Shape from Shading: Recognize the Mountains through a Global View

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Outline

- Introduction
- Fast marching algorithm
- Local uncertainties
- Exploiting global constraints
- Proposed approach
- Preliminary results
- Conclusion and future work

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Introduction

- A very old problem(dating back to 70's)
- Problem definition
  - Shape recovery from a single image
  - Many other assumptions

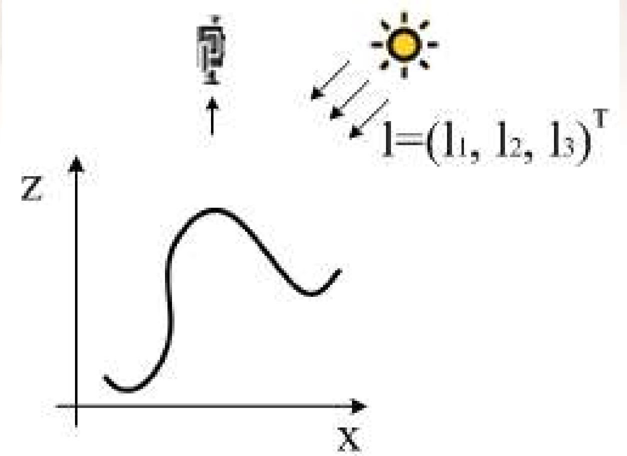
Shading image formulation

- Shading image formulation

$$I(p) = \rho\, \mathbf{l} \cdot \mathbf{n}(p)$$

$$I(p) = \frac{\rho(l_1 z_x + l_2 z_y + l_3)}{\sqrt{z_x^2 + z_y^2 + 1}}$$

- $I(p)$ intensity
- ρ albedo
- $\mathbf{l}$ light source direction
- $\mathbf{n}(p)$ surface normals



Different assumptions

- Classical assumptions
  - Lambertian
  - Point light source at infinity, known
  - Orthogonal view
  - Smooth surface
  - No shadows

- Recent concerns
  - Perspective view
  - $1/r^2$ Effect
  - With shadows

- Real image conditions(difficulties!)
  - Multiple light sources, diffuse
  - Light directions unknown
  - Albedo unknown
  - Shadows
  - Occluding contours

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Previous methods

- Minimization
  - A whole family of methods...
- Propagation
  - Characteristic strip
  - Fast marching
  - Viscosity solutions for PDE
- Others
  - Spectral graph
  - Belief propagation...

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Fast marching algorithm

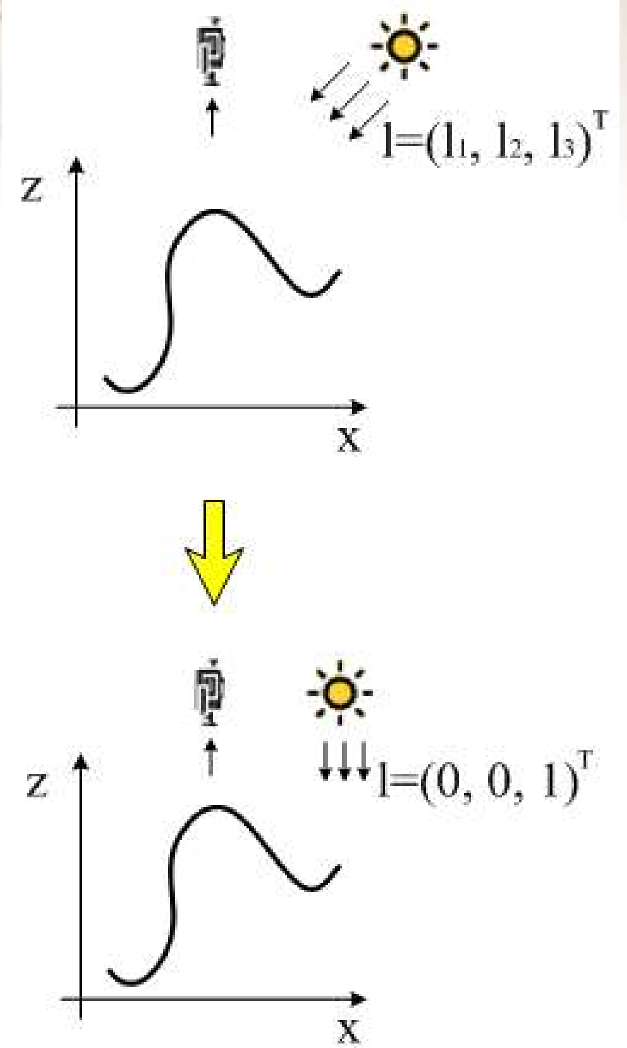
- Shading image formulation

$$I(p) = \rho\, \mathbf{l} \cdot \mathbf{n}(p)$$

$$I(p) = \frac{\rho(l_1 z_x + l_2 z_y + l_3)}{\sqrt{z_x^2 + z_y^2 + 1}}$$

- If $\mathbf{l} = (0,0,1)^T$, this reduces to

$$||\nabla z|| = \sqrt{z_x^2 + z_y^2} = \sqrt{\frac{1}{I^2} - 1}$$



Fast marching algorithm

- How to solve this Partial Derivative Equation (PDE)?

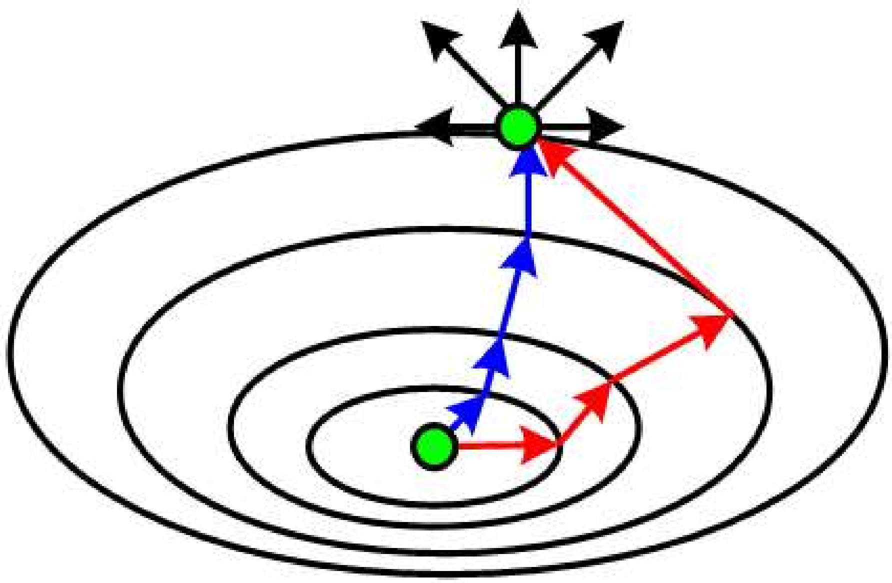
$$||\nabla z|| = \sqrt{z_x^2 + z_y^2} = \sqrt{\frac{1}{I^2} - 1}$$

- Propagate from a singular point
- This is equivalent to computing the shortest path from the singular point, with weight $\sqrt{\frac{1}{I^2} - 1}$ on every node.

The shortest path

$$||\nabla z|| = \sqrt{z_x^2 + z_y^2} = \sqrt{\frac{1}{I^2} - 1}$$

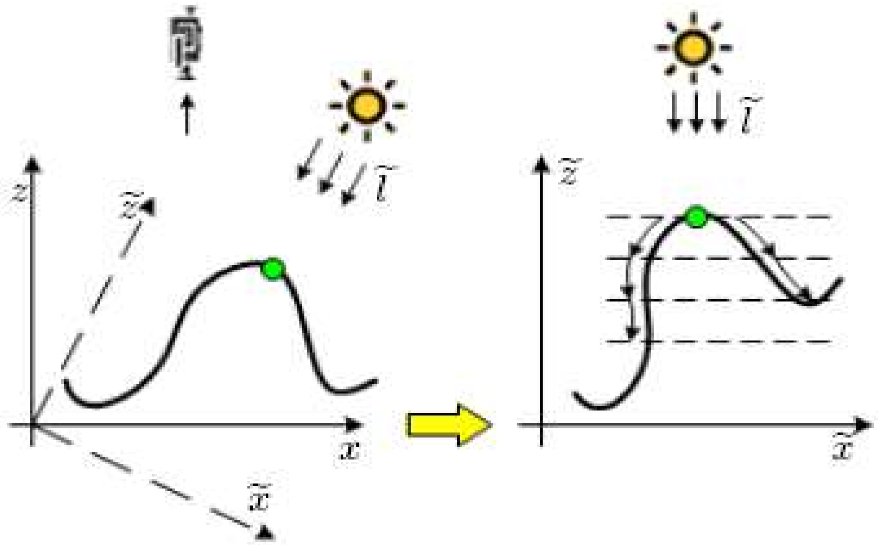
$$z(r) - z(r + \Delta s) \leq \Delta s ||\nabla z(r)||$$



Fast marching algorithm

- What if the light source is not vertial?
  - Assume $l_2 = 0$
  - Propagate in the new coordinate system

$$\tilde{l} = (0,0,1)^T, \tilde{p} = (\tilde{x}, \tilde{y}) = (-l_3\tilde{x} + l_1\tilde{z},\ y), \tilde{z} = l_1 x + l_3 z$$



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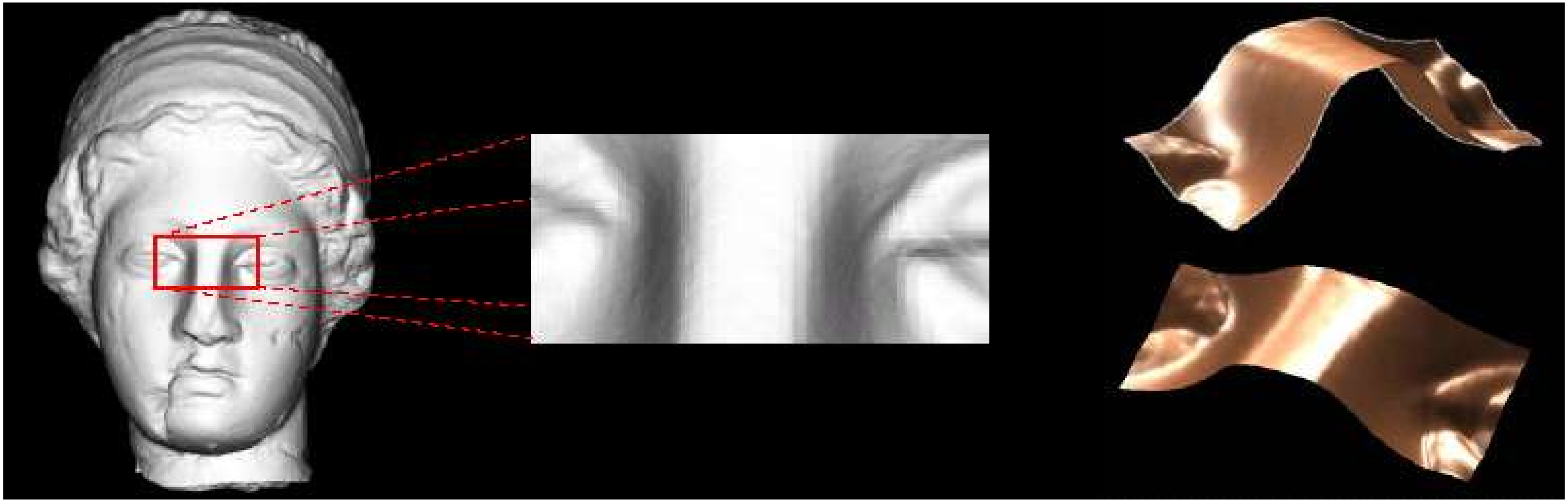
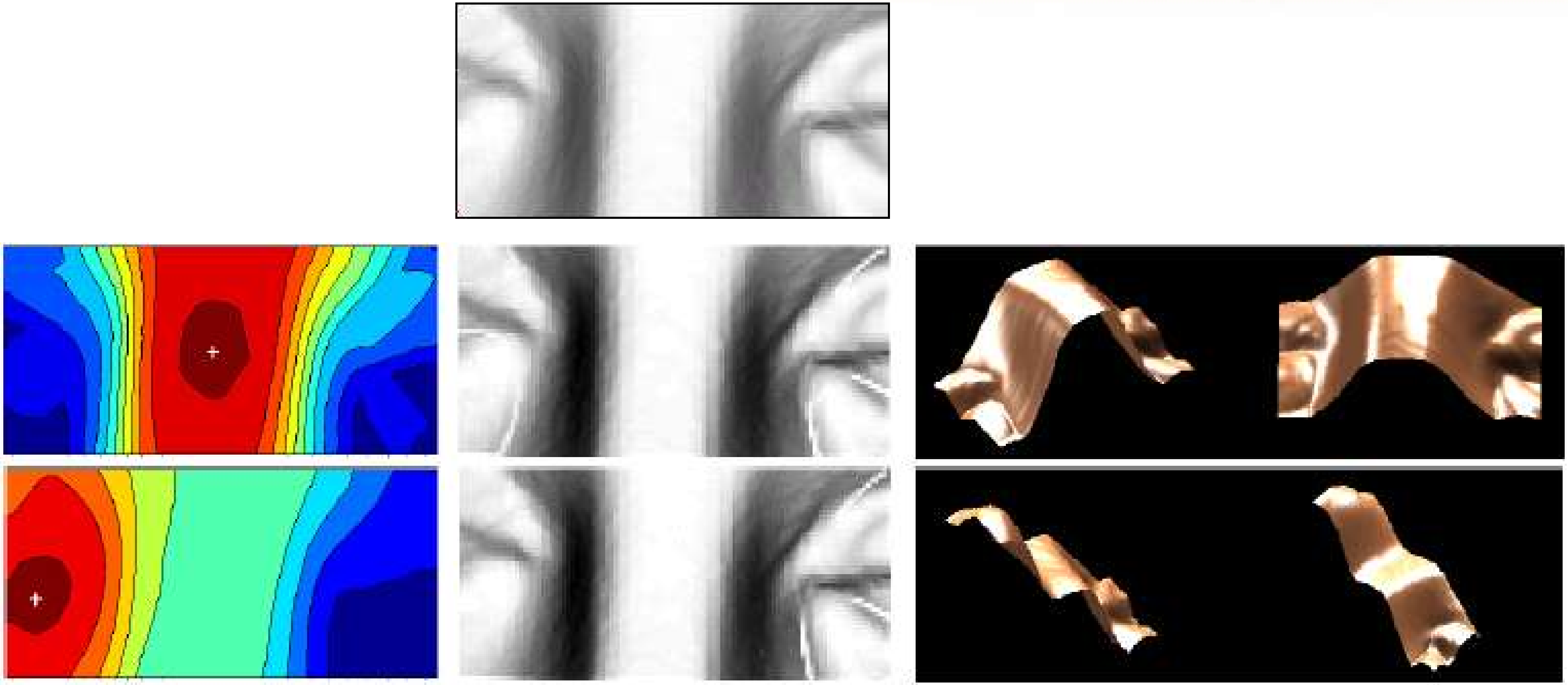
Local uncertainties

- Fast marching is good, but not solving everything
- Venus' nose



Local uncertainties

- Different results...

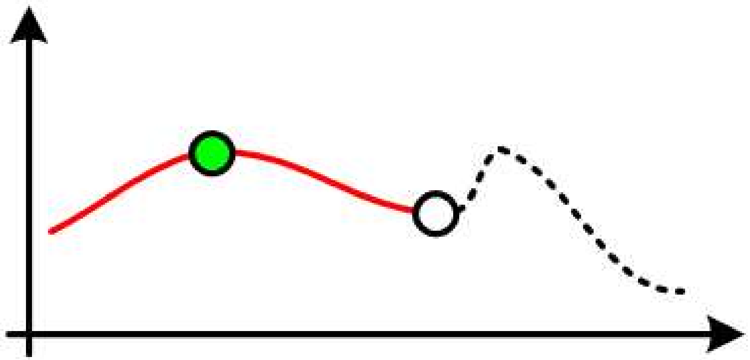


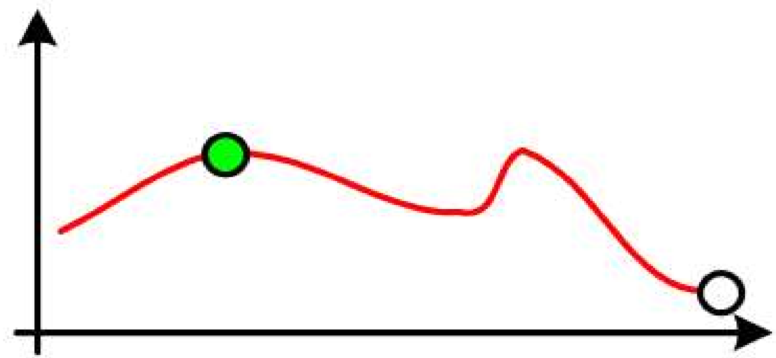
Local uncertainties

- What remains unknown after shortest path?
  - How far you can travel?
  - Are you going up or down?
  - Convex or concave?
- Common problems for propagation methods, not just for fast marching
- Let's see some simple cases...

Local uncertainties

- How far you can travel?

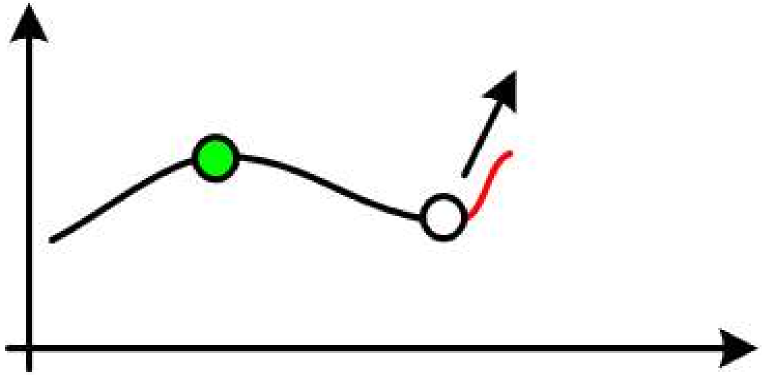


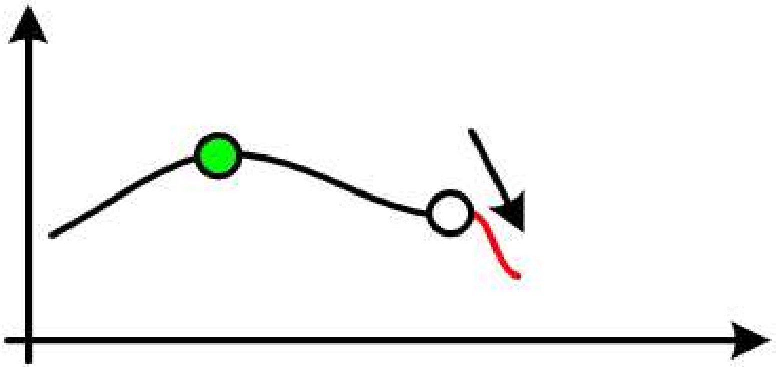


- Left or right?

Local uncertainties

- Are you going up or down?

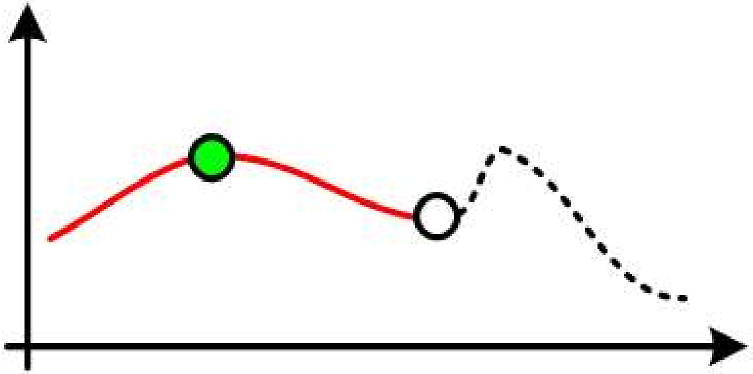


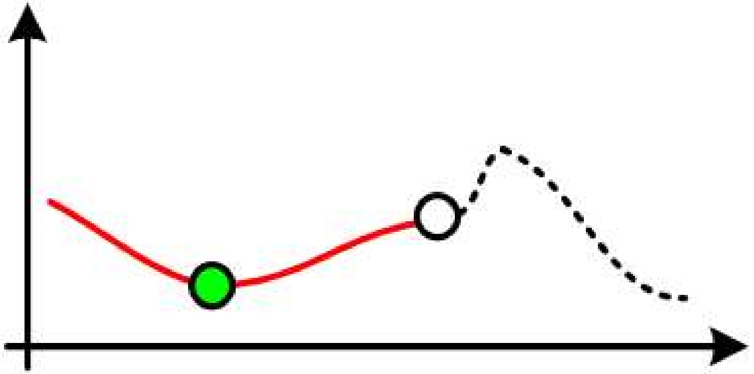


- Left or right?

Local uncertainties

- Convex or concave?





- Left or right?

Local uncertainties

- What remains unknown after shortest path?
  - How far you can travel?
  - Are you going up or down?
  - Convex or concave?
- Unsolvable locally!

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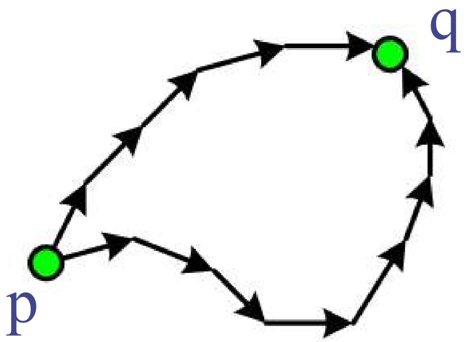
Exploiting global constraints

- Global integrability constraints
  - Continuous surface, no sudden 'jumps'
  - Local estimation of height differences
  - Integration along a loop must be 0, or different paths should have the same height difference.



- Smoothness constraints
  - Use propagation to generate local patches
  - Boundaries between patches must be smooth

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Proposed approach

- Configuration graph G=(V, E, W)
  - V Singular points $n = |V|$
  - E Edges connecting neighboring vertices $m = |E|$
  - W Height difference estimation by fast marching

$$W = \text{diag}(w_1, w_2, ..., w_m)$$

- Representing configurations
  - d +1/-1 defined on edges

$$d = (d_1, d_2, ..., d_m)^T \text{ with } d_i = \pm 1 (i = 1, 2, ..., m)$$

Configuration graph

- How do d solve the local uncertainties?
  - Are you going up or down?
    - easy, simply +1 for up, -1 for down
  - Convex or concave?
    - peaks: all edges going out +1, convex
    - valleys: all edges going out -1, concave
  - How far you can travel?
    - only start from peaks
    - always go down as far as you can

Constraints on the graph

- A little more definition
  - A Adjacency matrix $A \in \mathbb{R}^{m \times n}$

$$A_{ij} = \begin{cases} +1 & e_i = (v_j, v_k) \text{ for some } k \\ -1 & e_i = (v_k, v_j) \text{ for some } k \\ 0 & \text{otherwise} \end{cases}$$

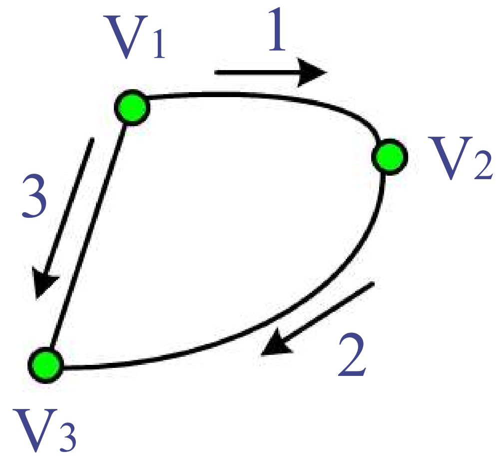
  - H Heights at vertices $h = (h_1, h_2, ..., h_n)^T$

Constraints on the graph

- Height difference constraints

$$Ah = Wd$$

- What are the constraints doing?
  - Check triangles & loops
  - Assume edge monotonous
  - V_2 should not be a peak or a valley
  - Why?
  - Global integrability constraints!
  - Check for every loop



Optimal configuration

- Optimize $||Ah - Wd||_2$

$$d_{opt} = \arg\min_{d,\, h} ||Ah - Wd||_2$$

- For a fixed d $h = A^+ W d$ $\quad A^+ = [A^T A]^{-1} A^T$
- Finally

$$\begin{aligned} d_{opt} &= \arg\min_{d,h} ||Ah - Wd||_2 \\ &= \arg\min_{d} d^T E d \end{aligned}$$

$$E = W^T (AA^+ - I)^T (AA^+ - I) W$$

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Max-cut problem

- Optimizing $d^T E d$ is simply a Max-cut!

$$d^T E d = \sum_{d_i d_j = 1} E_{ij} d_i d_j + \sum_{d_i d_j = -1} E_{ij} d_i d_j = 2 \sum E_{ij} - \sum_{d_i d_j = -1} E_{ij}$$

$$\arg\min_d d^T E d = \arg\max_d \sum_{d_i d_j = -1} E_{ij}$$

- Min-cut, N-cut is polynomial 
- Max-cut is NP-hard 
- But the graph is small...

Numerical approach

- Semi-Definite Programming(SDP)

$$\begin{aligned} \text{minimize} \quad & tr(CX) \\ \text{subject to} \quad & tr(A_i X) = b_i, \quad i = 1, 2, ..., p \\ & X \in S_+^n \end{aligned}$$

- Our problem

$$X = dd^T$$

$$\begin{aligned} \text{minimize} \quad & d^T E d = tr(EX) \\ \text{subject to} \quad & X_{ii} = tr(A_i X) = 1, \quad i = 1, 2, ..., m \\ & X \in S_+^m, A_i = e_i e_i^T \end{aligned}$$

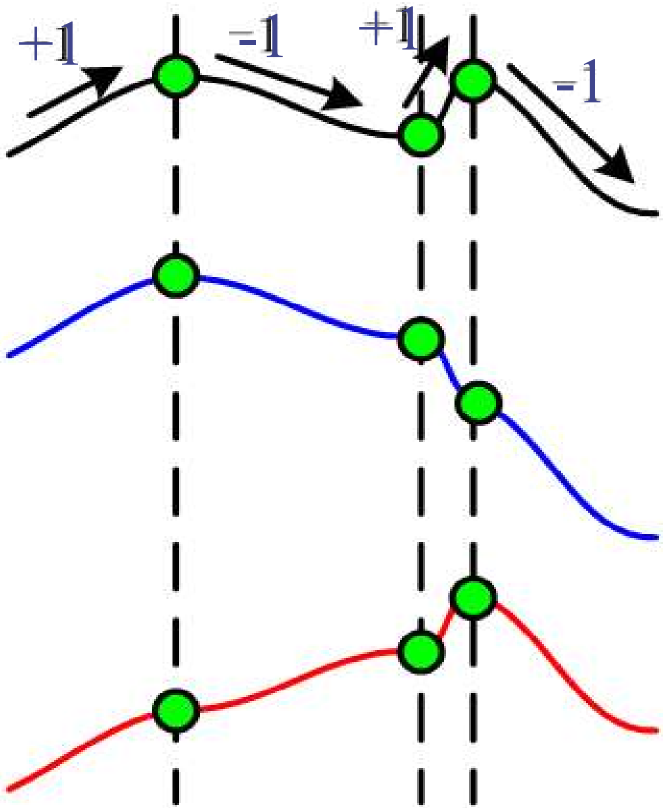
Shape recovery

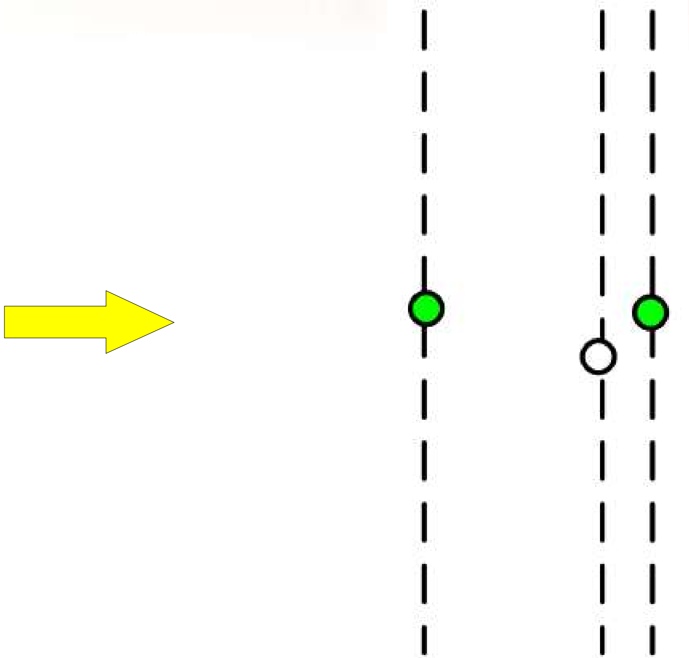
- We know d_{opt} then $h = A^{+} W d_{opt}$
- Also know which vertices are peaks $\mathcal{P}$
- Build patches around peaks
  - Fast marching
- Stitch the patches together

$$z(q) = \max_{p \in \mathcal{P}} \{ z(p) - D(p,q) \}$$

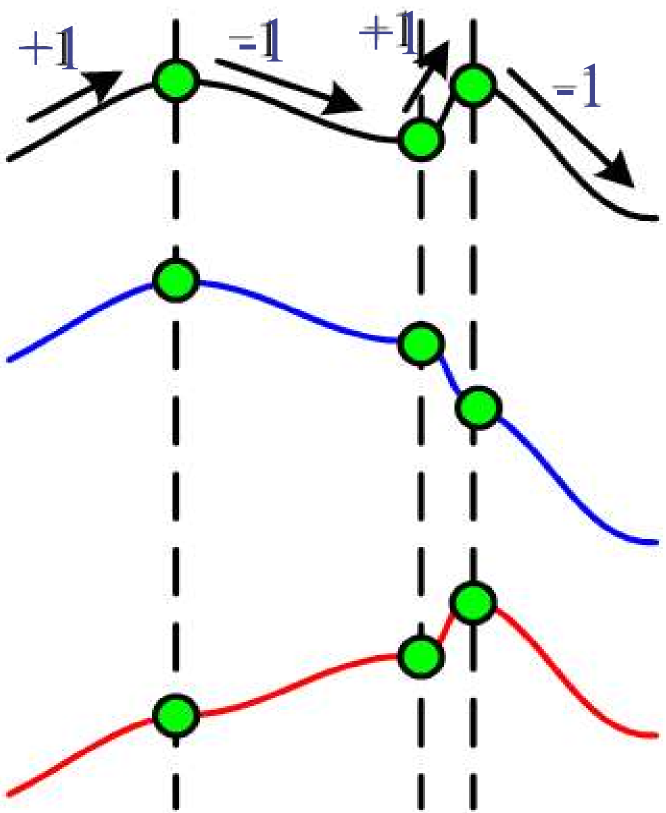
- Why does this work?

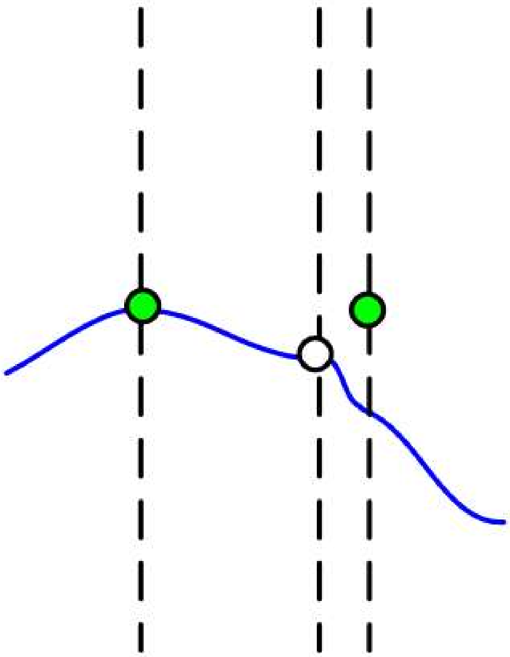
Shape recovery





Shape recovery





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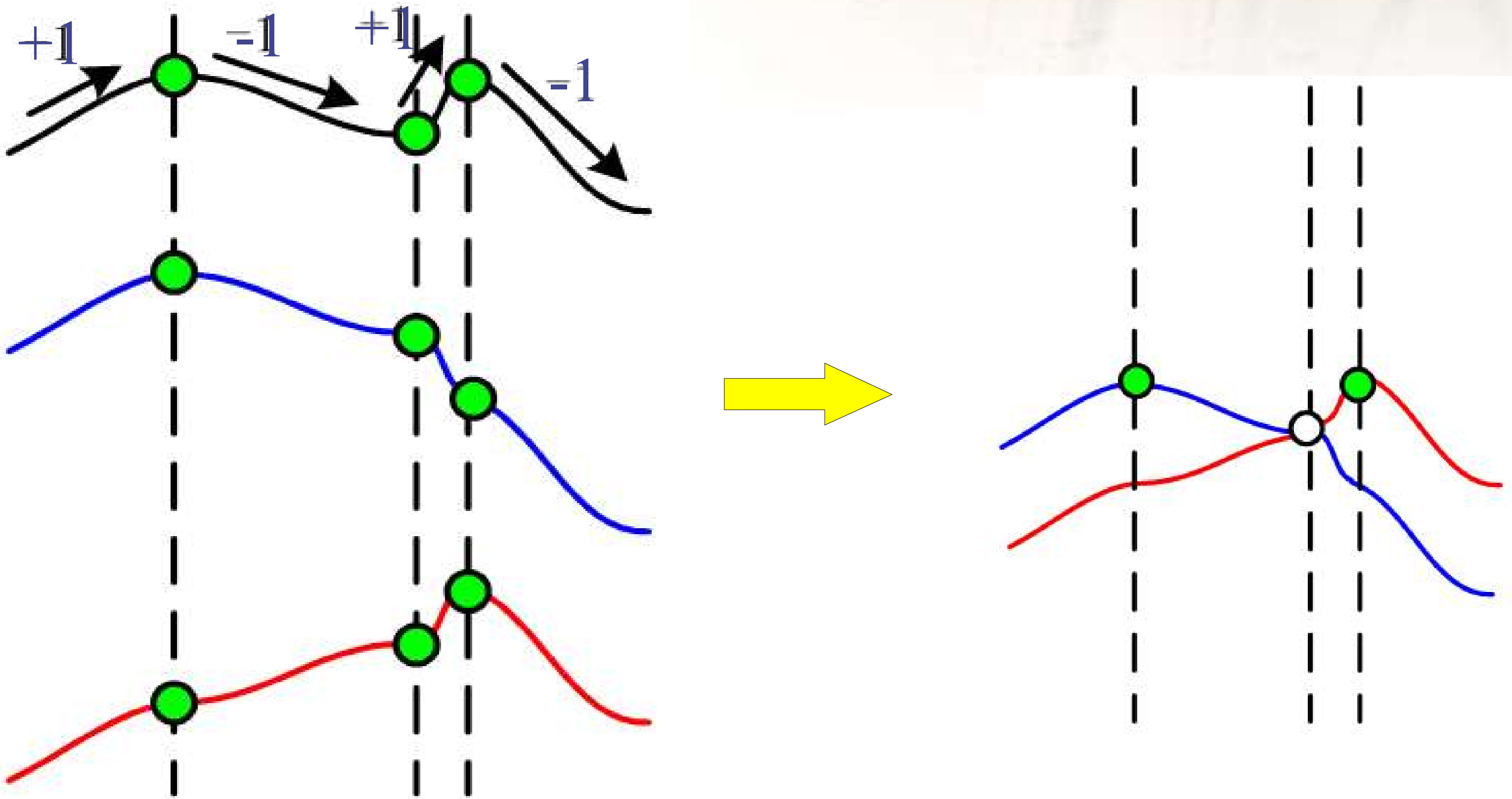

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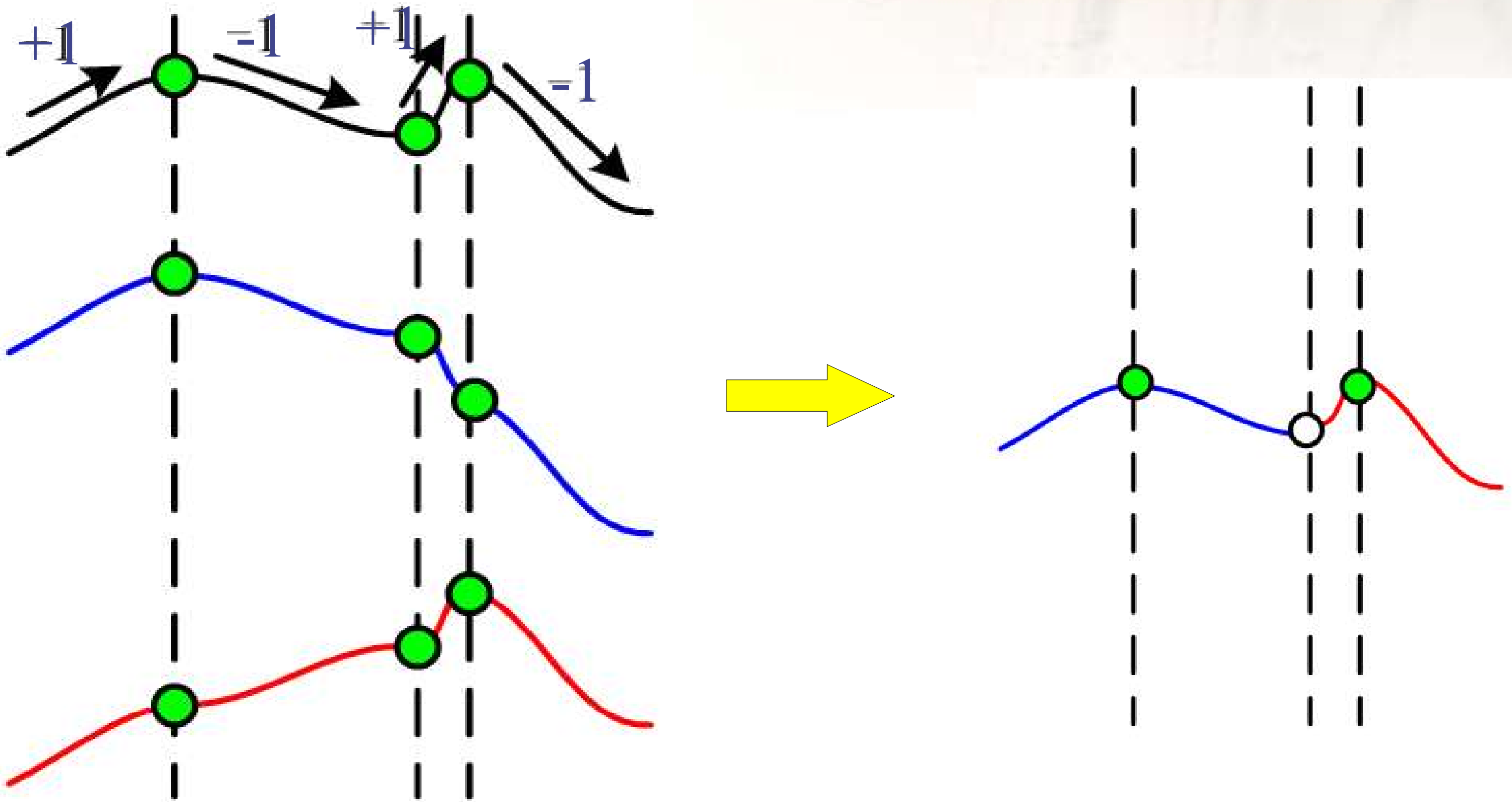

Algorithm overview

- Singular point detection
- Fast marching
- Graph formulation
  - Delaunay triangulation
  - Remove invalid edges
- Optimize $d^T E d$ by SDP
- Postprocessing
  - Identify peaks
- Shape recovery
  - Combine patches

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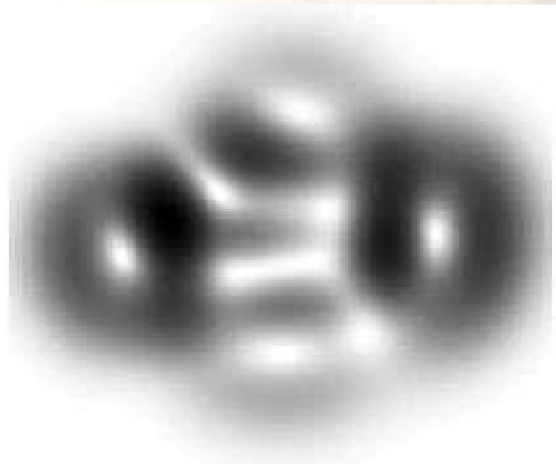
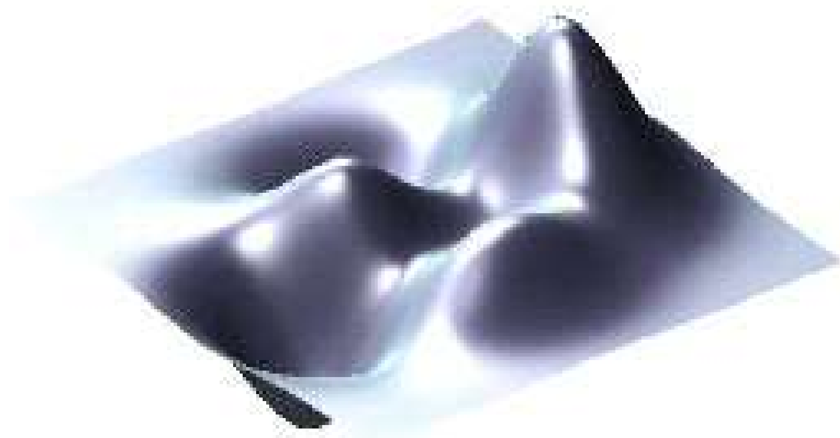
Preliminary results

Matlab PEAKS



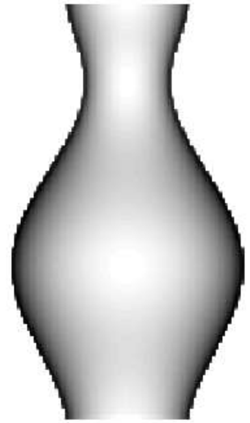
Ground truth



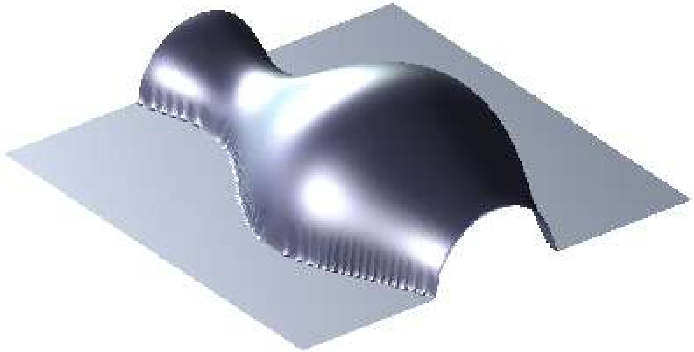
Reconstruction

Preliminary results

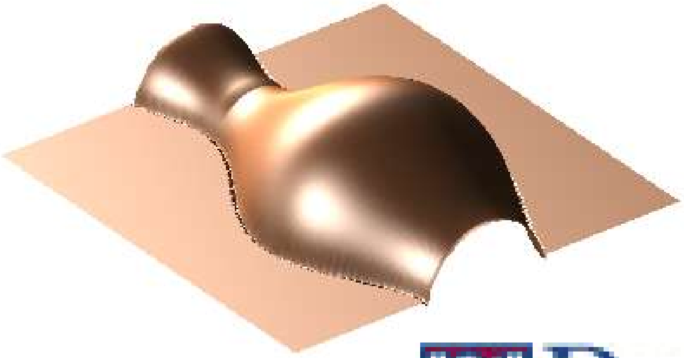
Vase



Ground truth



Reconstruction



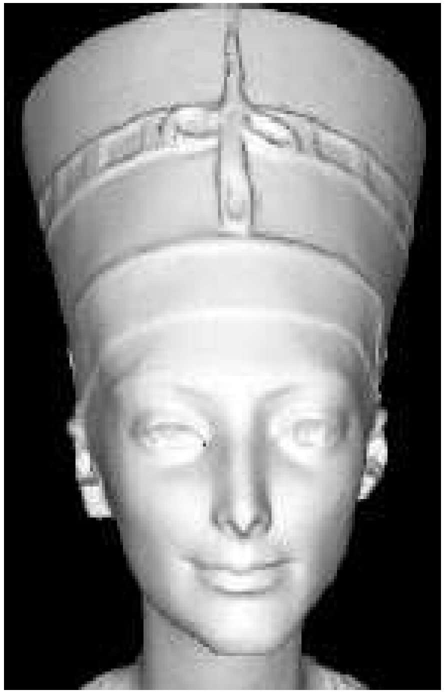
Preliminary results

Venus

Reconstructions

Preliminary results

Ancient woman



Reconstructions

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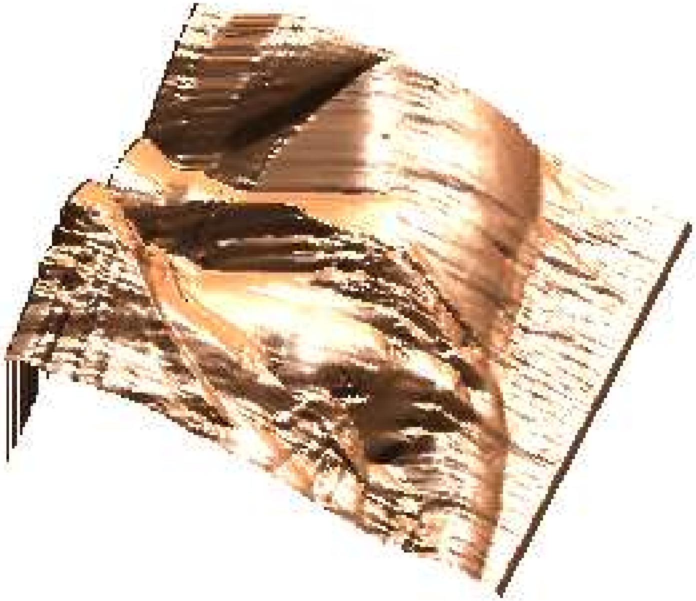
Preliminary results

Relief of Athena

Reconstruction



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Conclusion

- Global constraints are important and powerful
- Pros & Cons of our approach
  - \+ Address ambiguities directly
  - \+ Make decisions on structures, not pixels
  - \+ Also solve the self-shadow problem
  - \+ Simple and fast
  - \- Smoothness not in the framework
  - \- Mixing little peaks with global, big peaks
  - \- Relying on singular points

Future work

- Work on real images
- Consider multiple light sources
- Combine with shadows and occluding contours
- Combine with object models
- ...

Shape from Shading

Comments...