$$\underbrace{\begin{bmatrix} \hat{\mathbf{x}}_{\mathbf{2}}\mathbf{A_2}\mathbf{A_1}^{-1}\mathbf{x_1} & \hat{\mathbf{x}}_{\mathbf{2}}(\mathbf{a_2} - \mathbf{A_2}\mathbf{A_1}^{-1}\mathbf{a_1}) \\ \hat{\mathbf{x}}_{\mathbf{3}}\mathbf{A_3}\mathbf{A_1}^{-1}\mathbf{x_1} & \hat{\mathbf{x}}_{\mathbf{3}}(\mathbf{a_3} - \mathbf{A_3}\mathbf{A_1}^{-1}\mathbf{a_1}) \\ \vdots & \vdots \\ \hat{\mathbf{x}}_{\mathbf{K_d}}\mathbf{A_{K_d}}\mathbf{A_1}^{-1}\mathbf{x_1} & \hat{\mathbf{x}}_{\mathbf{K_d}}(\mathbf{a_{K_d}} - \mathbf{A_{K_d}}\mathbf{A_1}^{-1}\mathbf{a_1}) \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} 1 \\ \alpha \end{bmatrix} = 0 \tag{8}$$

The depth of the point with respect to the first view can be easily computed by performing the SVD decomposition of matrix $\mathbf{M}$.

1.3 Computation of the projective matrices

The goal is to determine the projective matrix $\mathbf{P_i}$ of the i^{th} camera. We assume that there are N points which are simultaneously viewed by the reference camera 1 and the one we aim to calibrate. We know $\mathbf{P_1}$, the image points in both views ($\mathbf{x_i^j}$ and $\mathbf{x_1^j}$ with $j = 1 \ldots N$), and the corresponding depths α_1^j with respect to the reference frame. The minimum required number of correspondences is 11 ($N >= 11$).

Lets return to the result of equation 7. Consider $\mathbf{\Psi_i} = \mathbf{A_i}\mathbf{A_1}^{-1}$ and $\phi_\mathbf{i} = \mathbf{a_i} - \mathbf{A_i}\mathbf{A_1}^{-1}\mathbf{a_1}$. Write the 3×3 matrix $\mathbf{\Psi_i}$ as a 9×1 vector ψ_i. Given a pair of corresponding image points $(\mathbf{x_i^j}, \mathbf{x_1^j})$, we can rewrite the result of equation 7 in the form of equation 9 where $\circ$ denotes the Kronecker product.

$$\begin{bmatrix} \hat{\mathbf{x}}_\mathbf{i}^\mathbf{j} \circ \mathbf{x_1^j} & \hat{\mathbf{x}}_\mathbf{i}^\mathbf{j} \end{bmatrix} \begin{bmatrix} \psi_\mathbf{i} \\ \phi_\mathbf{i} \end{bmatrix} = 0 \tag{9}$$

Consider the N points we can build matrix $\mathbf{G_i}$ and establish the following relation

$$\underbrace{\begin{bmatrix} \hat{\mathbf{x}}_\mathbf{i}^\mathbf{1} \circ \mathbf{x_1^1} & \hat{\mathbf{x}}_\mathbf{i}^\mathbf{1} \\ \hat{\mathbf{x}}_\mathbf{i}^\mathbf{2} \circ \mathbf{x_1^2} & \hat{\mathbf{x}}_\mathbf{i}^\mathbf{2} \\ \vdots & \vdots \\ \hat{\mathbf{x}}_\mathbf{i}^\mathbf{N} \circ \mathbf{x_1^N} & \hat{\mathbf{x}}_\mathbf{i}^\mathbf{N} \end{bmatrix}}_{\mathbf{G_i}} \begin{bmatrix} \psi_\mathbf{i} \\ \phi_\mathbf{i} \end{bmatrix} = 0 \tag{10}$$

Once again vectors $\psi_\mathbf{i}$ and $\phi_\mathbf{i}$ can be estimated by doing the SV decomposition of $\mathbf{G_i}$ (see comments on the multiview routine about the normalization). Since both $\mathbf{A_1}$ and $\mathbf{a_1}$ are known yields

$$\begin{aligned} \mathbf{A_i} &= \mathbf{\Psi_i}\mathbf{A_1} \\ \mathbf{a_i} &= \phi_\mathbf{i} - \mathbf{\Psi_i}\mathbf{a_1} \end{aligned} \tag{11}$$