ICCV 2003 Course on Omnidirectional Vision

lecture of

Tomáš Pajdla

with contributions from

B. Mičušík, M. Menem, H. Bakstein, and D. Martinec

Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering
Czech Technical University in Prague
Karlovo nám. 13, 12135 Prague
pajdla@cmp.felk.cvut.cz

Available at: http://cmp.felk.cvut.cz/~pajdla/Pajdla-Omni-Vision-ICCV-2003/

Outline of the lecture

1. Estimation of multiple view geometry of central dioptric & catadioptric omnidirectional cameras

2. Non-central cameras, their models, and stereo geometries

Part 1.

Estimation of multiple-view geometry of

central

dioptric & catadioptric

omnidirectional cameras

Central Omnidirectional Cameras

Catadioptric | Dioptric

Perspective cam.
Hyperbolic mirror
$360^\circ \times 180^\circ$

Orthographic cam.
Parabolic mirror
$360^\circ \times 140^\circ$

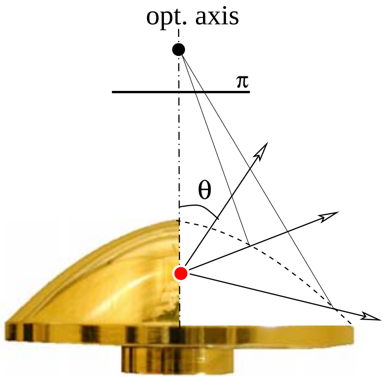
Nikon Coolpix
FC-E8 Lens
$360^\circ \times 183^\circ$

Canon EOS-1
Sigma Lens
$360^\circ \times 180^\circ$

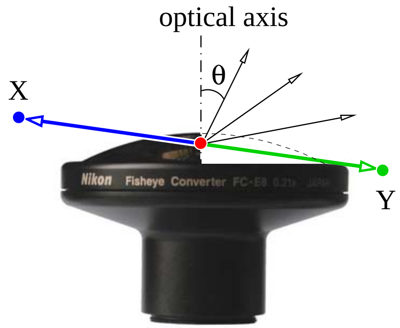
1.000$

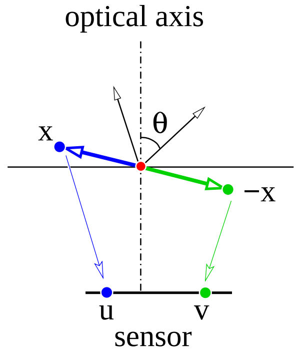
10.000$

Central

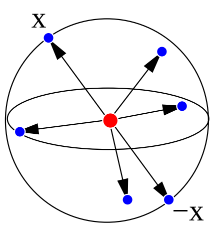


Omnidirectional





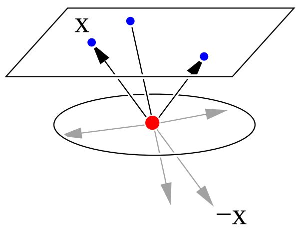
Spherical Model



Spherical model

$\mathbf{x}$, $-\mathbf{x}$ represent
two different image points

Rays are half-lines

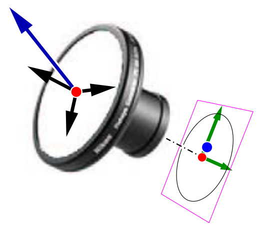


Perspective model

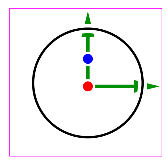
$\mathbf{x}$, $-\mathbf{x}$ represent
one image point

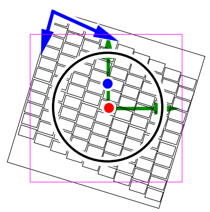
Image Formation

Camera

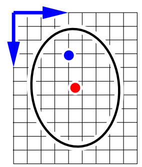


Sensor plane





Digital image



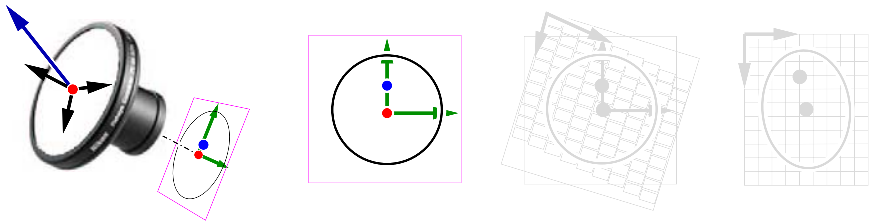
… From the camera to the sensor plane →

… From the sensor plane to the digital image →

Why two steps?

Scene coordinates — separated by non-linear projection from — image coordinates

From the camera to the sensor plane

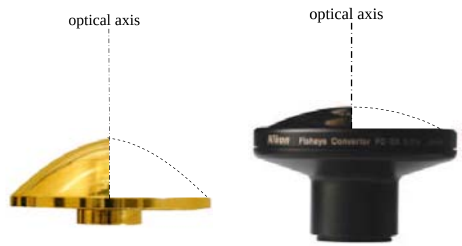


... From the camera to the sensor plane →

... From the sensor plane to the digital → image

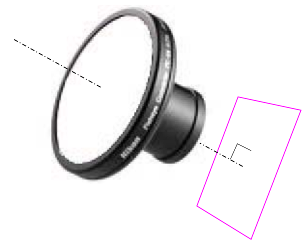
Sensor plane ⊥ Optical axis

1)



Mirrors & lenses are axially symmetric

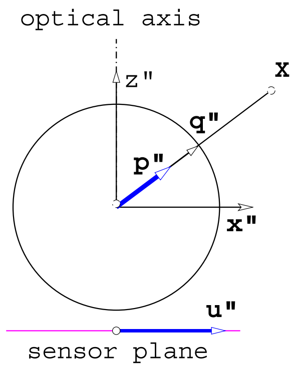
2)



Sensor plane ⊥ optical axis

From the camera to the sensor plane — general form

Spherical image point $\mathbf{q}'' \in S^3 = \left\{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| = 1\right\}$, represented by the directional vector $\mathbf{p}''$ of its projection ray, projects to a sensor plane ploint $\mathbf{u}''$ so that



$$\mathbf{p}'' = \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix}$$

where functions $f, h \colon \mathbb{R}^2 \to \mathbb{R}$ are rotationally symmetric, i.e. for every rotation $\mathtt{R}$ of the sensor plane plane

$$h(\mathtt{R}\,\mathbf{u}'') = h(\mathbf{u}'')$$
$$f(\mathtt{R}\,\mathbf{u}'') = f(\mathbf{u}'')$$

From the camera to the sensor plane — examples

Parabolic mirror | Hyperbolic mirror | Nikon FC-E8 Lens | Sigma Lens

$$\mathbf{p}'' = \begin{pmatrix} 1\mathbf{u}'' \\ \frac{a''^2 - \|\mathbf{u}''\|^2}{2a''} \end{pmatrix} \quad \begin{pmatrix} h(\mathbf{u}'')\,\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix} \quad \begin{pmatrix} 1\mathbf{u}'' \\ \frac{\|\mathbf{u}''\|}{\tan \frac{a''\|\mathbf{u}''\|}{1+b''\|\mathbf{u}''\|}} \end{pmatrix} \quad \begin{pmatrix} 1\mathbf{u}'' \\ \frac{\|\mathbf{u}''\|}{\tan\left(\frac{1}{b''}\operatorname{asin}\frac{b''\|\mathbf{u}''\|}{a''}\right)} \end{pmatrix}$$

↓

$$h(\mathbf{u}'') = \frac{b''^2\left(F''^2\sqrt{a''^2 + b''^2} + F''a''\sqrt{\|\mathbf{u}''\|^2 + F''^2}\right)}{F''^2 b''^2 - a''^2\|\mathbf{u}''\|^2}$$

$$f(\mathbf{u}'') = h(\mathbf{u}'')F'' - 2\sqrt{a''^2 + b''^2}$$

A generalization of the perspective & catadioptric projection

Perspective projection

$$\mathbf{p}'' = \begin{pmatrix} \mathbf{u}'' \\ 1 \end{pmatrix}$$

$$\mathbf{u}'' \longrightarrow \begin{pmatrix} 1\,\mathbf{u}'' \\ 1 \end{pmatrix}$$

Omnidirectional projection

$$\mathbf{p}'' = \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix}$$

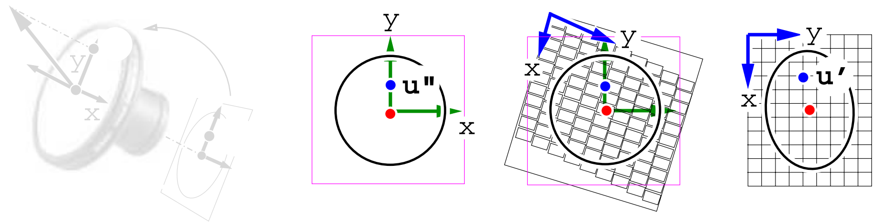
$$\mathbf{u}'' \longrightarrow \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix}$$

Catadioptric projection (Geyer & Daniilidis ECCV 2000)

$$h(\mathbf{u}'') = \frac{l(l+m) + \sqrt{\|\mathbf{u}''\|^2(1-l^2) + (l+m)^2}}{\|\mathbf{u}''\|^2 + (l+m)^2}$$

$$f(\mathbf{u}'') = h(\mathbf{u}'')\,(l+m) - l$$

From the sensor plane to the digital image



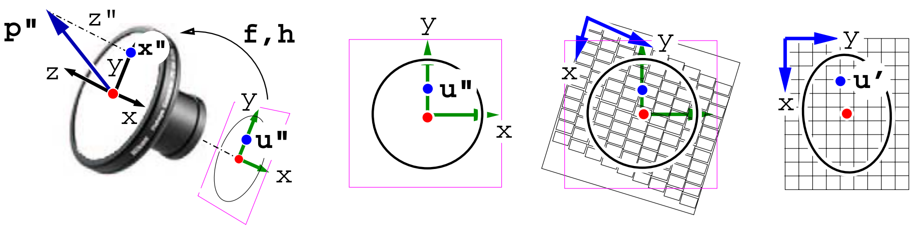
Sensor plane with Cartesian coord. s. | Digitized by skewed pixel grid | Digital image in computer

. . . Affine transformation →

$$\mathbf{u}'' = \mathtt{A}'\mathbf{u}' + \mathbf{t}'$$

$\mathtt{A}' \in \mathbb{R}^{2\times 2}$ regular, $\mathbf{t}' \in \mathbb{R}^2$

Complete image formation model



$$\exists\,\alpha'' > 0\colon \quad \tfrac{1}{\alpha''}\,\mathtt{P}\,\mathbf{X} = \mathbf{p}'' = \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix} = \begin{pmatrix} h(\mathtt{A}'\mathbf{u}' + \mathbf{t}')(\mathtt{A}'\mathbf{u}' + \mathbf{t}') \\ f(\mathtt{A}'\mathbf{u}' + \mathbf{t}') \end{pmatrix}$$

central projection — non-perspective optics — digitization

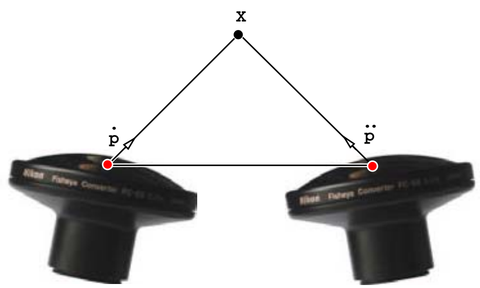
Parameters:

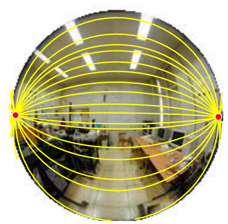
$\mathtt{P} \in \mathbb{R}^{3\times 4}$, rank $\mathtt{P} = 3$	. . .	projection matrix
$\mathtt{A}' \in \mathbb{R}^{2\times 2}$, rank $\mathtt{A}' = 2$	. . .	linear transformation
$\mathbf{t}' \in \mathbb{R}^2$	. . .	translation
$a'', b'', \ldots \in \mathbb{R}$	. . .	parameters of functions h, f

$f, h\colon \mathbb{R}^2 \to \mathbb{R}$:

$f, h(\mathtt{R}\mathbf{u}'') = f, h(\mathbf{u}'')$ for a rotation $\mathtt{R}$	. . .	rotational symmetry

Epipolar geometry





Epipolar constraint holds for every central camera

$$\dot{\mathbf{p}}^\top \mathtt{F}\, \ddot{\mathbf{p}} = 0$$

Epipolar curves are . . .

1. conics for central catadioptric cameras (Svoboda & Pajdla IJCV 2002)
2. non-conics for wide-angle dioptric cameras (Micusik & Pajdla CVPR 2003)

Calibration by epipolar geometry estimation

Let $C = \{\dot{\mathbf{u}} \leftrightarrow \ddot{\mathbf{u}}\}$ be a set of corresponding points in two omnidirectional images.

Find image formation parameters $\dot{\mathtt{A}}, \dot{\mathbf{t}}, a, b, \ldots$ and $\ddot{\mathtt{A}}, \ddot{\mathbf{t}}, a, b, \ldots$ so that there exists $\mathtt{F} \in \mathbb{R}^{3\times 3}$, rank $\mathtt{F} = 2$ such that for every correspondence $\dot{\mathbf{u}} \leftrightarrow \ddot{\mathbf{u}} \in C$ holds

$$\dot{\mathbf{p}}^\top \mathtt{F}\, \ddot{\mathbf{p}} = 0$$

for

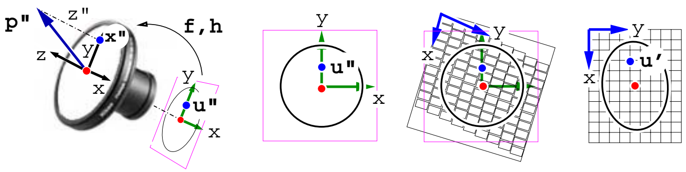
$$\dot{\mathbf{p}} = \begin{pmatrix} h(\dot{\mathtt{A}}\dot{\mathbf{u}}' + \dot{\mathbf{t}})(\dot{\mathtt{A}}\dot{\mathbf{u}}' + \dot{\mathbf{t}}) \\ f(\dot{\mathtt{A}}\dot{\mathbf{u}}' + \dot{\mathbf{t}}) \end{pmatrix} \qquad \ddot{\mathbf{p}} = \begin{pmatrix} h(\ddot{\mathtt{A}}\ddot{\mathbf{u}}' + \ddot{\mathbf{t}})(\ddot{\mathtt{A}}\ddot{\mathbf{u}}' + \ddot{\mathbf{t}}) \\ f(\ddot{\mathtt{A}}\ddot{\mathbf{u}}' + \ddot{\mathbf{t}}) \end{pmatrix}$$

Remember: $\mathtt{A}, \mathbf{t}, a, b, \ldots \quad \underset{\text{(in general)}}{\neq} \quad \mathtt{A}', \mathbf{t}', a'', b'', \ldots$

$\ldots\ \mathtt{A}', \mathbf{t}', a'', b'', \ldots$ cannot be often recovered.

(Recall that perpspective cameras also cannot be fully calibrated from epipolar geometry)

Two steps of the calibration



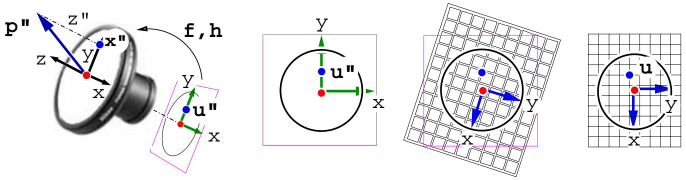
$\cdots\ \mathbf{u}'' = \mathtt{A}'\mathbf{u}' + \mathbf{t}' \rightarrow$

Step 1.

Image Coord. s. Calibration

$\mathbf{u} = \mathtt{A}_C\mathbf{u}' + \mathbf{t}_C$

$\mathtt{A}_C = \frac{1}{\rho}\mathtt{R}^{-1}\mathtt{A}',\ \mathbf{t}_C = \mathbf{t}',\ \rho > 0$



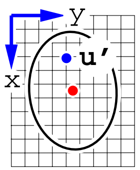
Metrically calibrated camera

$\cdots\ \mathbf{u}'' = \rho\,\mathtt{R}\mathbf{u} \rightarrow$

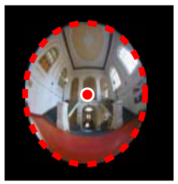
Step 2.

Calibration of non-linear f & h by Epipolar geometry estimation

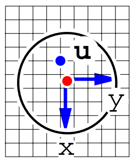
Step 1. – Calibration of image coordinate system



1. Complete circular field of view
   ⇒ complete ellipse in the image
2. Black background



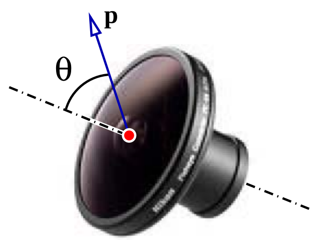
1. Detect contour
2. Fit ellipse
3. Find center



1. Move the origin to the center
2. Transform the ellipse to a circle

Step 2. – Calibration of non-linear f & h

i.e. . . . from an image point to its 3D ray

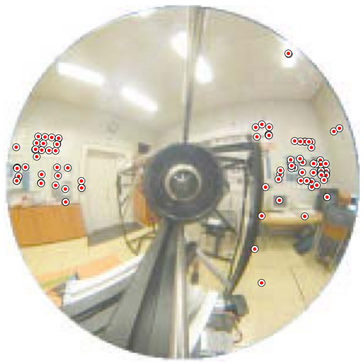


θ . . . angle w.r.t. the optical axis

$\|\mathbf{u}\|$ . . . image point radius

Para-catadioptric camera

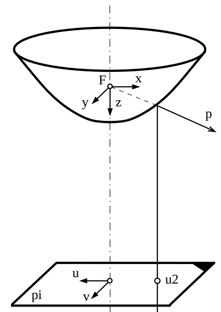
Camera



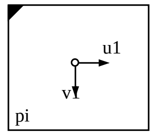
Points

Rays

Para-catadioptric camera - model



Coordinate system of the para-catadioptric camera. The origin is located in F.



The coordinate system in the calibrated image.

$$\mathbf{p} = \begin{pmatrix} -u \\ v \\ \frac{a^2-r^2}{2a} \end{pmatrix} \simeq \begin{pmatrix} -2\,a\,u \\ 2\,a\,v \\ a^2 - r^2 \end{pmatrix}$$

$$r = \sqrt{u^2+v^2}$$

Calibration based on epipolar geometry

$$\dot{\mathbf{p}}^\top \mathtt{F}\,\ddot{\mathbf{p}} = 0$$

$$\begin{pmatrix} -\dot{u} & \dot{v} & \frac{a^2-\dot{r}^2}{2a} \end{pmatrix} \mathtt{F} \begin{pmatrix} -\ddot{u} \\ \ddot{v} \\ \frac{a^2-\ddot{r}^2}{2a} \end{pmatrix} = 0$$

Denote $\mathbf{d} = \begin{pmatrix} f_1 & \dots & f_9 \end{pmatrix}^\top, \quad \mathtt{F} = \begin{pmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{pmatrix}$

Gather the point coordinates and radii into five design matrices $\longrightarrow$ a quartic (degree 4) equation in parameter a and linear in $\mathbf{d}$

$$\left(\mathtt{D}_1 + a\mathtt{D}_2 + a^2\mathtt{D}_3 + a^3\mathtt{D}_4 + a^4\mathtt{D}_5\right)\mathbf{d} = 0$$

$\mathtt{D}_1, \dots, \mathtt{D}_5 \in \mathbb{R}^{9\times 9}$ for 9 correspondences

Calibration based on epipolar geometry

$$\left(\mathtt{D}_1 + a\mathtt{D}_2 + a^2\mathtt{D}_3 + a^3\mathtt{D}_4 + a^4\mathtt{D}_5\right)\mathbf{d} = 0,$$

is known as the Polynomial Eigenvalue Problem (PEP) (Bai et al 2000) (`polyeig` in Matlab)

1. Generalization of (Fitzgibbon CVPR 2001) to omnidirectional cameras
2. Solution for 9 correspondences → RANSAC can be used

Algorithm

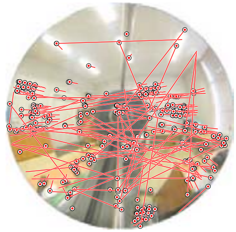
Algorithm for computing 3D rays and an essential matrix $\mathtt{F}$.

1. Find the ellipse corresponding to the view field of the camera. Transform the image so that the ellipse becomes a circle. Establish 9 point correspondences $\{\dot{\mathbf{u}} \leftrightarrow \ddot{\mathbf{u}}\}$ between two images.
2. Create matrices $\mathtt{D}_{1\ldots5} \in \mathbb{R}^{9\times9}$ and solve PEP. Use Matlab: $[\mathtt{H}\ \mathbf{a}] = \mathtt{polyeig}(\mathtt{D}_1, \mathtt{D}_2, \mathtt{D}_3, \mathtt{D}_4, \mathtt{D}_5)$, $\mathtt{H}$ is a 9×36 matrix with columns $\mathbf{d}$, $\mathbf{a}$ is a 36×1 vector with elements a.
3. Choose only real positive finite $a \neq 0$ (other solutions seem never be correct), 1–3 solutions remain. For every a there is a corresponding essential matrix $\mathtt{F}$.
4. Compute the angular error for all pairs $\{a \leftrightarrow \mathtt{F}\}$ as a sum of errors for all correspondences. The pair with the minimum error is the solution and a, and the essential matrix $\mathtt{F}$ are obtained.

For integrating the algorithm into the RANSAC, 9 points are selected from whole set of automatically detected correspondences and steps 1–4 are repeated till the model captured the highest number of matches is found.

Finding correspondences

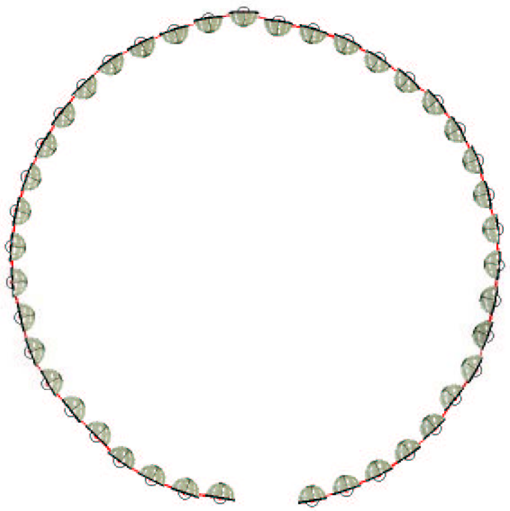

Pair of images with regions detected by Matas & Chum & Urban & Pajdla BMVC 2002



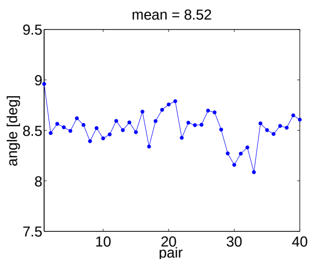
Tentative correspondences using similarity (Matas et al BMVC 2002) (many outliers)

Inliers satisfying epipolar geometry of para-catadioptric cameras (Micusik & Pajdla TR-18 2003)

Estimated camera trajectory



Reconstructed camera positions

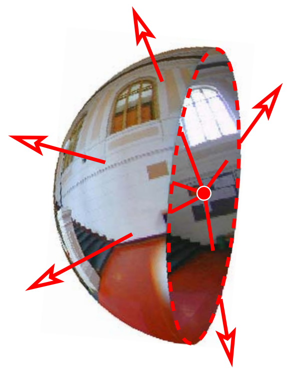
Estimated rotation angles

Dioptric camera

Camera

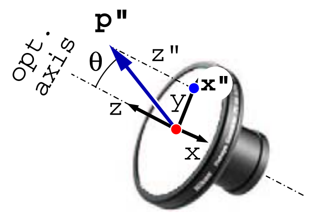
Image

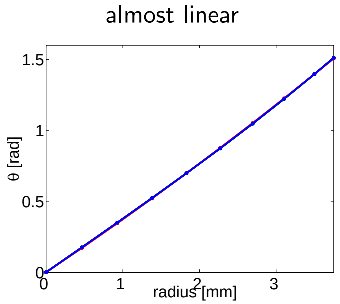
$360° \times 183°$



Rays

Dioptric cameras — models

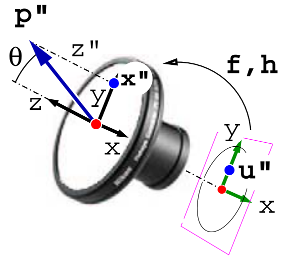




Nikon FC-E8 $\quad \theta = \dfrac{a'''\|\mathbf{x}''\|}{1 + b'''\|\mathbf{x}''\|^2}$

Sigma $\quad \theta = \dfrac{1}{b'''}\operatorname{asin}\dfrac{b'''\|\mathbf{x}''\|}{a'''}$

Dioptric cameras — models

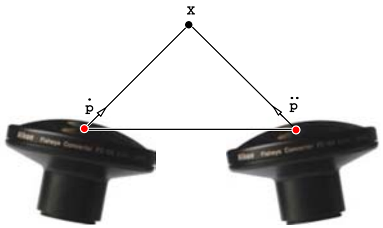


. . . Nikon FC-E8

$$\mathbf{p}'' = \begin{pmatrix} \mathbf{x}'' \\ z'' \end{pmatrix} = \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix} = \begin{pmatrix} 1\,\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix} = \begin{pmatrix} \mathbf{u}'' \\ \frac{\|\mathbf{u}''\|}{\tan\theta} \end{pmatrix} = \left\langle \begin{array}{l} \begin{pmatrix} \mathbf{u}'' \\ \frac{\|\mathbf{u}''\|}{\tan \frac{a''\|\mathbf{u}''\|}{1+b''\|\mathbf{u}''\|^2}} \end{pmatrix} \\ \begin{pmatrix} \mathbf{u}'' \\ \frac{\|\mathbf{u}''\|}{\tan\left(\frac{1}{b''}\operatorname{asin}\frac{b''\|\mathbf{u}''\|}{a''}\right)} \end{pmatrix} \end{array} \right.$$

. . . Sigma

Linearization



$$\dot{\mathbf{p}}^\top \mathtt{F}\, \ddot{\mathbf{p}} = 0$$

$$\begin{pmatrix} \dot{\mathbf{u}} \\ \frac{\|\dot{\mathbf{u}}\|}{\tan\theta} \end{pmatrix}^\top \mathtt{F} \begin{pmatrix} \ddot{\mathbf{u}} \\ \frac{\|\ddot{\mathbf{u}}\|}{\tan\theta} \end{pmatrix} = 0$$

does not lead to a simple (Polynomial Eigenvalue) Problem for

$$f(\|\mathbf{u}\|, a, b, \ldots) = \frac{\|\mathbf{u}\|}{\tan\theta(a, b, \ldots)}$$

is too much non-linear

$\Downarrow$

Linearization

Linearization

$$f(\|\mathbf{u}\|, a, b, \ldots) = \frac{\|\mathbf{u}\|}{\tan\theta(a, b, \ldots)}$$

$$\begin{aligned} \tilde{f}(\|\mathbf{u}\|, a, b, \ldots) &= f(\|\mathbf{u}\|, a_0, b_0, \ldots) \\ &+ f_a(\|\mathbf{u}\|, a_0, b_0, \ldots)(a - a_0) + f_b(\|\mathbf{u}\|, a_0, b_0, \ldots)(b - b_0) + \cdots \end{aligned}$$

$$\begin{aligned} \mathbf{p} &= \left[\begin{pmatrix} \mathbf{u} \\ f(.) - a_0 f_a(.) - b_0 f_b(.) + a f_a(.) + b f_b(.) \end{pmatrix}\right] \\ &= \left[\begin{pmatrix} \mathbf{u} \\ w \end{pmatrix} + a\begin{pmatrix} \mathbf{0} \\ s \end{pmatrix} + b\begin{pmatrix} \mathbf{0} \\ t \end{pmatrix}\right] \\ &= \mathbf{w} + a\mathbf{s} + b\mathbf{t} \end{aligned}$$

See Micusik & Pajdla CVPR 2003 for details. (derivatives, how to linearize)

Back to Polynomial Eigenvalue Problem (for Nikon FC-E8)

$$\dot{\mathbf{p}}^\top \mathtt{F}\, \ddot{\mathbf{p}} = 0$$

$$(\dot{\mathbf{w}} + a\,\dot{\mathbf{s}} + b\,\dot{\mathbf{t}})^\top \mathtt{F}\, (\ddot{\mathbf{w}} + a\,\ddot{\mathbf{s}} + b\,\ddot{\mathbf{t}}) = 0$$

Denote $\mathtt{F} = \begin{pmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{pmatrix}$

Gather the point coordinates and radii into three design matrices $\longrightarrow$ a quadratic equation in parameter a, b and linear in $\mathbf{d}(\mathtt{F}, b)$

$$(\mathtt{D}_1 + a\mathtt{D}_2 + a^2\mathtt{D}_3)\,\mathbf{d} = 0$$

. . . Quadratic Eigenvalue Problem (QEP) (Bai et al 2000) (`polyeig` in Matlab)

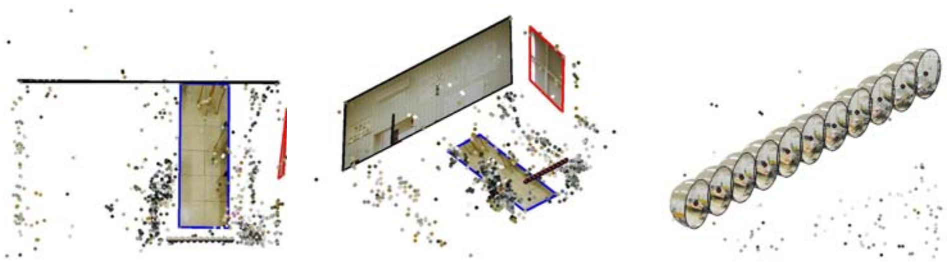
See Micusik & Pajdla CVPR 2003 for details.

Algorithm

Algorithm for computing 3D rays and an essential matrix is an extension of the algorithm for para-catadioptric camera by the linearization.

See Micusik & Pajdla SCIA 2003 for details.

3D Metric Reconstruction - I



Images → Calibration form EG's → Projective Factorization (Martinec & Pajdla ECCV 2002), see details in Micusik & Martinec & Pajdla TR-20 2003.

3D Metric Reconstruction - II

Images → Calibration form EG's → Projective Factorization (Martinec & Pajdla ECCV 2002), see details in Micusik & Martinec & Pajdla TR-20 2003.

3D Metric Reconstruction - II - video

Conclusions

1. Multiple view geometry of perspective cameras extended to omnidirectional cameras

$$\mathbf{p}'' = \begin{pmatrix} \mathbf{u}'' \\ 1 \end{pmatrix} \quad \longrightarrow \quad \mathbf{p}'' = \begin{pmatrix} h(\mathbf{u}'')\mathbf{u}'' \\ f(\mathbf{u}'') \end{pmatrix}$$

Perspective projection — Omnidirectional projection

2. Para-catadioptric camera → Polynomial Eigenvalue Problem
3. Other cameras → linearization → Polynomial Eigenvalue Problem
4. Complexity given by the number of parameters of the model rather than by the form of h, f.
5. Non-iterative solution → RANSAC (i.e. PEP is iterative but converges very fast).

Part 2.

Non-central cameras

models

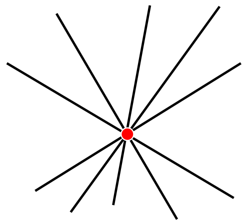
&

stereo geometries

Non-central cameras

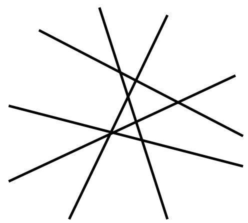
Space is projected to images along more general arrangements of lines called non-central cameras

Central camera



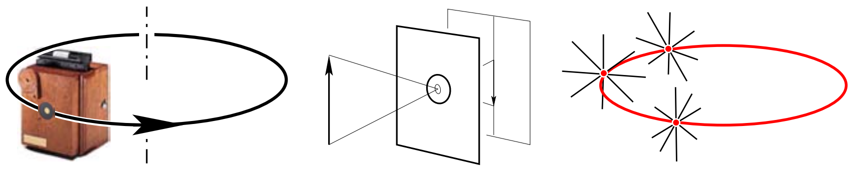
a set of rays
incident with one point

Non-central camera



(just) a set of rays

Example: Circular panoramas

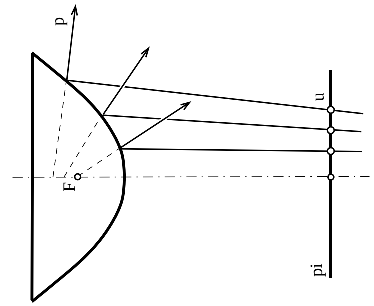


T.Pajdla, H.Bakstein, D.Večerka, 'Office 111' 2003

Advantages: large view field, higher precision, interesting . . .

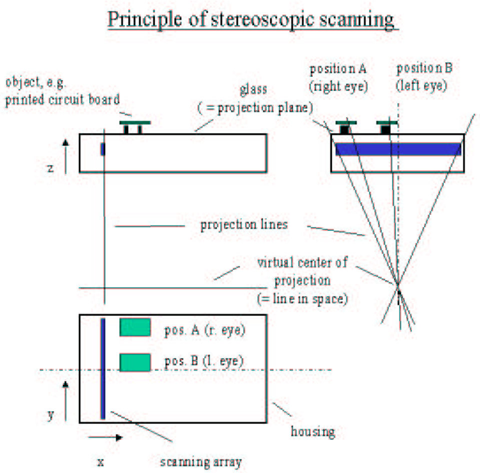
Example: Non-central catadioptric cameras

Real para-catadioptric camera

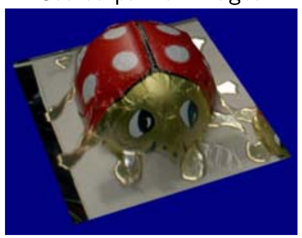


Flat-bed scanners are non-central cameras

. . . they can be used to do 3D reconstruction



Stereo pair of images



Reconstruction
(by Soft Imaging System GmbH.
http://www.soft-imaging.de)

Courtesy of Richard Schubert (http://www.stereoscopicscanning.de/)

Some history of non-central cameras

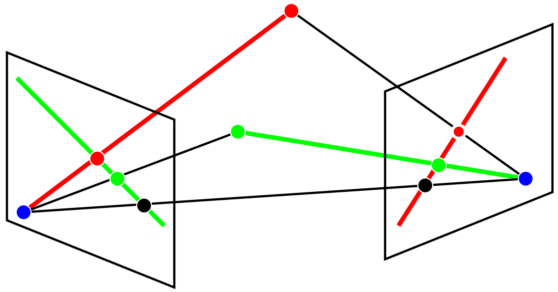
1843 Joseph Puchberger patented the 'slit camera' (similar to pushbroom camera).

1990 – ... Non-central cameras used in mosaicing (Ishiguro et. al 1992, Peleg et. al 1999, Shum et. al 1999, Huang et. al 2000, Nayar & Karmarkar 2000), reconstruction (Gupta & Hartley 1997), visualization (McMillan et. al 1995, Gortler et. al 1996, Levoy et. al 1996, Rademacher et. al 1998, Weinshall et. al 2002) ...

2001 T. Pajdla (Pajdla CVWW 2001) and S. Seitz (Seitz ICCV 2001) discoverd the generalization of epipolar planes to epipolar quadrics.

2001 – ... Non-central camera models developed camera models (Grossberg & Nayar ICCV 2001, Swaminathan et al ICCV 2001, Pless CVPR 2003, Neumann et al CVPR 2003, Micusik & Pajdla TR-19 2003), some stereo geometries analyzed (Pajdla IJCV 2001, Seitz & Kim IJCV 2001, Feldman et al ICCV 2003), . . .

Epipolar lines in central images are ‘independent’



Every point in space that projects on an epipolar line in the left image projects on the corresponding epipolar line in the right image

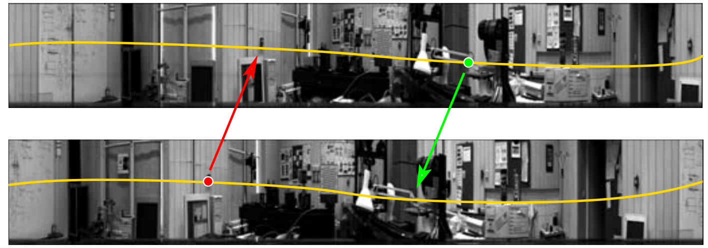
1. Search for correspondences can be done along epipolar lines ⟶ constraints
2. Each epipolar line is solved almost (epipoles) independently ⟶ easier search

Non-central cameras $\stackrel{?}{\longrightarrow}$ stereo geometry

Epipolar lines ⟶ stereo correspondence curves

left image

right image



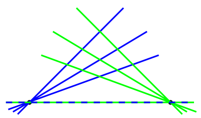
Every point in space that projects on the curve in the left image projects on the curve in the right image

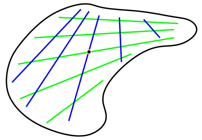
Stereo correspondence curves ⟹ stereo reconstruction with non-central cameras similar to stereo reconstruction with central cameras

Stereo geometry of non-central cameras

to have stereo correspondence curves

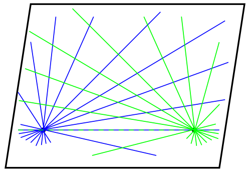
Epipolar planes ⟶ Stereo correspondence surfaces



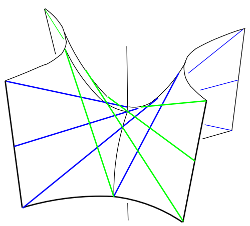


Result: (Pajdla CVWW 2001, Seitz ICCV 2001)

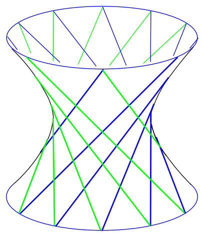
Interesting stereo correspondence surfaces are double ruled quadrics (all)



epipolar plane



hyperbolic paraboloid



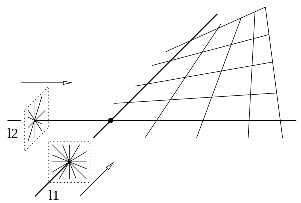
hyperboloid of one sheet

There are many stereo geometries

Double ruled quadrics can be arranged in space in many different ways

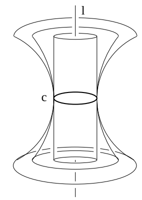
Examples

Pushbroom camera (Gupta & Hartley 1997)



two intersecting lines

Stereo panorama (Shum et. al 1999, Nayar & Karmarkar 2000)

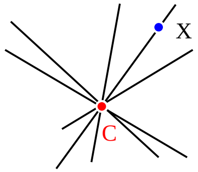


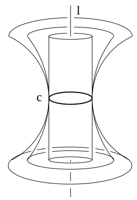
circle

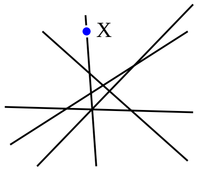
the situation can be somewhat complicated in general ⟶ current research (epilinear geometries, example)

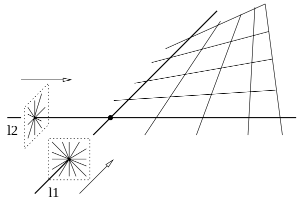
Hierarchy of cameras

central camera	all other cameras	?
all rays intersect at $\mathbf{C}$	$\longrightarrow$ some rays intersect $\longleftarrow$	no rays intersect



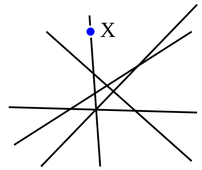






$\vdots$

Oblique camera



Definition

An oblique camera is a collection of lines such that every point in the projective space is contained in exactly one line.

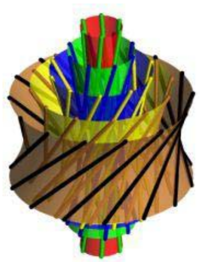
Observation Rays of an oblique camera do not intersect.

?

Do oblique cameras exist

Oblique cameras exist – an example

picture by Rolf Riesinger



picture by Hans Havlicek

A set of lines generated by the linear mapping σ (more)

point X *line*$[X\ \sigma(X)]$

$$span \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \longrightarrow span \begin{pmatrix} x & -y \\ y & x \\ z & w \\ w & -z \end{pmatrix}$$

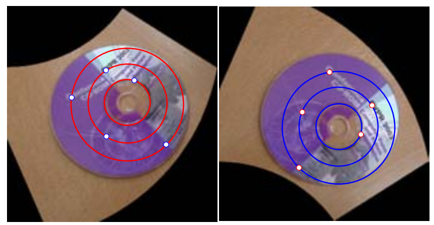
Lines are reguli of pairwise non-intersecting rotational hyperboloids

$$\mathbf{X}^T \begin{pmatrix} s & & & \\ & s & & \\ & & s-1 & \\ & & & s-1 \end{pmatrix} \mathbf{X} = 0,\ s \in [0,1]$$

Remark: OC are called spreads & wild spreads (not cospreads) exist!

Stereo geometry of oblique cameras

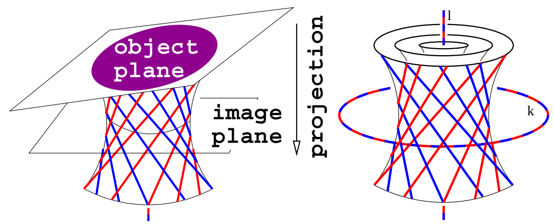
CD on a general plane in 3D seen by an Oblique Camera



Circular search curves do not intersect

Remarks:

Non-intersecting rotational hyperboloids



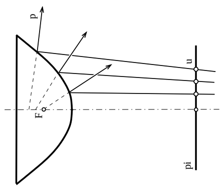
The best geometry: independent search curves

Oblique cameras can be realized

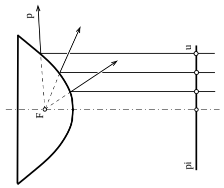
Non-central catadioptric cameras

Real para-catadioptric camera - calibration & reconstruction

real

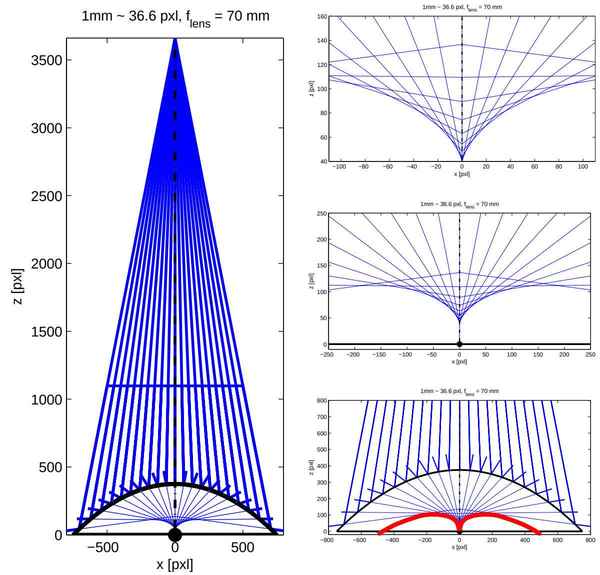


ideal



Non-central projection $\rightarrow$ Trajectory start $\neq$ Trajectory end

Rays are tangent to a caustic

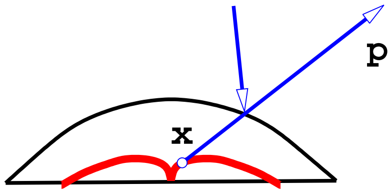


Rays reflected by the mirror are tangent to a caustic surface.

Ray = point $\mathbf{x}$ + direction vector $\mathbf{p}$

Camera model is a mapping from

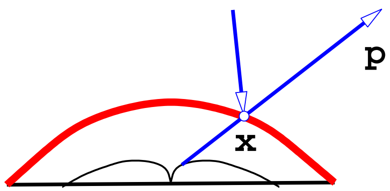
$\mathbf{u} \rightarrow$ rays $(\mathbf{x}, \mathbf{p})$



Point $\mathbf{x}$
is on the caustic.
(Grossberg & Nayar ICCV 2001
Swaminathan et al ICCV 2001)

Geometric, Radiometric, Photometric

Must be computed

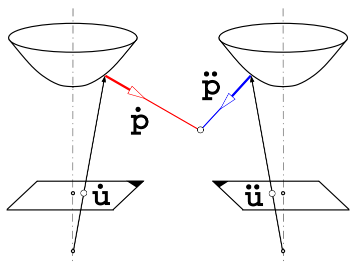


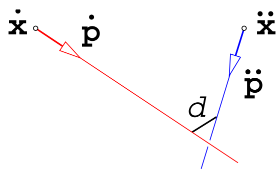
Point $\mathbf{x}$
is on the mirror
(Micusik & Pajdla TR-19 2003)

Geometric

Available

Calibration form a Stereo geometry





The corresponding rays $(\dot{\mathbf{x}}_w, \dot{\mathbf{p}}_w)$, $(\ddot{\mathbf{x}}_w, \ddot{\mathbf{p}}_w)$ intersect.

Tune parameters (bundle adjustment) of cameras, i.e.

$\dot{\mathtt{K}}, \dot{\mathtt{R}}_C, \dot{\mathbf{t}}_C, \ddot{a}$ and $\ddot{\mathtt{K}}, \ddot{\mathtt{R}}_C, \ddot{\mathbf{t}}_C, \dot{a}, \ddot{\mathtt{R}}_m, \ddot{\mathbf{t}}_m$

so that the mean distance

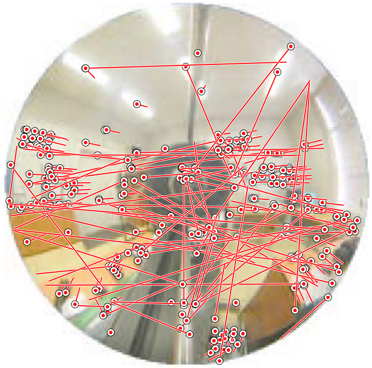
$$\operatorname*{mean}_{\mathcal{C}} d\left[(\dot{\mathbf{x}}_w, \dot{\mathbf{p}}_w), (\ddot{\mathbf{x}}_w, \ddot{\mathbf{p}}_w)\right]$$

between all the corresponding rays $\mathcal{C}$ is minimized.

Camera

Real para-catadioptric camera

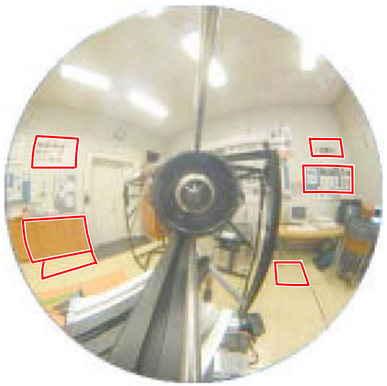
Correspondences



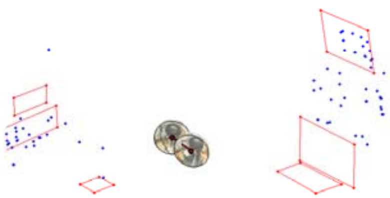
Tentative correspondences using similarity (Matas et al BMVC 2002) (many outliers)

Inliers satisfying epipolar geometry of central para-catadioptric camera model (Micusik & Pajdla TR-18 2003)

Calibration & Reconstruction



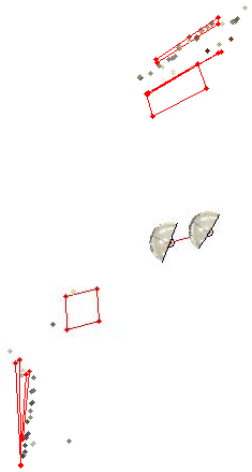
Marked polygons



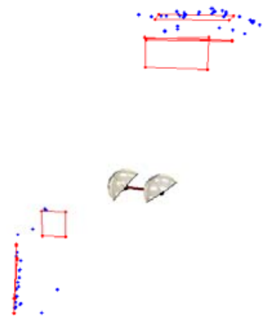
3D reconstruction

1) Non-central model initialized by the central one
2) Stereo geometry optimized with non-central model

Non-central vs. Central model



Central model (angles are wrong)

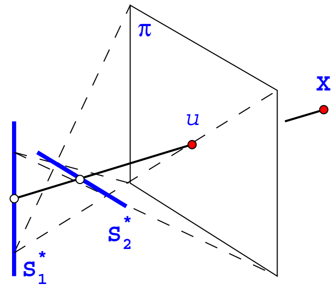


Non-central model (angles are correct)

were used to reconstruct the scene

Crossed-Slits (X-Slits) projection

defined by two lines (slits) through which all projection rays must pass (generalization of Pushbroom cameras) (Weinshall et al ECCV 2002, Feldman et al ICCV 2003)



Quadratic mapping: $\mathbb{P}^3 \to \mathbb{P}^2$

$$u = \begin{pmatrix} x \\ y \\ w \end{pmatrix} \propto \begin{pmatrix} \mathbf{X}^\top \mathtt{S}_1^* \mathtt{Q}_1 \mathtt{S}_2^* \mathbf{X} \\ \mathbf{X}^\top \mathtt{S}_1^* \mathtt{Q}_2 \mathtt{S}_2^* \mathbf{X} \\ \mathbf{X}^\top \mathtt{S}_1^* \mathtt{Q}_3 \mathtt{S}_2^* \mathbf{X} \end{pmatrix} \equiv \begin{pmatrix} \mathbf{X}^\top \mathtt{T}_1 \mathbf{X} \\ \mathbf{X}^\top \mathtt{T}_2 \mathbf{X} \\ \mathbf{X}^\top \mathtt{T}_3 \mathbf{X} \end{pmatrix}$$

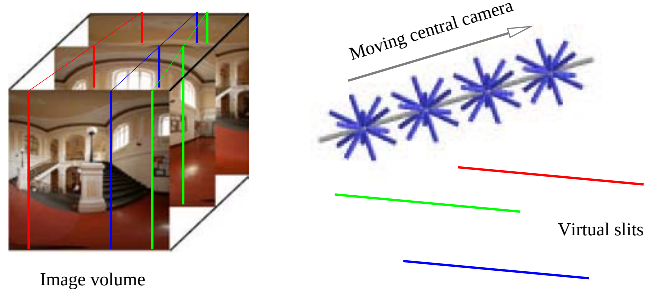
Dual Plücker matrices $\mathtt{S}_1^*$, $\mathtt{S}_2^*$ are defined by the slits.

Plücker matrices $\mathtt{Q}_1$, $\mathtt{Q}_2$, $\mathtt{Q}_3$ are defined by the image plane π.

Plücker matrices $\mathtt{Q}_i \propto \mathbf{x}_i \mathbf{y}_i^\top - \mathbf{y}_i \mathbf{x}_i^\top$, where $\mathbf{x}_i, \mathbf{y}_i$ are any 2 points that line on in $\mathbf{l}_i$.

Dual Plücker matrices $\mathtt{S}_i^* \propto \mathbf{u}_i \mathbf{v}_i^\top - \mathbf{v}_i \mathbf{u}_i^\top$, where $\mathbf{u}_i, \mathbf{v}_i$ are any 2 distinct planes that intersect in $\mathbf{l}_i$.

X-Slits cameras by sampling Image volumes



One sampling function → ones slit

X-Slits Image

X-Slits "Fundamental matrix" F

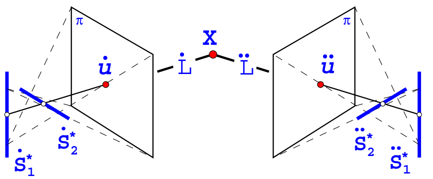


image point → Plücker matrix of its ray

$\dot{u} \to \dot{\mathtt{L}}(\dot{u}) = [\dot{l}_{ij}]$

$\ddot{u} \to \ddot{\mathtt{L}}(\ddot{u}) = [\ddot{l}_{kl}]$

$$\dot{\mathtt{L}} \text{ intersects } \ddot{\mathtt{L}} \quad \Leftrightarrow \quad \dot{l}_{12}\ddot{l}_{34} + \dot{l}_{34}\ddot{l}_{12} + \dot{l}_{13}\ddot{l}_{42} + \dot{l}_{42}\ddot{l}_{13} + \dot{l}_{14}\ddot{l}_{23} + \dot{l}_{23}\ddot{l}_{14} = 0$$

$$v(\ddot{u})^\top \mathtt{F}\, v(\dot{u}) = 0$$

using the Veronese mapping v: $(u_1, u_2, u_3)^\top \to (u_1^2,\, u_1u_2,\, u_1u_3,\, u_2^2,\, u_2u_3,\, u_3^2)^\top$

1. Maps points in one image to conics in the other image
2. rank $\mathtt{F} = 4$
3. $\mathtt{F}$ exists even if there are no 'epipolar quadrics'

X-Slits stereo geometry

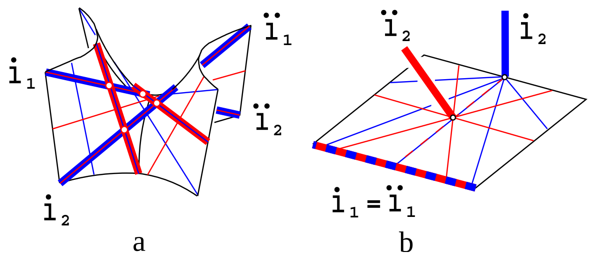
Observation: A general pair of X-Slits cameras does not have stereo correspondence surfaces.

Theorem (Feldman & Pajdla & Weinshall ICCV 2003):

A pair of X-Slits cameras posseses epipolar quadrics iff

(a) slits intersect in four pairwise disjoint points, or

(b) the cameras share a slit (correspondence curves are "image rows").



X-Slits stereo geometry — example

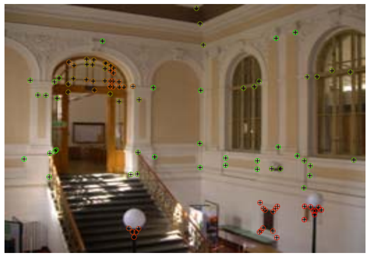
No correspondence curves . . . search curves are conics (hyperbolas)

More at ICCV 2003: Feldman & Pajdla & Weinshall ICCV 2003

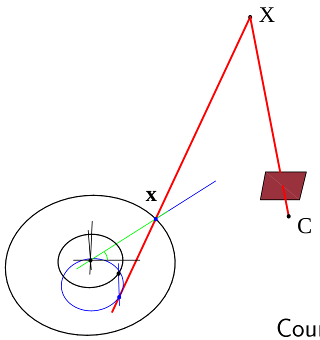
Applications

Reconstruction from Circular panorama & Perspective image

Circular panorama

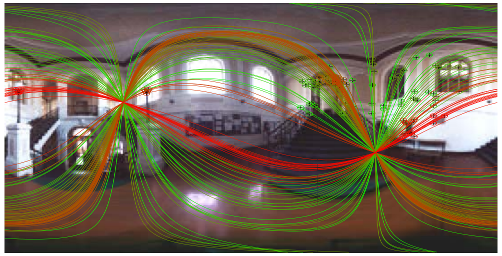


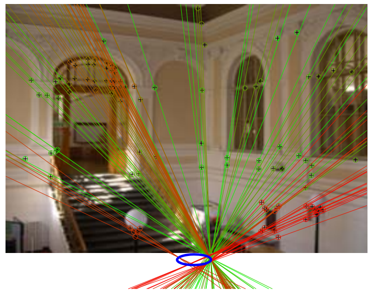
Perspective image



Courtesy of Marc Menem

Circular panorama & Perspective image

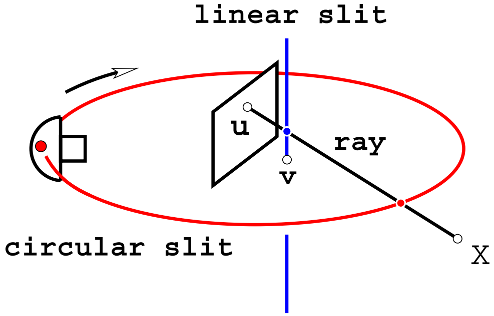




Reconstruction

Courtesy of Marc Menem

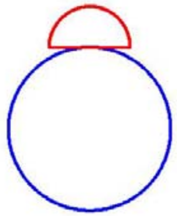
Application: Image Based Rendering with Non-central cameras



1. Central omnidirectional images aquired along a circular trajectory
2. At every viewpoint **v** inside the circle, a non-central image synthesized from acquired rays
3. by volume slicing . . . easy & fast
4. No 3D reconstruction needed, only pixel manipulation

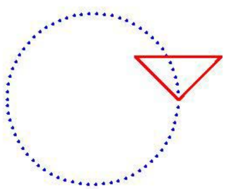
Application: IBR with Non-central cameras

Original sequence acquired by a central omni-camera along a circle



Application: IBR with Non-central cameras

Synthesized sequence as if taken by a (non-central) camera inside the circle



Application: Visualization with X-Slits

Synthesized sequence as if taken by a (non-central) camera inside the circle

Conclusions

1. Non-central cameras explain mosaics, panoramas, image volumes, . . .
2. Models of Non-central cameras developed (pramatrization on caustics, reflectors, Plücker coordinates)
3. Stereo geometry understood for X-Slits cameras and circular panoramas . . . not known for many others
4. Applications in Reconstruction, Image Based Rendering, . . .

References: Epipolar geometry of central omnidirectional cameras (back)

[Svoboda & Pajdla & Hlavac ECCV 1998] T. Svoboda, T. Pajdla, and V. Hlaváč. Epipolar geometry for panoramic cameras. ECCV 1998, vol. 1406 of Springer LNCS, pp. 218–232, June 1998.

[Svoboda & Pajdla IJCV 2002] T. Svoboda and T. Pajdla. Epipolar Geometry for Central Catadioptric Cameras. International Journal of Computer Vision, 49(1):23-37, Kluwer 2002. http://cmp.felk.cvut.cz/ pajdla/Pajdla-Omni-Vision-ICCV-2003/

[Micusik & Pajdla CVPR 2002] B. Micusik and T. Pajdla. Estimation of Omnidirectional Camera Model from Epipolar Geometry. CVPR 2003. Vol. 1, pp. 485–490. IEEE Press 2003. ftp://cmp.felk.cvut.cz/pub/cmp/articles/micusik/Micusik-CVPR2003.pdf

[Geyer & Daniilidis ECCV 2000] C. Geyer and K. Daniilidis. A unifying theory for central panoramic systems. ECCV 2000, pp. 445–461, LNCS 1843, Springer 2000.

[Micusik & Pajdla TR-18 2002] B. Micusik and T. Pajdla. Para-catadioptric camera auto-calibration from epipolar geometry. Research Report CTU-CMP-2003-18, CMP K13133 FEE Czech Technical University in Prague, 2003. http://cmp.felk.cvut.cz/ pajdla/Pajdla-Omni-Vision-ICCV-2003/

References: Omnidirectional camera calibration and epipolar geometry estimation (back)

[Bai et al 2000] Z. Bai, J. Demmel, J. Dongarra, A. Ruhe, and H. van der Vorst, editors. Templates for the Solution of Algebraic Eigenvalue Problems: A Practical Guide. SIAM 2000.

[Kang CVPR 2000] S. B. Kang. Catadioptric self-calibration. CVPR 2000, pp. I: 201–207, IEEE 2000.

[Fitzgibbon CVPR 2001] A. Fitzgibbon. Simultaneous linear estimation of multiple view geometry and lens distortion. CVPR 2001, pp. I: 125–132, IEEE 2001.

[Oliensis PAMI 2002] J. Oliensis. Exact two–image structure from motion. PAMI, 24(12):1618–1633, 2002.

[Micusik & Pajdla CVPR 2003] B. Mičušík, T. Pajdla. Estimation of omnidirectional camera model from epipolar geometry. CVPR 2003, volume 1, pp. 485–490, IEEE 2003.

[Micusik & Pajdla SCIA 2003] B. Mičušík, T. Pajdla. Omnidirectional camera model and epipolar geometry estimation by RANSAC with bucketing. SCIA 2003, vol. 1, pp. 83–90, LNCS. Springer-Verlag 2003. ftp://cmp.felk.cvut.cz/pub/cmp/articles/micusik/Micusik-SCIA2003.pdf

References: Wide-Baseline-Stereo (back)

[Matas et al BMVC 2002] J. Matas, O. Chum, M. Urban, and T. Pajdla. Robust wide baseline stereo from maximally stable extremal regions. BMVC, volume 1, pp. 384–393, London, UK, BMVA 2002.

References: Multiview reconstruction (back)

[Martinec & Pajdla ECCV 2002] D. Martinec and T. Pajdla. Structure from many perspective images with occlusions. ECCV 2002, II, pp. 355–369, Springer-Verlag. 2002. ftp://cmp.felk.cvut.cz/pub/cmp/articles/martinec/Martinec-ECCV2002.pdf

[Micusik & Martinec & Pajdla TR-20 2003] 3D Metric Reconstruction from Uncalibrated Omnidirectional Images. Research Report CTU–CMP–2003–20, CMP K13333 FEE Czech Technical University, Prague, Czech Republic, 2003. http://cmp.felk.cvut.cz/ pajdla/Pajdla-Omni-Vision-ICCV-2003/

References: Non-central cameras — scanner (back)

[Richard Schubert] R. Schubert. Using a flatbed scanner as a stereoscopic near-field camera. IEEE Computer Graphics and Applications, pp. 38-45, issue: March/April 2000.

References: Non-central cameras (back)

[Pless CVPR 2003] R. Pless. Using Many Cameras as One. CVPR 2003. pp. II:587–593, IEEE, June 2003.

[Neumann et al CVPR 2003] J. Neumann and C. Fermüller and Y. Aloimonos. Polydioptric Camera Design and 3D Motion Estimation. CVPR 2003. pp. II:292–3001, IEEE, June 2003.

[Feldman & Pajdla & Weinshall ICCV 2003] D. Feldman and T. Pajdla and D. Weinshall. On the Epipolar Geometry of the Crossed-Slits Projection. ICCV 2003, Nice, October 2003.

[Zomet & Feldman & Peleg & Weinshall PAMI 2003] A. Zomet and D. Feldman and S. Peleg and D. Weinshall. Mosaicing New Views: The Crossed-Slits Projection. IEEE PAMI, 25(6):741-754, June 2003.

[Pajdla CVWW 2001] T. Pajdla. Epipolar geometry of some non-classical cameras. In B Likar, editor, Proceedings of Computer Vision Winter Workshop, pp. 223–233, Ljubljana, Slovenia, Slovenian Pattern Recorgnition Society. February 2001. ftp://cmp.felk.cvut.cz/pub/cmp/articles/pajdla/Pajdla-CVWW2001.ps.gz

[Seitz ICCV 2001] S. Seitz. The space of all stereo images. ICCV 2001, pp. 26–33, July 2001.

[Pajdla IJCV 2001] T. Pajdla. Stereo with oblique cameras. IJCV, 47(1-3):161–170, May 2002. http://cmp.felk.cvut.cz/ pajdla/Pajdla-Omni-Vision-ICCV-2003/

References: Ray representation for non-central catadioptric cameras (back)

[Grossberg & Nayar ICCV 2001] M .D. Grossberg and S. K. Nayar. A General Imaging Model and a Method for Finding its Parameters. ICCV 2001. pp. II:108–115, IEEE, 2001.

[Swaminathan et al ICCV 2001] R. Swaminathan & M. D. Grossberg & S. K, Nayar Caustics of Catadioptric Cameras. ICCV 2001. pp. II:2–9, IEEE, 2001.

[Micusik & Pajdla TR-19 2003] B. Micusik and T. Pajdla. Non-central para-catadioptric camera model. Research Report CTU-CMP-2003-19, CMP K13133 FEE Czech Technical University in Prague, 2003. http://cmp.felk.cvut.cz/ pajdla/Pajdla-Omni-Vision-ICCV-2003/