Applications of Manifolds and Research Challenges

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Outline

• Concepts
• Illumination
• Appearance
• Simulation
• Faces
• Manifold Learning
• Wrap-up
Manifolds & Parametrization

- Two Points of View
  - Functions on surfaces
  - Functions defining surfaces
Desirable Properties

• Minimal Distortion
  - Angle
  - Area

• Smoothness
  - Differentiability
  - Continuity
Graphical Objects

- **Shape** $U$
  - Topology (domain)
    - Abstract Manifold
  - Geometry (function)
    - Embedding
- **Attributes** $f$
  - Functions (co-domain)

$O = (U, f)$
G.O. Manifold Setting

- **Canonical Surfaces**
  - Fixed Shape (defined *apriori*)
  - Variable Functions (complex)
    - ex: *Sphere*
- **Arbitrary Surfaces**
  - Complex Shape
  - Computation on Surfaces (attributes)
  - Building / Transforming (shape)
    - ex: *Triangle Meshes*
Applications

• Illumination
  - Canonical Manifold + Functions
• Appearance and Simulation
  - Pseudo-Manifold + Attributes
• Faces
  - Manifold + Geometric Deformation
• Surface Reconstruction
  - Pseudo-Manifold / Topology Estimation
The Sphere

- Construction [Grimm 2002]

Chart (squares), edge, and

Top cap

Bottom cap

A single chart on the sphere

Bdry path

Defining chart connectivity
Illumination

• Functions on the Sphere
  - Light Fields / BRDFs
• Applications
  - Capture / Synthesis
Illumination Maps

• Environment Maps
  - Area Sampling

• Light Maps
  - Stratification
Material Properties

- Spatially Varying BRDFs
Spherical Panoramas

- Panoramic Cameras
  - Processing

- Multi-Camera Assembly
  - Stitching / Blending
Omnidirectional Images

• Processing Large Spherical Imagery
  - Example: Sharpening

Obs: Metric Aware Operators
Polygonal Surfaces

- Building from Images

- Projective Map
Surface Properties

- Texture Atlas
  - Albedo
  - Normal Field
  - etc...
Painting

- Editing Ops
  - Color
  - Normals
Solving Equations on Manifolds

- Global Structure
  - Surface Points
  - Local Neighborhoods
Simulation

- Metric Aware Operators

\[ F_A(\theta, \varphi) \xrightarrow{\Phi(\theta, \varphi)} \tilde{F}_A = F_A \circ \Phi^{-1} \]

\[ F_B(\theta, \varphi) \xrightarrow{\Phi(\theta, \varphi)} \tilde{F}_B = F_B \circ \Phi^{-1} \]

\[ F_C(\theta, \varphi) \xrightarrow{\Phi(\theta, \varphi)} \tilde{F}_C = F_C \circ \Phi^{-1} \]
Fluids

- Vector Fields on Surfaces
Biological Processes

• Reaction Diffusion

• Examples:
Texture Synthesis

- Stationary / Quasi Stationary
Faces

- Geometry + Appearance

[ G. Borshukov et al  SIGGRAPH 2003 ]
Facial Expressions

- Deformations
Manifold Learning

- Estimate from Data Samples
  - Topology
  - Geometry
Surfaces

- Point Sets
N-Dimensional Case

- ex: Facial Expressions
Multiresolution

- Manifold T-Spline (Gu, et al)
Adaptation

• Hierarchical Surface Reconstruction
Challenges

• Representation
  - Simple / Emcompassing

• Operators
  - Efficient / Accurate

• Multi-Resolution
  - Hierarchical Atlas / Dynamic Setting

• API
  - Intuitive / General
Questions ?